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CHAPTER ONE

INTRODUCTION

The structure of NetworkX can be seen by the organization of its source code. The package provides classes for graph objects, generators to create standard graphs, IO routines for reading in existing datasets, algorithms to analyze the resulting networks and some basic drawing tools.

Most of the NetworkX API is provided by functions which take a graph object as an argument. Methods of the graph object are limited to basic manipulation and reporting. This provides modularity of code and documentation. It also makes it easier for newcomers to learn about the package in stages. The source code for each module is meant to be easy to read and reading this Python code is actually a good way to learn more about network algorithms, but we have put a lot of effort into making the documentation sufficient and friendly. If you have suggestions or questions please contact us by joining the NetworkX Google group.

Classes are named using CamelCase (capital letters at the start of each word). functions, methods and variable names are lower_case_underscore (lowercase with an underscore representing a space between words).

1.1 NetworkX Basics

After starting Python, import the networkx module with (the recommended way)

```python
>>> import networkx as nx
```

To save repetition, in the documentation we assume that NetworkX has been imported this way.

If importing networkx fails, it means that Python cannot find the installed module. Check your installation and your PYTHONPATH.

The following basic graph types are provided as Python classes:

- **Graph** This class implements an undirected graph. It ignores multiple edges between two nodes. It does allow self-loop edges between a node and itself.
- **DiGraph** Directed graphs, that is, graphs with directed edges. Provides operations common to directed graphs, (a subclass of Graph).
- **MultiGraph** A flexible graph class that allows multiple undirected edges between pairs of nodes. The additional flexibility leads to some degradation in performance, though usually not significant.
- **MultiDiGraph** A directed version of a MultiGraph.

Empty graph-like objects are created with

```python
>>> G = nx.Graph()
>>> G = nx.DiGraph()
>>> G = nx.MultiGraph()
>>> G = nx.MultiDiGraph()
```
All graph classes allow any hashable object as a node. Hashable objects include strings, tuples, integers, and more.

Arbitrary edge attributes such as weights and labels can be associated with an edge.

The graph internal data structures are based on an adjacency list representation and implemented using Python dictionary datastructures. The graph adjacency structure is implemented as a Python dictionary of dictionaries; the outer dictionary is keyed by nodes to values that are themselves dictionaries keyed by neighboring node to the edge attributes associated with that edge. This “dict-of-dicts” structure allows fast addition, deletion, and lookup of nodes and neighbors in large graphs. The underlying datastructure is accessed directly by methods (the programming interface “API”) in the class definitions. All functions, on the other hand, manipulate graph-like objects solely via those API methods and not by acting directly on the datastructure. This design allows for possible replacement of the ‘dicts-of-dicts’-based datastructure with an alternative datastructure that implements the same methods.

1.2 Graphs

The first choice to be made when using NetworkX is what type of graph object to use. A graph (network) is a collection of nodes together with a collection of edges that are pairs of nodes. Attributes are often associated with nodes and/or edges. NetworkX graph objects come in different flavors depending on two main properties of the network:

- Directed: Are the edges directed? Does the order of the edge pairs \((u, v)\) matter? A directed graph is specified by the “Di” prefix in the class name, e.g. DiGraph(). We make this distinction because many classical graph properties are defined differently for directed graphs.

- Multi-edges: Are multiple edges allowed between each pair of nodes? As you might imagine, multiple edges requires a different data structure, though clever users could design edge data attributes to support this functionality. We provide a standard data structure and interface for this type of graph using the prefix “Multi”, e.g., MultiGraph().

The basic graph classes are named: Graph, DiGraph, MultiGraph, and MultiDiGraph

1.2.1 Nodes and Edges

The next choice you have to make when specifying a graph is what kinds of nodes and edges to use.

If the topology of the network is all you care about then using integers or strings as the nodes makes sense and you need not worry about edge data. If you have a data structure already in place to describe nodes you can simply use that structure as your nodes provided it is hashable. If it is not hashable you can use a unique identifier to represent the node and assign the data as a node attribute.

Edges often have data associated with them. Arbitrary data can be associated with edges as an edge attribute. If the data is numeric and the intent is to represent a weighted graph then use the ‘weight’ keyword for the attribute. Some of the graph algorithms, such as Dijkstra’s shortest path algorithm, use this attribute name by default to get the weight for each edge.

Attributes can be assigned to an edge by using keyword/value pairs when adding edges. You can use any keyword to name your attribute and can then query the edge data using that attribute keyword.

Once you’ve decided how to encode the nodes and edges, and whether you have an undirected/directed graph with or without multiedges you are ready to build your network.

1.3 Graph Creation

NetworkX graph objects can be created in one of three ways:

- Graph generators—standard algorithms to create network topologies.
• Importing data from pre-existing (usually file) sources.

• Adding edges and nodes explicitly.

Explicit addition and removal of nodes/edges is the easiest to describe. Each graph object supplies methods to manipulate the graph. For example,

```python
>>> import networkx as nx
>>> G = nx.Graph()
>>> G.add_edge(1, 2)  # default edge data=1
>>> G.add_edge(2, 3, weight=0.9)  # specify edge data
```

Edge attributes can be anything:

```python
>>> import math
>>> G.add_edge('y', 'x', function=math.cos)
>>> G.add_node(math.cos)  # any hashable can be a node
```

You can add many edges at one time:

```python
>>> elist = [(1, 2), (2, 3), (1, 4), (4, 2)]
>>> G.add_edges_from(elist)
```

```python
>>> elist = [('a', 'b', 5.0), ('b', 'c', 3.0), ('a', 'c', 1.0), ('c', 'd', 7.3)]
>>> G.add_weighted_edges_from(elist)
```

See the Tutorial for more examples.

Some basic graph operations such as union and intersection are described in the operators module documentation.

Graph generators such as binomial_graph() and erdos_renyi_graph() are provided in the graph generators subpackage.

For importing network data from formats such as GML, GraphML, edge list text files see the reading and writing graphs subpackage.

### 1.4 Graph Reporting

Class views provide basic reporting of nodes, neighbors, edges and degree. These views provide iteration over the properties as well as membership queries and data attribute lookup. The views refer to the graph data structure so changes to the graph are reflected in the views. This is analogous to dictionary views in Python 3. If you want to change the graph while iterating you will need to use e.g. for e in list(G.edges):. The views provide set-like operations, e.g. union and intersection, as well as dict-like lookup and iteration of the data attributes using G.edges[u, v]['color'] and for e, datadict in G.edges.items():. Methods G.edges.items() and G.edges.values() are familiar from python dicts. In addition G.edges.data() provides specific attribute iteration e.g. for e, e_color in G.edges.data('color'):

The basic graph relationship of an edge can be obtained in two ways. One can look for neighbors of a node or one can look for edges. We jokingly refer to people who focus on nodes/neighbors as node-centric and people who focus on edges as edge-centric. The designers of NetworkX tend to be node-centric and view edges as a relationship between nodes. You can see this by our choice of lookup notation like G[u] providing neighbors (adjacency) while edge lookup is G.edges[u, v]. Most data structures for sparse graphs are essentially adjacency lists and so fit this perspective. In the end, of course, it doesn’t really matter which way you examine the graph. G.edges removes duplicate representations of undirected edges while neighbor reporting across all nodes will naturally report both directions.

Any properties that are more complicated than edges, neighbors and degree are provided by functions. For example nx.triangles(G, n) gives the number of triangles which include node n as a vertex. These functions are
grouped in the code and documentation under the term *algorithms*.

## 1.5 Algorithms

A number of graph algorithms are provided with NetworkX. These include shortest path, and breadth first search (see *traversal*), clustering and isomorphism algorithms and others. There are many that we have not developed yet too. If you implement a graph algorithm that might be useful for others please let us know through the NetworkX Google group or the Github Developer Zone.

As an example here is code to use Dijkstra’s algorithm to find the shortest weighted path:

```python
>>> import matplotlib.pyplot as plt
```
1.7 Data Structure

NetworkX uses a “dictionary of dictionaries of dictionaries” as the basic network data structure. This allows fast lookup with reasonable storage for large sparse networks. The keys are nodes so \( G[u] \) returns an adjacency dictionary keyed by neighbor to the edge attribute dictionary. A view of the adjacency data structure is provided by the dict-like object \( G.adj \) as e.g. for node, \( \text{nbrsdict in G.adj.items()} \):. The expression \( G[u][v] \) returns the edge attribute dictionary itself. A dictionary of lists would have also been possible, but not allow fast edge detection nor convenient storage of edge data.

Advantages of dict-of-dicts-of-dicts data structure:

- Find edges and remove edges with two dictionary look-ups.
- Prefer to “lists” because of fast lookup with sparse storage.
- Prefer to “sets” since data can be attached to edge.
- \( G[u][v] \) returns the edge attribute dictionary.
- \( n \text{ in } G \) tests if node \( n \) is in graph \( G \).
- \( \text{for } n \text{ in } G: \) iterates through the graph.
- \( \text{for } \text{nbr in } G[n]: \) iterates through neighbors.
As an example, here is a representation of an undirected graph with the edges \((A, B)\) and \((B, C)\).

```python
>>> G = nx.Graph()
>>> G.add_edge('A', 'B')
>>> G.add_edge('B', 'C')
>>> print(G.adj)
{'A': {'B': {}}, 'B': {'A': {}, 'C': {}}, 'C': {'B': {}}}
```

The data structure gets morphed slightly for each base graph class. For DiGraph two dict-of-dicts-of-dicts structures are provided, one for successors \((G.succ)\) and one for predecessors \((G.pred)\). For MultiGraph/MultiDiGraph we use a dict-of-dicts-of-dicts-of-dicts\(^1\) where the third dictionary is keyed by an edge key identifier to the fourth dictionary which contains the edge attributes for that edge between the two nodes.

Graphs provide two interfaces to the edge data attributes: adjacency and edges. So \(G[u][v]['width']\) is the same as \(G.edges[u, v]['width']\).

```python
>>> G = nx.Graph()
>>> G.add_edge(1, 2, color='red', weight=0.84, size=300)
>>> print(G[1][2]['size'])
300
>>> print(G.edges[1, 2]['color'])
red
```

\(^1\)"It's dictionaries all the way down."
NetworkX provides data structures and methods for storing graphs.

All NetworkX graph classes allow (hashable) Python objects as nodes and any Python object can be assigned as an edge attribute.

The choice of graph class depends on the structure of the graph you want to represent.

### 2.1 Which graph class should I use?

<table>
<thead>
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<th>NetworkX Class</th>
<th>Type</th>
<th>Self-loops allowed</th>
<th>Parallel edges allowed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph</td>
<td>undirected</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>DiGraph</td>
<td>directed</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>MultiGraph</td>
<td>undirected</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>MultiDiGraph</td>
<td>directed</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

### 2.2 Basic graph types

#### 2.2.1 Graph—Undirected graphs with self loops

**Overview**

```python
class Graph (incoming_graph_data=None, **attr)
```

A Graph stores nodes and edges with optional data, or attributes.

Graphs hold undirected edges. Self loops are allowed but multiple (parallel) edges are not.

Nodes can be arbitrary (hashable) Python objects with optional key/value attributes. By convention `None` is not used as a node.

Edges are represented as links between nodes with optional key/value attributes.

**Parameters**

- `incoming_graph_data (input graph (optional, default: None))` – Data to initialize graph. If None (default) an empty graph is created. The data can be any format that is supported by the `to_networkx_graph()` function, currently including edge list, dict of dicts, dict of lists, NetworkX graph, NumPy matrix or 2d ndarray, SciPy sparse matrix, or PyGraphviz graph.
• attr (keyword arguments, optional (default= no attributes)) – Attributes to add to graph as key=value pairs.

See also:

DiGraph, MultiGraph, MultiDiGraph, OrderedGraph

Examples

Create an empty graph structure (a “null graph”) with no nodes and no edges.

```python
>>> G = nx.Graph()
```

G can be grown in several ways.

Nodes:

Add one node at a time:

```python
>>> G.add_node(1)
```

Add the nodes from any container (a list, dict, set or even the lines from a file or the nodes from another graph).

```python
>>> G.add_nodes_from([2, 3])
>>> G.add_nodes_from(range(100, 110))
>>> H = nx.path_graph(10)
>>> G.add_nodes_from(H)
```

In addition to strings and integers any hashable Python object (except None) can represent a node, e.g. a customized node object, or even another Graph.

```python
>>> G.add_node(H)
```

Edges:

G can also be grown by adding edges.

Add one edge,

```python
>>> G.add_edge(1, 2)
```

a list of edges,

```python
>>> G.add_edges_from([(1, 2), (1, 3)])
```

or a collection of edges,

```python
>>> G.add_edges_from(H.edges)
```

If some edges connect nodes not yet in the graph, the nodes are added automatically. There are no errors when adding nodes or edges that already exist.

Attributes:

Each graph, node, and edge can hold key/value attribute pairs in an associated attribute dictionary (the keys must be hashable). By default these are empty, but can be added or changed using add_edge, add_node or direct manipulation of the attribute dictionaries named graph, node and edge respectively.
>>> G = nx.Graph(day="Friday")

Add node attributes using add_node(), add_nodes_from() or G.nodes

```python
>>> G.add_node(1, time='5pm')
>>> G.add_nodes_from([3], time='2pm')
>>> G.nodes[1]
{'time': '5pm'}
>>> G.nodes[1]['room'] = 714 # node must exist already to use G.nodes
>>> del G.nodes[1]['room'] # remove attribute
>>> list(G.nodes(data=True))
[(1, {'time': '5pm'}), (3, {'time': '2pm'})]
```

Add edge attributes using add_edge(), add_edges_from(), subscript notation, or G.edges.

```python
>>> G.add_edge(1, 2, weight=4.7)
>>> G.add_edges_from([(3, 4), (4, 5)], color='red')
>>> G.add_edges_from([(1, 2, {'color': 'blue'}), (2, 3, {'weight': 8})])
>>> G[1][2]['weight'] = 4.7
>>> G.edges[1, 2]['weight'] = 4
```

Warning: we protect the graph data structure by making G.edges a read-only dict-like structure. However, you can assign to attributes in e.g. G.edges[1, 2]. Thus, use 2 sets of brackets to add/change data attributes: G.edges[1, 2]['weight'] = 4 (For multigraphs: MG.edges[u, v, key][name] = value).

**Shortcuts:**

Many common graph features allow python syntax to speed reporting.

```python
>>> 1 in G   # check if node in graph
True
>>> [n for n in G if n < 3]  # iterate through nodes
[1, 2]
>>> len(G)   # number of nodes in graph
5
```

Often the best way to traverse all edges of a graph is via the neighbors. The neighbors are reported as an adjacency-dict G.adj or G.adjacency()

```python
>>> for n, nbrsdict in G.adjacency():
...     for nbr, etattr in nbrsdict.items():
...         if 'weight' in etattr:
...             # Do something useful with the edges
...             pass
```

But the edges() method is often more convenient:

```python
>>> for u, v, weight in G.edges.data('weight'):
...     if weight is not None:
...         # Do something useful with the edges
...         pass
```

**Reporting:**

Simple graph information is obtained using object-attributes and methods. Reporting typically provides views instead of containers to reduce memory usage. The views update as the graph is updated similarly to dict-views. The objects nodes, edges and adj provide access to data attributes via lookup
(e.g. `nodes[n]`, `edges[u, v]`, `adj[u][v]`) and iteration (e.g. `nodes.items()`, `nodes.data('color')`, `nodes.data('color', default='blue')`) and similarly for `edges`. Views exist for `nodes`, `edges`, `neighbors()` and `degree`.

For details on these and other miscellaneous methods, see below.

Subclasses (Advanced):

The `Graph` class uses a dict-of-dict-of-dict data structure. The outer dict (node_dict) holds adjacency information keyed by node. The next dict (adjlist_dict) represents the adjacency information and holds edge data keyed by neighbor. The inner dict (edge_attr_dict) represents the edge data and holds edge attribute values keyed by attribute names.

Each of these three dicts can be replaced in a subclass by a user defined dict-like object. In general, the dict-like features should be maintained but extra features can be added. To replace one of the dicts create a new graph class by changing the class(!) variable holding the factory for that dict-like structure. The variable names are `node_dict_factory`, `node_attr_dict_factory`, `adjlist_inner_dict_factory`, `adjlist_outer_dict_factory`, `edge_attr_dict_factory` and `graph_attr_dict_factory`.

- `node_dict_factory` [function, (default: dict)] Factory function to be used to create the dict containing node attributes, keyed by node id. It should require no arguments and return a dict-like object.
- `node_attr_dict_factory`: function, (default: dict) Factory function to be used to create the node attribute dict which holds attribute values keyed by attribute name. It should require no arguments and return a dict-like object.
- `adjlist_outer_dict_factory` [function, (default: dict)] Factory function to be used to create the outer-most dict in the data structure that holds adjacency info keyed by node. It should require no arguments and return a dict-like object.
- `adjlist_inner_dict_factory` [function, (default: dict)] Factory function to be used to create the adjacency list dict which holds edge data keyed by neighbor. It should require no arguments and return a dict-like object.
- `edge_attr_dict_factory` [function, (default: dict)] Factory function to be used to create the edge attribute dict which holds attribute values keyed by attribute name. It should require no arguments and return a dict-like object.
- `graph_attr_dict_factory` [function, (default: dict)] Factory function to be used to create the graph attribute dict which holds attribute values keyed by attribute name. It should require no arguments and return a dict-like object.

Typically, if your extension doesn’t impact the data structure all methods will inherit without issue except: `to_directed/to_undirected`. By default these methods create a DiGraph/Graph class and you probably want them to create your extension of a DiGraph/Graph. To facilitate this we define two class variables that you can set in your subclass.

- `to_directed_class` [callable, (default: DiGraph or MultiDiGraph)] Class to create a new graph structure in the `to_directed` method. If `None`, a NetworkX class (DiGraph or MultiDiGraph) is used.
- `to_undirected_class` [callable, (default: Graph or MultiGraph)] Class to create a new graph structure in the `to_undirected` method. If `None`, a NetworkX class (Graph or MultiGraph) is used.

Examples

Create a low memory graph class that effectively disallows edge attributes by using a single attribute dict for all edges. This reduces the memory used, but you lose edge attributes.

```python
>>> class ThinGraph(nx.Graph):
...     ...     all_edge_dict = {'weight': 1}
```

(continues on next page)
...     def single_edge_dict(self):
...         return self.all_edge_dict
...     edge_attr_dict_factory = single_edge_dict

>>> G = ThinGraph()
>>> G.add_edge(2, 1)
>>> G[2][1]
{'weight': 1}
>>> G.add_edge(2, 2)
>>> G[2][1] is G[2][2]
True

Please see ordered for more examples of creating graph subclasses by overwriting the base class dict with a dictionary-like object.

Methods

Adding and removing nodes and edges

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networkx.Graph.__init__

Graph.__init__((incoming_graph_data=None, **attr))
Initialize a graph with edges, name, or graph attributes.

Parameters

- **incoming_graph_data** (input graph (optional, default: None)) – Data to initialize graph. If None (default) an empty graph is created. The data can be an edge list, or any NetworkX graph object. If the corresponding optional Python packages are installed the data can also be a NumPy matrix or 2d ndarray, a SciPy sparse matrix, or a PyGraphviz graph.
- **attr** (keyword arguments, optional (default= no attributes)) – Attributes to add to graph as key=value pairs.

See also:

convert ()

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Examples

```python
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G = nx.Graph(name='my graph')
>>> e = [(1, 2), (2, 3), (3, 4)] # list of edges
>>> G = nx.Graph(e)

Arbitrary graph attribute pairs (key=value) may be assigned

```python
>>> G = nx.Graph(e, day="Friday")
>>> G.graph
{'day': 'Friday'}
```

```python
networkx.Graph.add_node
```

`Graph.add_node(node_for_adding, **attr)`

Add a single node `node_for_adding` and update node attributes.

**Parameters**

- `node_for_adding` *(node)* – A node can be any hashable Python object except None.
- `attr` *(keyword arguments, optional)* – Set or change node attributes using key=value.

**See also:**

* `add_nodes_from()`

**Examples**

```python
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_node(1)
>>> G.add_node('Hello')
>>> K3 = nx.Graph([(0, 1), (1, 2), (2, 0)])
>>> G.add_node(K3)
>>> G.number_of_nodes()
3

Use keywords set/change node attributes:

```python
>>> G.add_node(1, size=10)
>>> G.add_node(3, weight=0.4, UTM=('13S', 382871, 3972649))
```

**Notes**

A hashable object is one that can be used as a key in a Python dictionary. This includes strings, numbers, tuples of strings and numbers, etc.

On many platforms hashable items also include mutables such as NetworkX Graphs, though one should be careful that the hash doesn’t change on mutables.
**networkx.Graph.add_nodes_from**

Graph.add_nodes_from(nodes_for_adding, **attr)**
Add multiple nodes.

**Parameters**

- **nodes_for_adding** *(iterable container)* – A container of nodes (list, dict, set, etc.). OR A container of (node, attribute dict) tuples. Node attributes are updated using the attribute dict.
- **attr** *(keyword arguments, optional (default= no attributes))* – Update attributes for all nodes in nodes. Node attributes specified in nodes as a tuple take precedence over attributes specified via keyword arguments.

**See also:**
add_node()

**Examples**

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_nodes_from('Hello')
>>> K3 = nx.Graph([(0, 1), (1, 2), (2, 0)])
>>> G.add_nodes_from(K3)
>>> sorted(G.nodes(), key=str)
[0, 1, 2, 'H', 'e', 'l', 'o']
```

Use keywords to update specific node attributes for every node.

```python
>>> G.add_nodes_from([(1, 2), size=10])
>>> G.add_nodes_from([(3, 4), weight=0.4])
```

Use (node, attrdict) tuples to update attributes for specific nodes.

```python
>>> G.nodes[1]['size']
11
>>> H = nx.Graph()
>>> H.add_nodes_from(G.nodes(data=True))
>>> H.nodes[1]['size']
11
```

**networkx.Graph.remove_node**

Graph.remove_node(n)
Remove node n.

Removes the node n and all adjacent edges. Attempting to remove a non-existent node will raise an exception.

**Parameters**

- **n** *(node)* – A node in the graph

**Raises**

- NetworkXError – If n is not in the graph.

**See also:**
remove_nodes_from()
Examples

```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> list(G.edges)
[(0, 1), (1, 2)]
>>> G.remove_node(1)
>>> list(G.edges)
[]
```

`networkx.Graph.remove_nodes_from`

Graph.remove_nodes_from(nodes)
Remove multiple nodes.

Parameters

- **nodes** *(iterable container)* – A container of nodes (list, dict, set, etc.). If a node in the container is not in the graph it is silently ignored.

See also:

- `remove_node()`

Examples

```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> e = list(G.nodes)
>>> e
[0, 1, 2]
>>> G.remove_nodes_from(e)
>>> list(G.nodes)
[]
```

`networkx.Graph.add_edge`

Graph.add_edge(u_of_edge, v_of_edge, **attr)
Add an edge between u and v.

The nodes u and v will be automatically added if they are not already in the graph.

Edge attributes can be specified with keywords or by directly accessing the edge’s attribute dictionary. See examples below.

Parameters

- **u, v** *(nodes)* – Nodes can be, for example, strings or numbers. Nodes must be hashable (and not None) Python objects.

- **attr** *(keyword arguments, optional)* – Edge data (or labels or objects) can be assigned using keyword arguments.

See also:

- `add_edges_from()` add a collection of edges
Notes

Adding an edge that already exists updates the edge data.

Many NetworkX algorithms designed for weighted graphs use an edge attribute (by default `weight`) to hold a numerical value.

Examples

The following all add the edge e=(1, 2) to graph G:

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> e = (1, 2)
>>> G.add_edge(1, 2)  # explicit two-node form
>>> G.add_edge(*e)   # single edge as tuple of two nodes
>>> G.add_edges_from([(1, 2)])   # add edges from iterable container
```

Associate data to edges using keywords:

```python
>>> G.add_edge(1, 2, weight=3)
>>> G.add_edge(1, 3, weight=7, capacity=15, length=342.7)
```

For non-string attribute keys, use subscript notation.

```python
>>> G.add_edge(1, 2)
>>> G[1][2].update({0: 5})
>>> G.edges[1, 2].update({0: 5})
```

networkx.Graph.add_edges_from

Graph.add_edges_from(ebunch_to_add, **attr)
Add all the edges in ebunch_to_add.

Parameters

- **ebunch_to_add** *(container of edges)* – Each edge given in the container will be added to the graph. The edges must be given as 2-tuples (u, v) or 3-tuples (u, v, d) where d is a dictionary containing edge data.

- **attr** *(keyword arguments, optional)* – Edge data (or labels or objects) can be assigned using keyword arguments.

See also:

`add_edge()` add a single edge

`add_weighted_edges_from()` convenient way to add weighted edges

Notes

Adding the same edge twice has no effect but any edge data will be updated when each duplicate edge is added. Edge attributes specified in an ebunch take precedence over attributes specified via keyword arguments.
Examples

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edges_from([(0, 1), (1, 2)])  # using a list of edge tuples
>>> e = zip(range(0, 3), range(1, 4))
>>> G.add_edges_from(e)  # Add the path graph 0-1-2-3

Associate data to edges

```

```python
>>> G.add_edges_from([(1, 2), (2, 3)], weight=3)
>>> G.add_edges_from([(3, 4), (1, 4)], label='WN2898')
```

networkx.Graph.add_weighted_edges_from

**Graph.add_weighted_edges_from(ebunch_to_add, weight='weight', **attr)**

Add weighted edges in `ebunch_to_add` with specified weight `attr`.

**Parameters**

- `ebunch_to_add` *(container of edges)* – Each edge given in the list or container will be added to the graph. The edges must be given as 3-tuples (u, v, w) where w is a number.

- `weight` *(string, optional (default='weight'))* – The attribute name for the edge weights to be added.

- `attr` *(keyword arguments, optional (default= no attributes))* – Edge attributes to add/update for all edges.

**See also:**

- `add_edge()` add a single edge
- `add_edges_from()` add multiple edges

**Notes**

Adding the same edge twice for Graph/DiGraph simply updates the edge data. For MultiGraph/MultiDiGraph, duplicate edges are stored.

**Examples**

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_weighted_edges_from([(0, 1, 3.0), (1, 2, 7.5)])
```

networkx.Graph.remove_edge

**Graph.remove_edge(u, v)**

Remove the edge between u and v.

- **Parameters** u, v *(nodes)* – Remove the edge between nodes u and v.

- **Raises** `NetworkXError` – If there is not an edge between u and v.

**See also:**
remove_edges_from()  remove a collection of edges

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, etc
>>> G.remove_edge(0, 1)
```
```python
>>> e = (1, 2)
>>> G.remove_edge(*e)  # unpacks e from an edge tuple
```
```python
>>> e = (2, 3, {'weight':7})  # an edge with attribute data
>>> G.remove_edge(*e[:2])  # select first part of edge tuple
```

networkx.Graph.remove_edges_from

Graph.remove_edges_from(ebunch)
Remove all edges specified in ebunch.

Parameters  

- **ebunch** (list or container of edge tuples) – Each edge given in the list or container will be removed from the graph. The edges can be:
  - 2-tuples (u, v) edge between u and v.
  - 3-tuples (u, v, k) where k is ignored.

See also:

- remove_edge()  remove a single edge

Notes

Will fail silently if an edge in ebunch is not in the graph.

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> ebunch=[(1, 2), (2, 3)]
>>> G.remove_edges_from(ebunch)
```

networkx.Graph.update

Graph.update(edges=None, nodes=None)
Update the graph using nodes/edges/graphs as input.

Like dict.update, this method takes a graph as input, adding the graph’s nodes and edges to this graph. It can also take two inputs: edges and nodes. Finally it can take either edges or nodes. To specify only nodes the keyword nodes must be used.

The collections of edges and nodes are treated similarly to the add_edges_from/add_nodes_from methods. When iterated, they should yield 2-tuples (u, v) or 3-tuples (u, v, datadict).

Parameters
NetworkX Reference, Release 2.5rc1.dev20191214022633

- **edges** (*Graph object, collection of edges, or None*) – The first parameter can be a graph or some edges. If it has attributes *nodes* and *edges*, then it is taken to be a Graph-like object and those attributes are used as collections of nodes and edges to be added to the graph. If the first parameter does not have those attributes, it is treated as a collection of edges and added to the graph. If the first argument is None, no edges are added.

- **nodes** (*collection of nodes, or None*) – The second parameter is treated as a collection of nodes to be added to the graph unless it is None. If *edges* is None and *nodes* is None an exception is raised. If the first parameter is a Graph, then *nodes* is ignored.

Examples

```python
>>> G = nx.path_graph(5)
>>> G.update(nx.complete_graph(range(4,10)))
>>> from itertools import combinations
>>> edges = ((u, v, {'power': u * v})
... for u, v in combinations(range(10, 20), 2)
... if u * v < 225)
>>> nodes = [1000]           # for singleton, use a container
>>> G.update(edges, nodes)
```

Notes

It you want to update the graph using an adjacency structure it is straightforward to obtain the edges/nodes from adjacency. The following examples provide common cases, your adjacency may be slightly different and require tweaks of these examples.

```python
>>> # dict-of-set/list/tuple
>>> adj = {1: {2, 3}, 2: {1, 3}, 3: {1, 2}}
>>> e = [(u, v) for u, nbrs in adj.items() for v in nbrs]
>>> G.update(edges=e, nodes=adj)
```

```python
>>> DG = nx.DiGraph()
>>> # dict-of-dict-of-attribute
>>> adj = {1: {2: 1.3, 3: 0.7}, 2: {1: 1.4}, 3: {1: 0.7}}
>>> e = [(u, v, {'weight': d}) for u, nbrs in adj.items() for v, d in nbrs.items()]
>>> DG.update(edges=e, nodes=adj)
```

```python
>>> # dict-of-dict-of-dict
>>> adj = {1: {2: {'weight': 1.3}, 3: {'color': 0.7, 'weight':1.2}}}
>>> e = [(u, v, {'weight': d}) for u, nbrs in adj.items() for v, d in nbrs.items()]
>>> DG.update(edges=e, nodes=adj)
```

```python
>>> # predecessor adjacency (dict-of-set)
>>> pred = {1: {2, 3}, 2: {3}, 3: {3}}
>>> e = [(v, u) for u, nbrs in pred.items() for v in nbrs]
```

```python
>>> # MultiGraph dict-of-dict-of-dict-of-attribute
>>> MDG = nx.MultiDiGraph()
>>> adj = {1: {2: {0: {'weight': 1.3}, 1: {'weight': 1.2}}},
... 3: {2: {0: {'weight': 0.7}}}}
```
```
>>> e = [(u, v, ekey, d) for u, nbrs in adj.items()
...     for v, keydict in nbrs.items()
...     for ekey, d in keydict.items()]
>>> MDG.update(edges=e)
```

See also:

- `add_edges_from()` add multiple edges to a graph
- `add_nodes_from()` add multiple nodes to a graph

### networkx.Graph.clear

`Graph.clear()`

Remove all nodes and edges from the graph.

This also removes the name, and all graph, node, and edge attributes.

#### Examples

```
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.clear()
>>> list(G.nodes)
[]
>>> list(G.edges)
[]
```

### Reporting nodes edges and neighbors

- `Graph.nodes` A NodeView of the Graph as G.nodes or G.nodes().
- `Graph.__iter__()` Iterate over the nodes.
- `Graph.has_node(n)` Returns True if the graph contains the node n.
- `Graph.__contains__(n)` Returns True if n is a node, False otherwise.
- `Graph.edges` An EdgeView of the Graph as G.edges or G.edges().
- `Graph.has_edge(u, v)` Returns True if the edge (u, v) is in the graph.
- `Graph.get_edge_data(u, v[, default])` Returns the attribute dictionary associated with edge (u, v).
- `Graph.neighbors(n)` Returns an iterator over all neighbors of node n.
- `Graph.adj` Graph adjacency object holding the neighbors of each node.
- `Graph.__getitem__(n)` Returns a dict of neighbors of node n.
- `Graph.adjacency()` Returns an iterator over (node, adjacency dict) tuples for all nodes.
- `Graph.nbunch_iter([nbunch])` Returns an iterator over nodes contained in nbunch that are also in the graph.

2.2. Basic graph types
**networkx.Graph.nodes**

**property**  
Graph.nodes

A NodeView of the Graph as G.nodes or G.nodes().

Can be used as G.nodes for data lookup and for set-like operations. Can also be used as G.nodes(data='color', default=None) to return a NodeDataView which reports specific node data but no set operations. It presents a dict-like interface as well with G.nodes.items() iterating over (node, nodedata) 2-tuples and G.nodes[3]['foo'] providing the value of the foo attribute for node 3. In addition, a view G.nodes.data('foo') provides a dict-like interface to the foo attribute of each node. G.nodes.data('foo', default=1) provides a default for nodes that do not have attribute foo.

**Parameters**

- **data** *(string or bool, optional (default=False))* – The node attribute returned in 2-tuple (n, ddict[data]). If True, return entire node attribute dict as (n, ddict). If False, return just the nodes n.

- **default** *(value, optional (default=None))* – Value used for nodes that don’t have the requested attribute. Only relevant if data is not True or False.

**Returns**

Allows set-like operations over the nodes as well as node attribute dict lookup and calling to get a NodeDataView. A NodeDataView iterates over (n, data) and has no set operations. A NodeView iterates over n and includes set operations.

When called, if data is False, an iterator over nodes. Otherwise an iterator of 2-tuples (node, attribute value) where the attribute is specified in data. If data is True then the attribute becomes the entire data dictionary.

**Return type**  
NodeView

**Notes**

If your node data is not needed, it is simpler and equivalent to use the expression for n in G, or list(G).

**Examples**

There are two simple ways of getting a list of all nodes in the graph:

```python
>>> G = nx.path_graph(3)
>>> list(G.nodes)
[0, 1, 2]
>>> list(G)
[0, 1, 2]
```

To get the node data along with the nodes:

```python
>>> G.add_node(1, time='5pm')
>>> G.nodes[0]['foo'] = 'bar'
>>> list(G.nodes(data=True))
[(0, {'foo': 'bar'}), (1, {'time': '5pm'}), (2, {})]
>>> list(G.nodes.data())
[(0, {'foo': 'bar'}), (1, {'time': '5pm'}), (2, {})]
```
If some of your nodes have an attribute and the rest are assumed to have a default attribute value you can create a dictionary from node/attribute pairs using the default keyword argument to guarantee the value is never None:

```python
>>> G = nx.Graph()
>>> G.add_node(0)
>>> G.add_node(1, weight=2)
>>> G.add_node(2, weight=3)
>>> dict(G.nodes(data='weight', default=1))
{0: 1, 1: 2, 2: 3}
```
Examples

```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.has_node(0)
True
```

It is more readable and simpler to use

```python
>>> 0 in G
True
```

**networkx.Graph.__contains__**

`Graph.__contains__(n)`

Returns True if `n` is a node, False otherwise. Use: ‘`n in G`’.

**Examples**

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> 1 in G
True
```

**networkx.Graph.edges**

`property Graph.edges`

An `EdgeView` of the Graph as `G.edges` or `G.edges()`.

```python
g = Graph()
g.edges()  # returns edge-iterator
```

edges(self, nbunch=None, data=False, default=None)

The `EdgeView` provides set-like operations on the edge-tuples as well as edge attribute lookup. When called, it also provides an `EdgeDataView` object which allows control of access to edge attributes (but does not provide set-like operations). Hence, `G.edges[u, v][\text{color}]` provides the value of the color attribute for edge `(u, v)` while `for (u, v, c) in G.edges.data(\text{color}, default='red'):` iterates through all the edges yielding the color attribute with default 'red' if no color attribute exists.

**Parameters**

- **nbunch** (`single node, container, or all nodes (default= all nodes)`) – The view will only report edges incident to these nodes.
- **data** (`string or bool, optional (default=False)`) – The edge attribute returned in 3-tuple `(u, v, ddict[data])`. If True, return edge attribute dict in 3-tuple `(u, v, ddict)`. If False, return 2-tuple `(u, v)`.
- **default** (`value, optional (default=None)`) – Value used for edges that don’t have the requested attribute. Only relevant if data is not True or False.

**Returns edges** – A view of edge attributes, usually it iterates over `(u, v)` or `(u, v, d)` tuples of edges, but can also be used for attribute lookup as `edges[u, v][\text{foo}]`.

**Return type** `EdgeView`
Notes

Nodes in nbunch that are not in the graph will be (quietly) ignored. For directed graphs this returns the out-edges.

Examples

```python
>>> G = nx.path_graph(3)  # or MultiGraph, etc
>>> G.add_edge(2, 3, weight=5)
>>> [e for e in G.edges]
[(0, 1), (1, 2), (2, 3)]
>>> G.edges.data()  # default data is {} (empty dict)
EdgeDataView([(0, 1, {}), (1, 2, {}), (2, 3, {'weight': 5})])
>>> G.edges.data('weight', default=1)
EdgeDataView([(0, 1, 1), (1, 2, 1), (2, 3, 5)])
>>> G.edges([0, 3])  # only edges incident to these nodes
EdgeDataView([(0, 1), (3, 2)])
>>> G.edges(0)  # only edges incident to a single node (use G.adj[0]?)
EdgeDataView([(0, 1)])
```

networkx.Graph.has_edge

Graph.has_edge(u, v)

Returns True if the edge (u, v) is in the graph.

This is the same as v in G[u] without KeyError exceptions.

Parameters u, v (nodes) – Nodes can be, for example, strings or numbers. Nodes must be hashable (and not None) Python objects.

Returns edge_ind – True if edge is in the graph, False otherwise.

Return type bool

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.has_edge(0, 1)  # using two nodes
True
>>> e = (0, 1)
>>> G.has_edge(*e)  # e is a 2-tuple (u, v)
True
>>> e = (0, 1, {'weight':7})
>>> G.has_edge(*e[:2])  # e is a 3-tuple (u, v, data_dictionary)
True
```

The following syntax are equivalent:

```python
>>> G.has_edge(0, 1)
True
>>> 1 in G[0]  # though this gives KeyError if 0 not in G
True
```
networkx.Graph.get_edge_data

Graph.get_edge_data(u, v, default=None)
Returns the attribute dictionary associated with edge (u, v).

This is identical to G[u][v] except the default is returned instead of an exception if the edge doesn’t exist.

Parameters

• u, v (nodes)
• default (any Python object (default=None)) – Value to return if the edge (u, v) is not found.

Returns edge_dict – The edge attribute dictionary.

Return type dictionary

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G[0][1]
{}  # default edge data is {}

Warning: Assigning to G[u][v] is not permitted. But it is safe to assign attributes G[u][v]['foo']

>>> G[0][1]['weight'] = 7
>>> G[0][1]['weight']
7
>>> G[1][0]['weight']
7

>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.get_edge_data(0, 1)  # default edge data is {}
{}
>>> e = (0, 1)
>>> G.get_edge_data(*e)  # tuple form
{}
>>> G.get_edge_data('a', 'b', default=0)  # edge not in graph, return 0
0
```

networkx.Graph.neighbors

Graph.neighbors(n)
Returns an iterator over all neighbors of node n.

This is identical to iter(G[n])

Parameters n (node) – A node in the graph

Returns neighbors – An iterator over all neighbors of node n

Return type iterator

Raises NetworkXError – If the node n is not in the graph.
Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [n for n in G.neighbors(0)]
[1]
```

Notes

Alternate ways to access the neighbors are G.adj[n] or G[n]:

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge('a', 'b', weight=7)
>>> G['a']
AtlasView({'b': {'weight': 7}})
>>> G = nx.path_graph(4)
>>> [n for n in G[0]]
[1]
```

networkx.Graph.adj

**property** `Graph.adj`

Graph adjacency object holding the neighbors of each node.

This object is a read-only dict-like structure with node keys and neighbor-dict values. The neighbor-dict is keyed by neighbor to the edge-data-dict. So `G.adj[3][2]['color'] = 'blue'` sets the color of the edge $(3, 2)$ to "blue".

Iterating over G.adj behaves like a dict. Useful idioms include `for nbr, datadict in G.adj[n].items()`.

The neighbor information is also provided by subscripting the graph. So `for nbr, foo_value in G[node].data('foo', default=1):` works.

For directed graphs, `G.adj` holds outgoing (successor) info.

networkx.Graph.__getitem__

**Graph.__getitem__(n)**

Returns a dict of neighbors of node n. Use: ‘G[n]’.

**Parameters**

- `n (node)` – A node in the graph.

**Returns**

`adj_dict` – The adjacency dictionary for nodes connected to n.

**Return type**

dictionary

Notes

G[n] is the same as G.adj[n] and similar to G.neighbors(n) (which is an iterator over G.adj[n])

2.2. Basic graph types
Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G[0]
AtlasView({1: {}})
```

**networkx.Graph.adjacency**

Graph.adjacency()

Returns an iterator over (node, adjacency dict) tuples for all nodes.

For directed graphs, only outgoing neighbors/adjacencies are included.

- **Returns** adj_iter – An iterator over (node, adjacency dictionary) for all nodes in the graph.
- **Return type** iterator

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [(n, nbrdict) for n, nbrdict in G.adjacency()]
[(0, {1: {}}), (1, {0: {}, 2: {}}), (2, {1: {}, 3: {}}), (3, {2: {}})]
```

**networkx.Graph.nbunch_iter**

Graph.nbunch_iter(nbunch=None)

Returns an iterator over nodes contained in nbunch that are also in the graph.

The nodes in nbunch are checked for membership in the graph and if not are silently ignored.

- **Parameters** nbunch – single node, container, or all nodes (default=all nodes) – The view will only report edges incident to these nodes.
- **Returns** niter – An iterator over nodes in nbunch that are also in the graph. If nbunch is None, iterate over all nodes in the graph.
- **Return type** iterator
- **Raises** NetworkXError – If nbunch is not a node or or sequence of nodes. If a node in nbunch is not hashable.

See also:

- `Graph.__iter__()`

Notes

When nbunch is an iterator, the returned iterator yields values directly from nbunch, becoming exhausted when nbunch is exhausted.

To test whether nbunch is a single node, one can use “if nbunch in self:”; even after processing with this routine.

If nbunch is not a node or a (possibly empty) sequence/iterator or None, a NetworkXError is raised. Also, if any object in nbunch is not hashable, a NetworkXError is raised.
Counting nodes edges and neighbors

<table>
<thead>
<tr>
<th>Method</th>
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<tr>
<td><code>Graph.order()</code></td>
<td>Returns the number of nodes in the graph.</td>
</tr>
<tr>
<td><code>Graph.number_of_nodes()</code></td>
<td>Returns the number of nodes in the graph.</td>
</tr>
<tr>
<td><code>Graph.__len__()</code></td>
<td>Returns the number of nodes in the graph.</td>
</tr>
<tr>
<td><code>Graph.degree</code></td>
<td>A DegreeView for the Graph as G.degree or G.degree().</td>
</tr>
<tr>
<td><code>Graph.size([weight])</code></td>
<td>Returns the number of edges or total of all edge weights.</td>
</tr>
<tr>
<td><code>Graph.number_of_edges([u, v])</code></td>
<td>Returns the number of edges between two nodes.</td>
</tr>
</tbody>
</table>

networkx.Graph.order

`Graph.order()`

Returns the number of nodes in the graph.

Returns \( nnodes \) — The number of nodes in the graph.

Return type: `int`

See also:
- `number_of_nodes()`, `__len__()`

Examples

```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.order()
3
```

networkx.Graph.number_of_nodes

`Graph.number_of_nodes()`

Returns the number of nodes in the graph.

Returns \( nnodes \) — The number of nodes in the graph.

Return type: `int`

See also:
- `order()`, `__len__()`

Examples

```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.number_of_nodes()
3
```

networkx.Graph.__len__

`Graph.__len__()`

Returns the number of nodes in the graph. Use: `len(G)`.

2.2. Basic graph types
Returns \texttt{nnodes} – The number of nodes in the graph.

Return type \texttt{int}

See also:

\texttt{number_of_nodes()}, \texttt{order()}

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> len(G)
4
```

\texttt{networkx.Graph.degree}

\texttt{property Graph.degree}

A DegreeView for the Graph as \texttt{G.degree} or \texttt{G.degree()}.

The node degree is the number of edges adjacent to the node. The weighted node degree is the sum of the edge weights for edges incident to that node.

This object provides an iterator for (node, degree) as well as lookup for the degree for a single node.

Parameters

- \texttt{nbunch} (\texttt{single node, container, or all nodes (default= all nodes)}) – The view will only report edges incident to these nodes.
- \texttt{weight} (\texttt{string or None, optional (default=None)}) – The name of an edge attribute that holds the numerical value used as a weight. If \texttt{None}, then each edge has weight 1. The degree is the sum of the edge weights adjacent to the node.

Returns

- \texttt{If a single node is requested}
- \texttt{deg (int)} – Degree of the node
- \texttt{OR if multiple nodes are requested}
- \texttt{nd_view (A DegreeView object capable of iterating (node, degree) pairs)}

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.degree[0]  # node 0 has degree 1
1
>>> list(G.degree([0, 1, 2]))
[(0, 1), (1, 2), (2, 2)]
```

\texttt{networkx.Graph.size}

\texttt{Graph.size(weight=\texttt{None})}

Returns the number of edges or total of all edge weights.
**Parameters** `weight` *(string or None, optional (default=None))* – The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.

**Returns**

`size` – The number of edges or (if weight keyword is provided) the total weight sum.

If weight is None, returns an int. Otherwise a float (or more general numeric if the weights are more general).

**Return type** numeric

See also:

`number_of_edges()`

**Examples**

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.size()
3

>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge('a', 'b', weight=2)
>>> G.add_edge('b', 'c', weight=4)
>>> G.size()
2
>>> G.size(weight='weight')
6.0
```

`networkx.Graph.number_of_edges`

`Graph.number_of_edges(u=None, v=None)`

Returns the number of edges between two nodes.

**Parameters** `u`, `v` *(nodes, optional (default=all edges))* – If `u` and `v` are specified, return the number of edges between `u` and `v`. Otherwise return the total number of all edges.

**Returns** `nedges` – The number of edges in the graph. If `nodes u` and `v` are specified return the number of edges between those nodes. If the graph is directed, this only returns the number of edges from `u` to `v`.

**Return type** int

See also:

`size()`

**Examples**

For undirected graphs, this method counts the total number of edges in the graph:

```python
>>> G = nx.path_graph(4)
>>> G.number_of_edges()
3
```

If you specify two nodes, this counts the total number of edges joining the two nodes:
For directed graphs, this method can count the total number of directed edges from \( u \) to \( v \):

```python
>>> G = nx.DiGraph()
>>> G.add_edge(0, 1)
>>> G.add_edge(1, 0)
>>> G.number_of_edges(0, 1)
1
```

Making copies and subgraphs

<table>
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<th>Method</th>
<th>Description</th>
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<td><code>Graph.copy([as_view])</code></td>
<td>Returns a copy of the graph.</td>
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<tr>
<td><code>Graph.to_undirected([as_view])</code></td>
<td>Returns an undirected copy of the graph.</td>
</tr>
<tr>
<td><code>Graph.to_directed([as_view])</code></td>
<td>Returns a directed representation of the graph.</td>
</tr>
<tr>
<td><code>Graph.subgraph(nodes)</code></td>
<td>Returns a SubGraph view of the subgraph induced on nodes.</td>
</tr>
<tr>
<td><code>Graph.edge_subgraph(edges)</code></td>
<td>Returns the subgraph induced by the specified edges.</td>
</tr>
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</table>

networkx.Graph.copy

`Graph.copy(as_view=False)`

Returns a copy of the graph.

The copy method by default returns an independent shallow copy of the graph and attributes. That is, if an attribute is a container, that container is shared by the original and the copy. Use Python’s `copy.deepcopy` for new containers.

If `as_view` is True then a view is returned instead of a copy.

Notes

All copies reproduce the graph structure, but data attributes may be handled in different ways. There are four types of copies of a graph that people might want.

Deepcopy – A “deepcopy” copies the graph structure as well as all data attributes and any objects they might contain. The entire graph object is new so that changes in the copy do not affect the original object. (see Python’s `copy.deepcopy`)

Data Reference (Shallow) – For a shallow copy the graph structure is copied but the edge, node and graph attribute dicts are references to those in the original graph. This saves time and memory but could cause confusion if you change an attribute in one graph and it changes the attribute in the other. NetworkX does not provide this level of shallow copy.

Independent Shallow – This copy creates new independent attribute dicts and then does a shallow copy of the attributes. That is, any attributes that are containers are shared between the new graph and the original. This is exactly what `dict.copy()` provides. You can obtain this style copy using:

```python
>>> G = nx.path_graph(5)
>>> H = G.copy()
>>> H = G.copy(as_view=False)
```
Fresh Data – For fresh data, the graph structure is copied while new empty data attribute dicts are created. The resulting graph is independent of the original and it has no edge, node or graph attributes. Fresh copies are not enabled. Instead use:

```
>>> H = G.__class__()
>>> H.add_nodes_from(G)
>>> H.add_edges_from(G.edges)
```

View – Inspired by dict-views, graph-views act like read-only versions of the original graph, providing a copy of the original structure without requiring any memory for copying the information.

See the Python copy module for more information on shallow and deep copies, https://docs.python.org/2/library/copy.html.

**Parameters**

- `as_view` *(bool, optional (default=False)) – If True, the returned graph-view provides a read-only view of the original graph without actually copying any data.*

**Returns**

- `G` – A copy of the graph.

**Return type**

- `Graph`

**See also:**

- `to_directed()` return a directed copy of the graph.

**Examples**

```
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> H = G.copy()
```

```
networkx.Graph.to_undirected
```

**Graph.to_undirected** *(as_view=False)*

Returns an undirected copy of the graph.

**Parameters**

- `as_view` *(bool (optional, default=False)) – If True return a view of the original undirected graph.*

**Returns**

- `G` – A deepcopy of the graph.

**Return type**

- `Graph/MultiGraph`

**See also:**

- `Graph()`, `copy()`, `add_edge()`, `add_edges_from()`

**Notes**

This returns a “deepcopy” of the edge, node, and graph attributes which attempts to completely copy all of the data and references.

This is in contrast to the similar `G = nx.DiGraph(D)` which returns a shallow copy of the data.

2.2. Basic graph types
See the Python copy module for more information on shallow and deep copies, https://docs.python.org/2/library/copy.html.

Warning: If you have subclassed DiGraph to use dict-like objects in the data structure, those changes do not transfer to the Graph created by this method.

**Examples**

```python
>>> G = nx.path_graph(2)    # or MultiGraph, etc
>>> H = G.to_directed()
>>> list(H.edges)
[(0, 1), (1, 0)]
>>> G2 = H.to_undirected()
>>> list(G2.edges)
[(0, 1)]
```

`networkx.Graph.to_directed`

`Graph.to_directed(as_view=False)`

Returns a directed representation of the graph.

**Returns G** — A directed graph with the same name, same nodes, and with each edge \((u, v, \text{data})\) replaced by two directed edges \((u, v, \text{data})\) and \((v, u, \text{data})\).

**Return type** `DiGraph`

**Notes**

This returns a “deepcopy” of the edge, node, and graph attributes which attempts to completely copy all of the data and references.

This is in contrast to the similar `D=DiGraph(G)` which returns a shallow copy of the data.

See the Python copy module for more information on shallow and deep copies, https://docs.python.org/2/library/copy.html.

Warning: If you have subclassed Graph to use dict-like objects in the data structure, those changes do not transfer to the DiGraph created by this method.

**Examples**

```python
>>> G = nx.Graph()    # or MultiGraph, etc
>>> G.add_edge(0, 1)
>>> H = G.to_directed()
>>> list(H.edges)
[(0, 1), (1, 0)]
```

If already directed, return a (deep) copy

```python
>>> G = nx.DiGraph()    # or MultiDiGraph, etc
>>> G.add_edge(0, 1)
>>> H = G.to_directed()
>>> list(H.edges)
[(0, 1)]
```
networkx.Graph.subgraph

Graph.subgraph(nodes)

Returns a SubGraph view of the subgraph induced on nodes.

The induced subgraph of the graph contains the nodes in nodes and the edges between those nodes.

Parameters nodes (list, iterable) – A container of nodes which will be iterated through once.

Returns G – A subgraph view of the graph. The graph structure cannot be changed but node/edge attributes can and are shared with the original graph.

Return type SubGraph View

Notes

The graph, edge and node attributes are shared with the original graph. Changes to the graph structure is ruled out by the view, but changes to attributes are reflected in the original graph.

To create a subgraph with its own copy of the edge/node attributes use: G.subgraph(nodes).copy()

For an inplace reduction of a graph to a subgraph you can remove nodes: G.remove_nodes_from([n for n in G if n not in set(nodes)])

Subgraph views are sometimes NOT what you want. In most cases where you want to do more than simply look at the induced edges, it makes more sense to just create the subgraph as its own graph with code like:

```python
# Create a subgraph SG based on a (possibly multigraph) G
SG = G.__class__()
SG.add_nodes_from((n, G.nodes[n]) for n in largest_wcc)
if SG.is_multigraph():
    SG.add_edges_from((n, nbr, key, d)
        for n, nbrs in G.adj.items() if n in largest_wcc
        for nbr, keydict in nbrs.items() if nbr in largest_wcc
        for key, d in keydict.items())
else:
    SG.add_edges_from((n, nbr, d)
        for n, nbrs in G.adj.items() if n in largest_wcc
        for nbr, d in nbrs.items() if nbr in largest_wcc)
SG.graph.update(G.graph)
```

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> H = G.subgraph([(0, 1, 2)])
>>> list(H.edges)
[(0, 1), (1, 2)]
```

networkx.Graph.edge_subgraph

Graph.edge_subgraph(edges)

Returns the subgraph induced by the specified edges.

The induced subgraph contains each edge in edges and each node incident to any one of those edges.

Parameters edges (iterable) – An iterable of edges in this graph.
Returns  G – An edge-induced subgraph of this graph with the same edge attributes.

Return type  Graph

Notes

The graph, edge, and node attributes in the returned subgraph view are references to the corresponding attributes in the original graph. The view is read-only.

To create a full graph version of the subgraph with its own copy of the edge or node attributes, use:

```python
>>> G.edge_subgraph(edges).copy()
```

Examples

```python
>>> G = nx.path_graph(5)
>>> H = G.edge_subgraph([(0, 1), (3, 4)])
>>> list(H.nodes)
[0, 1, 3, 4]
>>> list(H.edges)
[(0, 1), (3, 4)]
```

2.2.2 DiGraph—Directed graphs with self loops

Overview

class DiGraph (incoming_graph_data=None, **attr)

Base class for directed graphs.

A DiGraph stores nodes and edges with optional data, or attributes.

DiGraphs hold directed edges. Self loops are allowed but multiple (parallel) edges are not.

Nodes can be arbitrary (hashable) Python objects with optional key/value attributes. By convention None is not used as a node.

Edges are represented as links between nodes with optional key/value attributes.

Parameters

- **incoming_graph_data (input graph (optional, default: None))** – Data to initialize graph. If None (default) an empty graph is created. The data can be any format that is supported by the to_networkx_graph() function, currently including edge list, dict of dicts, dict of lists, NetworkX graph, NumPy matrix or 2d ndarray, SciPy sparse matrix, or PyGraphviz graph.

- **attr (keyword arguments, optional (default= no attributes))** – Attributes to add to graph as key=value pairs.

See also:

Graph, MultiGraph, MultiDiGraph, OrderedDiGraph
Examples

Create an empty graph structure (a “null graph”) with no nodes and no edges.

```python
>>> G = nx.DiGraph()
```

G can be grown in several ways.

Nodes:

Add one node at a time:

```python
>>> G.add_node(1)
```

Add the nodes from any container (a list, dict, set or even the lines from a file or the nodes from another graph).

```python
>>> G.add_nodes_from([2, 3])
>>> G.add_nodes_from(range(100, 110))
>>> H = nx.path_graph(10)
>>> G.add_nodes_from(H)
```

In addition to strings and integers any hashable Python object (except None) can represent a node, e.g. a customized node object, or even another Graph.

```python
>>> G.add_node(H)
```

Edges:

G can also be grown by adding edges.

Add one edge,

```python
>>> G.add_edge(1, 2)
```

a list of edges,

```python
>>> G.add_edges_from([(1, 2), (1, 3)])
```

or a collection of edges,

```python
>>> G.add_edges_from(H.edges)
```

If some edges connect nodes not yet in the graph, the nodes are added automatically. There are no errors when adding nodes or edges that already exist.

Attributes:

Each graph, node, and edge can hold key/value attribute pairs in an associated attribute dictionary (the keys must be hashable). By default these are empty, but can be added or changed using add_edge, add_node or direct manipulation of the attribute dictionaries named graph, node and edge respectively.

```python
>>> G = nx.DiGraph(day="Friday")
>>> G.graph
{'day': 'Friday'}
```

Add node attributes using add_node(), add_nodes_from() or G.nodes
Add edge attributes using add_edge(), add_edges_from(), subscript notation, or G.edges.

```python
>>> G.add_edge(1, 2, weight=4.7)
>>> G.add_edges_from(((3, 4), (4, 5)), color='red')
>>> G.add_edges_from(((1, 2, {'color': 'blue'}), (2, 3, {'weight': 8})))
>>> G[1][2]['weight'] = 4.7
>>> G.edges[1, 2]['weight'] = 4
```

Warning: we protect the graph data structure by making `G.edges[1, 2]` a read-only dict-like structure. However, you can assign to attributes in e.g. `G.edges[1, 2]`. Thus, use 2 sets of brackets to add/change data attributes: `G.edges[1, 2]['weight'] = 4` (For multigraphs: `MG.edges[u, v, key][name] = value`).

**Shortcuts:**

Many common graph features allow python syntax to speed reporting.

```python
>>> 1 in G  # check if node in graph
True
>>> [n for n in G if n < 3]  # iterate through nodes
[1, 2]
>>> len(G)  # number of nodes in graph
5
```

Often the best way to traverse all edges of a graph is via the neighbors. The neighbors are reported as an adjacency-dict `G.adj` or `G.adjacency()`

```python
>>> for n, nbrdict in G.adjacency():
...     for nbr, eattr in nbrdict.items():
...         if 'weight' in eattr:
...             # Do something useful with the edges
...             pass
```

But the edges reporting object is often more convenient:

```python
>>> for u, v, weight in G.edges(data='weight'):
...     if weight is not None:
...         # Do something useful with the edges
...         pass
```

**Reporting:**

Simple graph information is obtained using object-attributes and methods. Reporting usually provides views instead of containers to reduce memory usage. The views update as the graph is updated similarly to dict-views. The objects `nodes`, `edges` and `adj` provide access to data attributes via lookup (e.g. `nodes[n]`, `edges[u, v]`, `adj[u][v]`) and iteration (e.g. `nodes.items()`, `nodes.data('color')`, `nodes.data('color', default='blue')`) and similarly for `edges` Views exist for `nodes`, `edges`, `neighbors()`/`adj` and `degree`.

For details on these and other miscellaneous methods, see below.
Subclasses (Advanced):

The Graph class uses a dict-of-dict-of-dict data structure. The outer dict (node_dict) holds adjacency information keyed by node. The next dict (adjlist_dict) represents the adjacency information and holds edge data keyed by neighbor. The inner dict (edge_attr_dict) represents the edge data and holds edge attribute values keyed by attribute names.

Each of these three dicts can be replaced in a subclass by a user defined dict-like object. In general, the dict-like features should be maintained but extra features can be added. To replace one of the dicts create a new graph class by changing the class(!) variable holding the factory for that dict-like structure. The variable names are node_dict_factory, node_attr_dict_factory, adjlist_inner_dict_factory, adjlist_outer_dict_factory, edge_attr_dict_factory and graph_attr_dict_factory.

**node_dict_factory** [function, (default: dict)] Factory function to be used to create the dict containing node attributes, keyed by node id. It should require no arguments and return a dict-like object

**node_attr_dict_factory**: function, (default: dict) Factory function to be used to create the node attribute dict which holds attribute values keyed by attribute name. It should require no arguments and return a dict-like object

**adjlist_outer_dict_factory** [function, (default: dict)] Factory function to be used to create the outer-most dict in the data structure that holds adjacency info keyed by node. It should require no arguments and return a dict-like object.

**adjlist_inner_dict_factory** [function, optional (default: dict)] Factory function to be used to create the adjacency list dict which holds edge data keyed by neighbor. It should require no arguments and return a dict-like object.

**edge_attr_dict_factory** [function, optional (default: dict)] Factory function to be used to create the edge attribute dict which holds attribute values keyed by attribute name. It should require no arguments and return a dict-like object.

**graph_attr_dict_factory** [function, (default: dict)] Factory function to be used to create the graph attribute dict which holds attribute values keyed by attribute name. It should require no arguments and return a dict-like object.

Typically, if your extension doesn’t impact the data structure all methods will inherited without issue except: **to_directed/to_undirected**. By default these methods create a DiGraph/Graph class and you probably want them to create your extension of a DiGraph/Graph. To facilitate this we define two class variables that you can set in your subclass.

**to_directed_class** [callable, (default: DiGraph or MultiDiGraph)] Class to create a new graph structure in the **to_directed** method. If **None**, a NetworkX class (DiGraph or MultiDiGraph) is used.

**to_undirected_class** [callable, (default: Graph or MultiGraph)] Class to create a new graph structure in the **to_undirected** method. If **None**, a NetworkX class (Graph or MultiGraph) is used.

**Examples**

Create a low memory graph class that effectively disallows edge attributes by using a single attribute dict for all edges. This reduces the memory used, but you lose edge attributes.

```python
>>> class ThinGraph(nx.Graph):
...     all_edge_dict = {'weight': 1}
...     def single_edge_dict(self):
...         return self.all_edge_dict
...     edge_attr_dict_factory = self.single_edge_dict
>>> G = ThinGraph()
>>> G.add_edge(2, 1)
```

(continues on next page)
>>> G[2][1]
{'weight': 1}
>>> G.add_edge(2, 2)
>>> G[2][1] is G[2][2]
True

Please see ordered for more examples of creating graph subclasses by overwriting the base class dict with a dictionary-like object.

Methods

Adding and removing nodes and edges

networkx.DiGraph.__init__

DiGraph.__init__(incoming_graph_data=None, **attr)
Initialize a graph with edges, name, or graph attributes.

Parameters

- **incoming_graph_data** (input graph (optional, default: None)) – Data to initialize graph. If None (default) an empty graph is created. The data can be an edge list, or any NetworkX graph object. If the corresponding optional Python packages are installed the data can also be a NumPy matrix or 2d ndarray, a SciPy sparse matrix, or a PyGraphviz graph.
- **attr** (keyword arguments, optional (default= no attributes)) – Attributes to add to graph as key=value pairs.

See also:

convert()
Examples

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G = nx.Graph(name='my graph')
>>> e = [(1, 2), (2, 3), (3, 4)]  # list of edges
>>> G = nx.Graph(e)
```

Arbitrary graph attribute pairs (key=value) may be assigned

```python
>>> G = nx.Graph(e, day="Friday")
>>> G.graph
{'day': 'Friday'}
```

debug

networkx.DiGraph.add_node

DiGraph.add_node(node_for_adding, **attr)

Add a single node node_for_adding and update node attributes.

Parameters

- node_for_adding (node) – A node can be any hashable Python object except None.
- attr (keyword arguments, optional) – Set or change node attributes using key=value.

See also:

add_nodes_from()

Examples

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_node(1)
>>> G.add_node('Hello')
>>> K3 = nx.Graph([(0, 1), (1, 2), (2, 0)])
>>> G.add_node(K3)
>>> G.number_of_nodes()
3
```

Use keywords set/change node attributes:

```python
>>> G.add_node(1, size=10)
>>> G.add_node(3, weight=0.4, UTM=('13S', 382871, 3972649))
```

Notes

A hashable object is one that can be used as a key in a Python dictionary. This includes strings, numbers, tuples of strings and numbers, etc.

On many platforms hashable items also include mutables such as NetworkX Graphs, though one should be careful that the hash doesn’t change on mutables.
networkx.DiGraph.add_nodes_from

DiGraph.add_nodes_from(nodes_for_adding, **attr)
Add multiple nodes.

Parameters

- **nodes_for_adding** *(iterable container)* – A container of nodes (list, dict, set, etc.). OR A container of (node, attribute dict) tuples. Node attributes are updated using the attribute dict.

- **attr** *(keyword arguments, optional (default= no attributes))* – Update attributes for all nodes in nodes. Node attributes specified in nodes as a tuple take precedence over attributes specified via keyword arguments.

See also:

*add_node()*

Examples

```python
>>> G = nx.Graph()     # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_nodes_from('Hello')
>>> K3 = nx.Graph([(0, 1), (1, 2), (2, 0)])
>>> G.add_nodes_from(K3)
>>> sorted(G.nodes(), key=str)
[0, 1, 2, 'H', 'e', 'l', 'o']
```

Use keywords to update specific node attributes for every node.

```python
>>> G.add_nodes_from([(1, dict(size=11)), (2, {'color':'blue'})])
>>> G.nodes[1]['size']
11
```

networkx.DiGraph.remove_node

DiGraph.remove_node(n)
Remove node n.

Removes the node n and all adjacent edges. Attempting to remove a non-existent node will raise an exception.

Parameters **n** *(node)* – A node in the graph

Raises **NetworkXError** – If n is not in the graph.

See also:

*remove_nodes_from()*
Examples

```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> list(G.edges)
[(0, 1), (1, 2)]
>>> G.remove_node(1)
>>> list(G.edges)
[]
```

networkx.DiGraph.remove_nodes_from

DiGraph.remove_nodes_from(nodes)
Remove multiple nodes.

Parameters

- **nodes** *(iterable container)* — A container of nodes (list, dict, set, etc.). If a node in the
  container is not in the graph it is silently ignored.

See also:

- remove_node()

Examples

```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> e = list(G.nodes)
>>> e
[0, 1, 2]
>>> G.remove_nodes_from(e)
>>> list(G.nodes)
[]
```

networkx.DiGraph.add_edge

DiGraph.add_edge(u_of_edge, v_of_edge, **attr)
Add an edge between u and v.

The nodes u and v will be automatically added if they are not already in the graph.

Edge attributes can be specified with keywords or by directly accessing the edge’s attribute dictionary. See examples below.

Parameters

- **u, v** *(nodes)* — Nodes can be, for example, strings or numbers. Nodes must be hashable (and not None) Python objects.

- **attr** *(keyword arguments, optional)* — Edge data (or labels or objects) can be assigned using keyword arguments.

See also:

- add_edges_from() add a collection of edges
Notes

Adding an edge that already exists updates the edge data.

Many NetworkX algorithms designed for weighted graphs use an edge attribute (by default `weight`) to hold a numerical value.

Examples

The following all add the edge e=(1, 2) to graph G:

```
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> e = (1, 2)
>>> G.add_edge(1, 2)  # explicit two-node form
>>> G.add_edge(*e)   # single edge as tuple of two nodes
>>> G.add_edges_from([(1, 2)])  # add edges from iterable container
```

Associate data to edges using keywords:

```
>>> G.add_edge(1, 2, weight=3)
>>> G.add_edge(1, 3, weight=7, capacity=15, length=342.7)
```

For non-string attribute keys, use subscript notation.

```
>>> G.add_edge(1, 2)
>>> G[1][2].update({0: 5})
>>> G.edges[1, 2].update({0: 5})
```

networkx.DiGraph.add_edges_from

```
DiGraph.add_edges_from(ebunch_to_add, **attr)
```

Add all the edges in `ebunch_to_add`.

Parameters

- `ebunch_to_add (container of edges)` – Each edge given in the container will be added to the graph. The edges must be given as 2-tuples (u, v) or 3-tuples (u, v, d) where d is a dictionary containing edge data.

- `attr (keyword arguments, optional)` – Edge data (or labels or objects) can be assigned using keyword arguments.

See also:

- `add_edge()` add a single edge
- `add_weighted_edges_from()` convenient way to add weighted edges

Notes

Adding the same edge twice has no effect but any edge data will be updated when each duplicate edge is added. Edge attributes specified in an `ebunch` take precedence over attributes specified via keyword arguments.
Examples

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edges_from([(0, 1), (1, 2)])  # using a list of edge tuples
>>> e = zip(range(0, 3), range(1, 4))
>>> G.add_edges_from(e)  # Add the path graph 0-1-2-3

Associate data to edges

```python
>>> G.add_edges_from([(1, 2), (2, 3)], weight=3)
>>> G.add_edges_from([(3, 4), (1, 4)], label='WN2898')
```

**networkx.DiGraph.add_weighted_edges_from**

DiGraph.add_weighted_edges_from(ebunch_to_add, weight='weight', **attr)

Add weighted edges in ebunch_to_add with specified weight attr

Parameters

- **ebunch_to_add** (container of edges) – Each edge given in the list or container will be added to the graph. The edges must be given as 3-tuples (u, v, w) where w is a number.
- **weight** (string, optional (default= 'weight')) – The attribute name for the edge weights to be added.
- **attr** (keyword arguments, optional (default= no attributes)) – Edge attributes to add/update for all edges.

See also:

- add_edge() add a single edge
- add_edges_from() add multiple edges

Notes

Adding the same edge twice for Graph/DiGraph simply updates the edge data. For MultiGraph/MultiDiGraph, duplicate edges are stored.

Examples

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_weighted_edges_from([(0, 1, 3.0), (1, 2, 7.5)])
```

**networkx.DiGraph.remove_edge**

DiGraph.remove_edge(u, v)

Remove the edge between u and v.

Parameters **u, v** (nodes) – Remove the edge between nodes u and v.

Raises **NetworkXError** – If there is not an edge between u and v.

See also:
remove_edges_from()  remove a collection of edges

Examples

>>> G = nx.Graph()  # or DiGraph, etc
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.remove_edge(0, 1)
>>> e = (1, 2)
>>> G.remove_edge(*e)  # unpacks e from an edge tuple
>>> e = (2, 3, {'weight':7})  # an edge with attribute data
>>> G.remove_edge(*e[:2])  # select first part of edge tuple

networkx.DiGraph.remove_edges_from

DiGraph.remove_edges_from(ebunch)  Remove all edges specified in ebunch.

Parameters  ebunch (list or container of edge tuples) – Each edge given in the list or container will be removed from the graph. The edges can be:

• 2-tuples (u, v) edge between u and v.
• 3-tuples (u, v, k) where k is ignored.

See also:

remove_edge()  remove a single edge

Notes

Will fail silently if an edge in ebunch is not in the graph.

Examples

>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> ebunch = [(1, 2), (2, 3)]
>>> G.remove_edges_from(ebunch)

networkx.DiGraph.update

DiGraph.update(edges=None, nodes=None)  Update the graph using nodes/edges/graphs as input.

Like dict.update, this method takes a graph as input, adding the graph’s nodes and edges to this graph. It can also take two inputs: edges and nodes. Finally it can take either edges or nodes. To specify only nodes the keyword nodes must be used.

The collections of edges and nodes are treated similarly to the add_edges_from/add_nodes_from methods. When iterated, they should yield 2-tuples (u, v) or 3-tuples (u, v, datadict).

Parameters
• **edges** (*Graph object, collection of edges, or None*) – The first parameter can be a graph or some edges. If it has attributes *nodes* and *edges*, then it is taken to be a Graph-like object and those attributes are used as collections of nodes and edges to be added to the graph. If the first parameter does not have those attributes, it is treated as a collection of edges and added to the graph. If the first argument is None, no edges are added.

• **nodes** (*collection of nodes, or None*) – The second parameter is treated as a collection of nodes to be added to the graph unless it is None. If *edges* is None and *nodes* is None an exception is raised. If the first parameter is a Graph, then *nodes* is ignored.

**Examples**

```python
>>> G = nx.path_graph(5)
>>> G.update(nx.complete_graph(range(4,10)))
>>> from itertools import combinations
>>> edges = ((u, v, {'power': u * v})
...           for u, v in combinations(range(10, 20), 2)
...           if u * v < 225)
>>> nodes = [1000]  # for singleton, use a container
>>> G.update(edges=edges, nodes=nodes)
```

**Notes**

It you want to update the graph using an adjacency structure it is straightforward to obtain the edges/nodes from adjacency. The following examples provide common cases, your adjacency may be slightly different and require tweaks of these examples.

```python
>>> # dict-of-set/list/tuple
>>> adj = {1: {2: 3}, 2: {1, 3}, 3: {1, 2}}
>>> e = [(u, v) for u, nbrs in adj.items() for v in nbrs]
>>> G.update(edges=e, nodes=adj)
```

```python
>>> DG = nx.DiGraph()
>>> # dict-of-dict-of-attribute
>>> adj = {1: {2: 1.3, 3: 0.7}, 2: {1: 1.4, 3: 1: 0.7}}
>>> e = [(u, v, {'weight': d}) for u, nbrs in adj.items() for v, d in nbrs.items()]
>>> DG.update(edges=e, nodes=adj)
```

```python
>>> # dict-of-dict-of-dict
>>> adj = {1: {2: {'weight': 1.3}, 3: {'color': 0.7, 'weight':1.2}}}
>>> e = [(u, v, {'weight': d}) for u, nbrs in adj.items() for v, d in nbrs.items()]
>>> DG.update(edges=e, nodes=adj)
```

```python
>>> # predecessor adjacency (dict-of-set)
>>> pred = {1: {2, 3}, 2: {3}, 3: {3}}
>>> e = [(v, u) for u, nbrs in pred.items() for v in nbrs]
```

```python
>>> # MultiGraph dict-of-dict-of-dict-of-attribute
>>> MDG = nx.MultiDiGraph()
>>> adj = {1: {2: {0: {'weight': 1.3}, 1: {'weight': 1.2}}},
...        3: {2: {0: {'weight': 0.7}}}}
```

(continues on next page)
```
>>> e = [(u, v, ekey, d) for u, nbrs in adj.items()
...     for v, keydict in nbrs.items()
...     for ekey, d in keydict.items()]
>>> MDG.update(edges=e)
```

See also:

- **add_edges_from()** add multiple edges to a graph
- **add_nodes_from()** add multiple nodes to a graph

### networkx.DiGraph.clear

**DiGraph.clear()**

Remove all nodes and edges from the graph.

This also removes the name, and all graph, node, and edge attributes.

#### Examples

```
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.clear()
>>> list(G.nodes)
[]
>>> list(G.edges)
[]
```

### Reporting nodes edges and neighbors

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<td>Returns an iterator over successor nodes of n.</td>
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<td>Graph adjacency object holding the successors of each node.</td>
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networkx.DiGraph.nodes

**property DiGraph.nodes**

A NodeView of the Graph as G.nodes or G.nodes().

Can be used as G.nodes for data lookup and for set-like operations. Can also be used as G.nodes(data='color', default=None) to return a NodeDataView which reports specific node data but no set operations. It presents a dict-like interface as well with G.nodes.items() iterating over (node, nodedata) 2-tuples and G.nodes[3]['foo'] providing the value of the foo attribute for node 3. In addition, a view G.nodes.data('foo') provides a dict-like interface to the foo attribute of each node. G.nodes.data('foo', default=1) provides a default for nodes that do not have attribute foo.

**Parameters**

- **data** (string or bool, optional (default=False)) – The node attribute returned in 2-tuple (n, ddict[data]). If True, return entire node attribute dict as (n, ddict). If False, return just the nodes n.

- **default** (value, optional (default=None)) – Value used for nodes that don’t have the requested attribute. Only relevant if data is not True or False.

**Returns**

Allows set-like operations over the nodes as well as node attribute dict lookup and calling to get a NodeDataView. A NodeDataView iterates over (n, data) and has no set operations. A NodeView iterates over n and includes set operations.

When called, if data is False, an iterator over nodes. Otherwise an iterator of 2-tuples (node, attribute value) where the attribute is specified in data. If data is True then the attribute becomes the entire data dictionary.

**Return type** NodeView

**Notes**

If your node data is not needed, it is simpler and equivalent to use the expression for n in G, or list(G).

**Examples**

There are two simple ways of getting a list of all nodes in the graph:

```python
>>> G = nx.path_graph(3)
>>> list(G.nodes)
[0, 1, 2]
>>> list(G)
[0, 1, 2]
```

To get the node data along with the nodes:
NetworkX Reference, Release 2.5rc1.dev20191214022633

```python
>>> G.add_node(1, time='5pm')
>>> G.nodes[0]['foo'] = 'bar'
>>> list(G.nodes(data=True))
[(0, {'foo': 'bar'}), (1, {'time': '5pm'}), (2, {})]
>>> list(G.nodes.data())
[(0, {'foo': 'bar'}), (1, {'time': '5pm'}), (2, {})]

>>> list(G.nodes(data='foo'))
[(0, 'bar'), (1, None), (2, None)]
>>> list(G.nodes.data('foo'))
[(0, 'bar'), (1, None), (2, None)]

>>> list(G.nodes(data='time'))
[(0, None), (1, '5pm'), (2, None)]
>>> list(G.nodes.data('time'))
[(0, None), (1, '5pm'), (2, None)]

>>> list(G.nodes(data='time', default='Not Available'))
[(0, 'Not Available'), (1, '5pm'), (2, 'Not Available')]
>>> list(G.nodes.data('time', default='Not Available'))
[(0, 'Not Available'), (1, '5pm'), (2, 'Not Available')]
```

If some of your nodes have an attribute and the rest are assumed to have a default attribute value you can create a dictionary from node/attribute pairs using the default keyword argument to guarantee the value is never None:

```python
>>> G = nx.Graph()
>>> G.add_node(0)
>>> G.add_node(1, weight=2)
>>> G.add_node(2, weight=3)
>>> dict(G.nodes(data='weight', default=1))
{0: 1, 1: 2, 2: 3}
```

networkx.DiGraph.__iter__

DiGraph.__iter__()

Iterate over the nodes. Use: ‘for n in G’.

Returns niter – An iterator over all nodes in the graph.

Return type iterator

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [n for n in G]
[0, 1, 2, 3]
>>> list(G)
[0, 1, 2, 3]
```
networkx.DiGraph.has_node

DiGraph.

\texttt{has\_node}(n)

Returns True if the graph contains the node \texttt{n}.

Identical to \texttt{n in G}

\textbf{Parameters} \texttt{n (node)}

\textbf{Examples}

```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.has_node(0)
True
```

It is more readable and simpler to use

```python
>>> 0 in G
True
```

networkx.DiGraph.__contains__

DiGraph.

\texttt{\_contains\_}(n)

Returns True if \texttt{n} is a node, False otherwise. Use: \texttt{`n in G'}.  

\textbf{Examples}

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> 1 in G
True
```

networkx.DiGraph.edges

\textbf{property} \texttt{DiGraph.edges}

An OutEdgeView of the DiGraph as \texttt{G.edges} or \texttt{G.edges()}.  

\texttt{edges(self, nbunch=None, data=False, default=None)}

The OutEdgeView provides set-like operations on the edge-tuples as well as edge attribute lookup. When called, it also provides an EdgeDataView object which allows control of access to edge attributes (but does not provide set-like operations). Hence, \texttt{G.edges[u, v][`color'] provides the value of the color attribute for edge \texttt{(u, v)} while for \texttt{(u, v, c) in G.edges.data(`color', default='red')}: iterates through all the edges yielding the color attribute with default 'red' if no color attribute exists.

\textbf{Parameters}

\begin{itemize}
  \item \texttt{nbunch} (\texttt{single node, container, or all nodes (default= all nodes)}) – The view will only report edges incident to these nodes.
  \item \texttt{data} (\texttt{string or bool, optional (default=False)}) – The edge attribute returned in 3-tuple \texttt{(u, v, ddict[data]). If True, return edge attribute dict in 3-tuple \texttt{(u, v, ddict). If False, return 2-tuple \texttt{(u, v)}.}
\end{itemize}
NetworkX Reference, Release 2.5rc1.dev20191214022633

- **default** *(value, optional (default=None)) – Value used for edges that don’t have the requested attribute. Only relevant if data is not True or False.*

**Returns edges** – A view of edge attributes, usually it iterates over (u, v) or (u, v, d) tuples of edges, but can also be used for attribute lookup as edges[u, v]['foo'].

**Return type** OutEdgeView

See also:

in_edges, out_edges

**Notes**

Nodes in nbunch that are not in the graph will be (quietly) ignored. For directed graphs this returns the out-edges.

**Examples**

```python
>>> G = nx.DiGraph()  # or MultiDiGraph, etc
>>> nx.add_path(G, [0, 1, 2])
>>> G.add_edge(2, 3, weight=5)
>>> [e for e in G.edges]
[(0, 1), (1, 2), (2, 3)]
>>> G.edges.data()  # default data is {} (empty dict)
OutEdgeDataView([(0, 1, {}), (1, 2, {}), (2, 3, {'weight': 5})])
>>> G.edges.data('weight', default=1)
OutEdgeDataView([(0, 1, 1), (1, 2, 1), (2, 3, 5)])
>>> G.edges([0, 2])  # only edges incident to these nodes
OutEdgeDataView([(0, 1), (2, 3)])
>>> G.edges(0)  # only edges incident to a single node (use G.adj[0]?)
OutEdgeDataView([(0, 1)])
```

**networkx.DiGraph.out_edges**

**property DiGraph.out_edges**

An OutEdgeView of the DiGraph as G.edges or G.edges().

**edges(self, nbunch=None, data=False, default=None)**

The OutEdgeView provides set-like operations on the edge-tuples as well as edge attribute lookup. When called, it also provides an EdgeDataView object which allows control of access to edge attributes (but does not provide set-like operations). Hence, G.edges[u, v]['color'] provides the value of the color attribute for edge (u, v) while for (u, v, c) in G.edges.data('color', default='red'): iterates through all the edges yielding the color attribute with default 'red' if no color attribute exists.

**Parameters**

- **nbunch** *(single node, container, or all nodes (default= all nodes)) – The view will only report edges incident to these nodes.*
- **data** *(string or bool, optional (default=False)) – The edge attribute returned in 3-tuple (u, v, ddict[data]). If True, return edge attribute dict in 3-tuple (u, v, ddict). If False, return 2-tuple (u, v).*
- **default** *(value, optional (default=None)) – Value used for edges that don’t have the requested attribute. Only relevant if data is not True or False.*
Returns edges – A view of edge attributes, usually it iterates over (u, v) or (u, v, d) tuples of edges, but can also be used for attribute lookup as edges[u, v]['foo'].

Return type OutEdgeView

See also:
in_edges, out_edges

Notes
Nodes in nbunch that are not in the graph will be (quietly) ignored. For directed graphs this returns the out-edges.

Examples

```python
>>> G = nx.DiGraph()  # or MultiDiGraph, etc
>>> nx.add_path(G, [0, 1, 2])
>>> G.add_edge(2, 3, weight=5)
>>> [e for e in G.edges]
[(0, 1), (1, 2), (2, 3)]
>>> G.edges.data()  # default data is {} (empty dict)
OutEdgeDataView([(0, 1), (1, 2), (2, 3), {'weight': 5}])
>>> G.edges.data('weight', default=1)
OutEdgeDataView([(0, 1), (1, 2), (3, 2, 1)])
>>> G.edges([0, 2])  # only edges incident to these nodes
OutEdgeDataView([(0, 1), (2, 3)])
```

networkx.DiGraph.in_edges

property DiGraph.in_edges
An InEdgeView of the Graph as G.in_edges or G.in_edges().
in_edges(self, nbunch=None, data=False, default=None):

Parameters

- **nbunch** (single node, container, or all nodes (default=all nodes)) – The view will only report edges incident to these nodes.
- **data** (string or bool, optional (default=False)) – The edge attribute returned in 3-tuple (u, v, ddict[data]). If True, return edge attribute dict in 3-tuple (u, v, ddict). If False, return 2-tuple (u, v).
- **default** (value, optional (default=None)) – Value used for edges that don’t have the requested attribute. Only relevant if data is not True or False.

Returns in_edges – A view of edge attributes, usually it iterates over (u, v) or (u, v, d) tuples of edges, but can also be used for attribute lookup as edges[u, v]['foo'].

Return type InEdgeView

See also:
edges
networkx.DiGraph.has_edge

DiGraph.\texttt{has\_edge}(u, v)

Returns True if the edge \((u, v)\) is in the graph.

This is the same as \(v \text{ in } G[u]\) without KeyError exceptions.

\textbf{Parameters} \(u, v\) (\textit{nodes}) – Nodes can be, for example, strings or numbers. Nodes must be hashable (and not None) Python objects.

\textbf{Returns} \texttt{edge\_ind} – True if edge is in the graph, False otherwise.

\textbf{Return type} \texttt{bool}

\textbf{Examples}

\begin{verbatim}
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.has_edge(0, 1)  # using two nodes
True
>>> e = (0, 1)
>>> G.has_edge(*e)  # \(e\) is a 2-tuple \((u, v)\)
True
>>> e = (0, 1, {'weight':7})
>>> G.has_edge(*e[:2])  # \(e\) is a 3-tuple \((u, v, \text{data\_dictionary})\)
True

The following syntax are equivalent:

\begin{verbatim}
>>> G.has_edge(0, 1)
True
>>> 1 \text{ in } G[0]  # though this gives KeyError if 0 not in G
True
\end{verbatim}

\end{verbatim}

networkx.DiGraph.get_edge_data

DiGraph.\texttt{get\_edge\_data}(u, v, default=None)

Returns the attribute dictionary associated with edge \((u, v)\).

This is identical to \(G[u][v]\) except the default is returned instead of an exception if the edge doesn’t exist.

\textbf{Parameters}

\begin{itemize}
  \item \(u, v\) (\textit{nodes})
  \item \texttt{default} (\textit{any Python object (default=None)}) – Value to return if the edge \((u, v)\) is not found.
\end{itemize}

\textbf{Returns} \texttt{edge\_dict} – The edge attribute dictionary.

\textbf{Return type} \texttt{dictionary}

\textbf{Examples}

\begin{verbatim}
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G[0][1]
()

Warning: Assigning to \(G[u][v]\) is not permitted. But it is safe to assign attributes \(G[u][v][\textquotesingle\text{foo}\textquotesingle]\)
>>> G[0][1]['weight'] = 7
>>> G[0][1]['weight']
7
>>> G[1][0]['weight']
7

>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.get_edge_data(0, 1)  # default edge data is {}
{}
>>> e = (0, 1)
>>> G.get_edge_data(*e)  # tuple form
{}
>>> G.get_edge_data('a', 'b', default=0)  # edge not in graph, return 0
0

networkx.DiGraph.neighbors

DiGraph.neighbors(n)
Returns an iterator over successor nodes of n.
A successor of n is a node m such that there exists a directed edge from n to m.

Parameters
n (node) – A node in the graph

Raises
NetworkXError – If n is not in the graph.

See also:
predecessors()

Notes
neighbors() and successors() are the same.

networkx.DiGraph.adj

property DiGraph.adj
Graph adjacency object holding the neighbors of each node.
This object is a read-only dict-like structure with node keys and neighbor-dict values. The neighbor-dict is keyed by neighbor to the edge-data-dict. So G.adj[3][2]['color'] = 'blue' sets the color of the edge (3, 2) to "blue".

Iterating over G.adj behaves like a dict. Useful idioms include for nbr, datadict in G.adj[n].items():

The neighbor information is also provided by subscripting the graph. So for nbr, foovalue in G[node].data('foo', default=1): works.

For directed graphs, G.adj holds outgoing (successor) info.

networkx.DiGraph.__getitem__

DiGraph.__getitem__ (n)
Returns a dict of neighbors of node n. Use: ‘G[n]’.

2.2. Basic graph types
Parameters n (node) – A node in the graph.

Returns adj_dict – The adjacency dictionary for nodes connected to n.

Return type dictionary

Notes

G[n] is the same as G.adj[n] and similar to G.neighbors(n) (which is an iterator over G.adj[n])

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G[0]
AtlasView({1: {}})
```

networkx.DiGraph.successors

DiGraph.successors(n)

Returns an iterator over successor nodes of n.

A successor of n is a node m such that there exists a directed edge from n to m.

Parameters n (node) – A node in the graph

Raises NetworkXError – If n is not in the graph.

See also:

 predecessors()

Notes

neighbors() and successors() are the same.

networkx.DiGraph.succ

property DiGraph.succ

Graph adjacency object holding the successors of each node.

This object is a read-only dict-like structure with node keys and neighbor-dict values. The neighbor-dict is keyed by neighbor to the edge-data-dict. So G.succ[3][2]['color'] = 'blue' sets the color of the edge (3, 2) to "blue".

Iterating over G.succ behaves like a dict. Useful idioms include for nbr, datadict in G.succ[n].items(). A data-view not provided by dicts also exists: for nbr, foovalue in G.succ[node].data('foo'): and a default can be set via a default argument to the data method.

The neighbor information is also provided by subscripting the graph. So for nbr, foovalue in G[node].data('foo', default=1): works.

For directed graphs, G.adj is identical to G.succ.
**networkx.DiGraph.predecessors**

DiGraph.

predecessors(n)

Returns an iterator over predecessor nodes of n.

A predecessor of n is a node m such that there exists a directed edge from m to n.

**Parameters**

n (node) – A node in the graph

**Raises**

NetworkXError – If n is not in the graph.

**See also:**

successors()

**networkx.DiGraph.pred**

property DiGraph.

pred

Graph adjacency object holding the predecessors of each node.

This object is a read-only dict-like structure with node keys and neighbor-dict values. The neighbor-dict is keyed by neighbor to the edge-data-dict. So G.pred[2][3]['color'] = 'blue' sets the color of the edge (3, 2) to "blue".

Iterating over G.pred behaves like a dict. Useful idioms include for nbr, datadict in G.pred[n].items(). A data-view not provided by dicts also exists: for nbr, foovalue in G.pred[node].data('foo'): A default can be set via a default argument to the data method.

**networkx.DiGraph.adjacency**

DiGraph.

adjacency()

Returns an iterator over (node, adjacency dict) tuples for all nodes.

For directed graphs, only outgoing neighbors/adjacencies are included.

**Returns**

adj_iter – An iterator over (node, adjacency dictionary) for all nodes in the graph.

**Return type**

iterator

**Examples**

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [(n, nbrdict) for n, nbrdict in G.adjacency()]
[(0, {1: {}}), (1, {0: {}, 2: {}}), (2, {1: {}, 3: {}}), (3, {2: {}})]
```

**networkx.DiGraph.nbunch_iter**

DiGraph.

nbunch_iter(nbunch=None)

Returns an iterator over nodes contained in nbunch that are also in the graph.

The nodes in nbunch are checked for membership in the graph and if not are silently ignored.

**Parameters**

nbunch (single node, container, or all nodes (default= all nodes)) – The view will only report edges incident to these nodes.

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Returns niter – An iterator over nodes in nbunch that are also in the graph. If nbunch is None, iterate over all nodes in the graph.

Return type iterator

Raises NetworkXError – If nbunch is not a node or or sequence of nodes. If a node in nbunch is not hashable.

See also:
Graph.__iter__()  

Notes

When nbunch is an iterator, the returned iterator yields values directly from nbunch, becoming exhausted when nbunch is exhausted.

To test whether nbunch is a single node, one can use “if nbunch in self:”, even after processing with this routine.

If nbunch is not a node or a (possibly empty) sequence/iterator or None, a NetworkXError is raised. Also, if any object in nbunch is not hashable, a NetworkXError is raised.

Counting nodes edges and neighbors

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<th>Description</th>
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<td>Returns the number of nodes in the graph.</td>
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<td>DiGraph.number_of_nodes()</td>
<td>Returns the number of nodes in the graph.</td>
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<tr>
<td>DiGraph.degree</td>
<td>A DegreeView for the Graph as G.degree or G.degree().</td>
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<td>DiGraph.in_degree</td>
<td>An InDegreeView for (node, in_degree) or in_degree for single node.</td>
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<td>DiGraph.size([weight])</td>
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<td>Returns the number of edges between two nodes.</td>
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</table>

networkx.DiGraph.order

DiGraph.order()

Returns the number of nodes in the graph.

Returns nnodes – The number of nodes in the graph.

Return type int

See also:
number_of_nodes(), __len__()

Examples

```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.order()
3
```
networkx.DiGraph.number_of_nodes

DiGraph.

number_of_nodes()  
Returns the number of nodes in the graph.

Returns nnodes – The number of nodes in the graph.

Return type int

See also:

order(), __len__()

Examples

```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.number_of_nodes()
3
```

networkx.DiGraph.__len__

DiGraph.

__len__()  
Returns the number of nodes in the graph. Use: `len(G)`.

Returns nnodes – The number of nodes in the graph.

Return type int

See also:

number_of_nodes(), order()

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> len(G)
4
```

networkx.DiGraph.degree

property DiGraph.

degree  
A DegreeView for the Graph as G.degree or G.degree().

The node degree is the number of edges adjacent to the node. The weighted node degree is the sum of the edge weights for edges incident to that node.

This object provides an iterator for (node, degree) as well as lookup for the degree for a single node.

Parameters

- nbunch (single node, container, or all nodes (default= all nodes)) – The view will only report edges incident to these nodes.
- weight (string or None, optional (default= None)) – The name of an edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1. The degree is the sum of the edge weights adjacent to the node.
networkx.DiGraph.in_degree

property DiGraph.in_degree
An InDegreeView for (node, in_degree) or in_degree for single node.

The node in_degree is the number of edges pointing to the node. The weighted node degree is the sum of the edge weights for edges incident to that node.

This object provides an iteration over (node, in_degree) as well as lookup for the degree for a single node.

Parameters

- nbunch (single node, container, or all nodes (default=all nodes)) – The view will only report edges incident to these nodes.
- weight (string or None, optional (default=None)) – The name of an edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1. The degree is the sum of the edge weights adjacent to the node.

Returns

- If a single node is requested
- deg (int) – In-degree of the node
- OR if multiple nodes are requested
- nd_iter (iterator) – The iterator returns two-tuples of (node, in-degree).

See also:

degree, out_degree

Examples
>>> G = nx.DiGraph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.in_degree(0) # node 0 with degree 0
0
>>> list(G.in_degree([0, 1, 2]))
[(0, 0), (1, 1), (2, 1)]

networkx.DiGraph.out_degree

property DiGraph.out_degree
An OutDegreeView for (node, out_degree)

The node out_degree is the number of edges pointing out of the node. The weighted node degree is the sum of the edge weights for edges incident to that node.

This object provides an iterator over (node, out_degree) as well as lookup for the degree for a single node.

Parameters

• nbunch (single node, container, or all nodes (default=all nodes)) – The view will only report edges incident to these nodes.
• weight (string or None, optional (default=None)) – The name of an edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1. The degree is the sum of the edge weights adjacent to the node.

Returns

• If a single node is requested

  • deg (int) – Out-degree of the node
• OR if multiple nodes are requested

  • nd_iter (iterator) – The iterator returns two-tuples of (node, out-degree).

See also:
degree, in_degree

Examples

>>> G = nx.DiGraph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.out_degree(0) # node 0 with degree 1
1
>>> list(G.out_degree([0, 1, 2]))
[(0, 1), (1, 1), (2, 1)]

networkx.DiGraph.size

DiGraph.size(weight=None)

Returns the number of edges or total of all edge weights.

Parameters weight (string or None, optional (default=None)) – The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.
Returns

size – The number of edges or (if weight keyword is provided) the total weight sum.

If weight is None, returns an int. Otherwise a float (or more general numeric if the weights are
more general).

Return type numeric

See also:

number_of_edges()

Examples

>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.size()
3

>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge('a', 'b', weight=2)
>>> G.add_edge('b', 'c', weight=4)
>>> G.size()
2
>>> G.size(weight='weight')
6.0

networkx.DiGraph.number_of_edges

DiGraph.number_of_edges(u=None, v=None)

Returns the number of edges between two nodes.

Parameters u, v (nodes, optional (default=all edges)) – If u and v are specified, return the number
of edges between u and v. Otherwise return the total number of all edges.

Returns nedges – The number of edges in the graph. If nodes u and v are specified return the
number of edges between those nodes. If the graph is directed, this only returns the number of
edges from u to v.

Return type int

See also:

size()

Examples

For undirected graphs, this method counts the total number of edges in the graph:

>>> G = nx.path_graph(4)
>>> G.number_of_edges()
3

If you specify two nodes, this counts the total number of edges joining the two nodes:

>>> G.number_of_edges(0, 1)
1
For directed graphs, this method can count the total number of directed edges from $u$ to $v$:

```python
>>> G = nx.DiGraph()
>>> G.add_edge(0, 1)
>>> G.add_edge(1, 0)
>>> G.number_of_edges(0, 1)
1
```

Making copies and subgraphs

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<td><code>DiGraph.reverse([copy])</code></td>
<td>Returns the reverse of the graph.</td>
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`networkx.DiGraph.copy`

DiGraph.**copy** *(as_view=False)*

Returns a copy of the graph.

The copy method by default returns an independent shallow copy of the graph and attributes. That is, if an attribute is a container, that container is shared by the original and the copy. Use Python’s `copy.deepcopy` for new containers.

If `as_view` is True then a view is returned instead of a copy.

**Notes**

All copies reproduce the graph structure, but data attributes may be handled in different ways. There are four types of copies of a graph that people might want.

Deepcopy – A “deepcopy” copies the graph structure as well as all data attributes and any objects they might contain. The entire graph object is new so that changes in the copy do not affect the original object. (see Python’s `copy.deepcopy`)

Data Reference (Shallow) – For a shallow copy the graph structure is copied but the edge, node and graph attribute dicts are references to those in the original graph. This saves time and memory but could cause confusion if you change an attribute in one graph and it changes the attribute in the other. NetworkX does not provide this level of shallow copy.

Independent Shallow – This copy creates new independent attribute dicts and then does a shallow copy of the attributes. That is, any attributes that are containers are shared between the new graph and the original. This is exactly what `dict.copy()` provides. You can obtain this style copy using:

```python
>>> G = nx.path_graph(5)
>>> H = G.copy()
>>> H = G.copy(as_view=False)
>>> H = nx.Graph(G)
>>> H = G.__class__(G)
```
Fresh Data – For fresh data, the graph structure is copied while new empty data attribute dicts are created. The resulting graph is independent of the original and it has no edge, node or graph attributes. Fresh copies are not enabled. Instead use:

```python
>>> H = G.__class__()
>>> H.add_nodes_from(G)
>>> H.add_edges_from(G.edges)
```

View – Inspired by dict-views, graph-views act like read-only versions of the original graph, providing a copy of the original structure without requiring any memory for copying the information.

See the Python copy module for more information on shallow and deep copies, https://docs.python.org/2/library/copy.html.

**Parameters**

- **as_view** (bool, optional (default=False)) – If True, the returned graph-view provides a read-only view of the original graph without actually copying any data.

**Returns**

- **G** – A copy of the graph.

**Return type**

- **Graph**

**See also:**

- `to_directed()` return a directed copy of the graph.

**Examples**

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> H = G.copy()
```

`networkx.DiGraph.to_undirected`

**DiGraph.to_undirected**(reciprocal=False, as_view=False)

Returns an undirected representation of the digraph.

**Parameters**

- **reciprocal** (bool (optional)) – If True only keep edges that appear in both directions in the original digraph.

- **as_view** (bool (optional, default=False)) – If True return an undirected view of the original directed graph.

**Returns**

- **G** – An undirected graph with the same name and nodes and with edge (u, v, data) if either (u, v, data) or (v, u, data) is in the digraph. If both edges exist in digraph and their edge data is different, only one edge is created with an arbitrary choice of which edge data to use. You must check and correct for this manually if desired.

**Return type**

- **Graph**

**See also:**

- `Graph()`, `copy()`, `add_edge()`, `add_edges_from()`
Notes

If edges in both directions \((u, v)\) and \((v, u)\) exist in the graph, attributes for the new undirected edge will be a combination of the attributes of the directed edges. The edge data is updated in the (arbitrary) order that the edges are encountered. For more customized control of the edge attributes use add_edge().

This returns a “deepcopy” of the edge, node, and graph attributes which attempts to completely copy all of the data and references.

This is in contrast to the similar \(G=\text{DiGraph}(D)\) which returns a shallow copy of the data.

See the Python copy module for more information on shallow and deep copies, https://docs.python.org/2/library/copy.html.

Warning: If you have subclassed DiGraph to use dict-like objects in the data structure, those changes do not transfer to the Graph created by this method.

Examples

```python
>>> G = nx.path_graph(2)  # or MultiGraph, etc
>>> H = G.to_directed()
>>> list(H.edges)
[(0, 1), (1, 0)]
>>> G2 = H.to_undirected()
>>> list(G2.edges)
[(0, 1)]
```

networkx.DiGraph.to_directed

\texttt{DiGraph.to\_directed}(\texttt{as\_view=False})

Returns a directed representation of the graph.

Returns \(G\) – A directed graph with the same name, same nodes, and with each edge \((u, v, data)\) replaced by two directed edges \((u, v, data)\) and \((v, u, data)\).

Return type \texttt{DiGraph}

Notes

This returns a “deepcopy” of the edge, node, and graph attributes which attempts to completely copy all of the data and references.

This is in contrast to the similar \(D=\text{DiGraph}(G)\) which returns a shallow copy of the data.

See the Python copy module for more information on shallow and deep copies, https://docs.python.org/2/library/copy.html.

Warning: If you have subclassed Graph to use dict-like objects in the data structure, those changes do not transfer to the DiGraph created by this method.

Examples
```python
>>> G = nx.Graph()  # or MultiGraph, etc
>>> G.add_edge(0, 1)
>>> H = G.to_directed()
>>> list(H.edges)
[(0, 1), (1, 0)]
```

If already directed, return a (deep) copy

```python
>>> G = nx.DiGraph()  # or MultiDiGraph, etc
>>> G.add_edge(0, 1)
>>> H = G.to_directed()
>>> list(H.edges)
[(0, 1)]
```

networkx.DiGraph.subgraph

**DiGraph.subgraph(nodes)**

Returns a SubGraph view of the subgraph induced on `nodes`.

The induced subgraph of the graph contains the nodes in `nodes` and the edges between those nodes.

**Parameters**

- `nodes` *(list, iterable)* – A container of nodes which will be iterated through once.

**Returns**

- `G` – A subgraph view of the graph. The graph structure cannot be changed but node/edge attributes can and are shared with the original graph.

**Return type**

SubGraph View

**Notes**

The graph, edge and node attributes are shared with the original graph. Changes to the graph structure is ruled out by the view, but changes to attributes are reflected in the original graph.

To create a subgraph with its own copy of the edge/node attributes use: `G.subgraph(nodes).copy()`

For an inplace reduction of a graph to a subgraph you can remove nodes: `G.remove_nodes_from([n for n in G if n not in set(nodes)])`

Subgraph views are sometimes NOT what you want. In most cases where you want to do more than simply look at the induced edges, it makes more sense to just create the subgraph as its own graph with code like:

```python
# Create a subgraph SG based on a (possibly multigraph) G
SG = G.__class__()  # Create an empty Graph
SG.add_nodes_from((n, G.nodes[n]) for n in largest_wcc)
if SG.is_multigraph():
    SG.add_edges_from((n, nbr, key, d)
                      for n, nbrs in G.adj.items() if n in largest_wcc
                      for nbr, keydict in nbrs.items() if nbr in largest_wcc
                      for key, d in keydict.items())
else:
    SG.add_edges_from((n, nbr, d)
                       for n, nbrs in G.adj.items() if n in largest_wcc
                       for nbr, d in nbrs.items() if nbr in largest_wcc)
SG.graph.update(G.graph)
```
Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> H = G.subgraph([0, 1, 2])
>>> list(H.edges)
[(0, 1), (1, 2)]
```

networkx.DiGraph.edge_subgraph

DiGraph.edge_subgraph(edges)

Returns the subgraph induced by the specified edges.

The induced subgraph contains each edge in `edges` and each node incident to any one of those edges.

Parameters
edges (iterable) – An iterable of edges in this graph.

Returns
G – An edge-induced subgraph of this graph with the same edge attributes.

Return type
Graph

Notes

The graph, edge, and node attributes in the returned subgraph view are references to the corresponding attributes in the original graph. The view is read-only.

To create a full graph version of the subgraph with its own copy of the edge or node attributes, use:

```python
>>> G.edge_subgraph(edges).copy()
```

Examples

```python
>>> G = nx.path_graph(5)
>>> H = G.edge_subgraph([(0, 1), (3, 4)])
>>> list(H.nodes)
[0, 1, 3, 4]
>>> list(H.edges)
[(0, 1), (3, 4)]
```

networkx.DiGraph.reverse

DiGraph.reverse(copy=True)

Returns the reverse of the graph.

The reverse is a graph with the same nodes and edges but with the directions of the edges reversed.

Parameters

`copy` (bool optional (default=True)) – If True, return a new DiGraph holding the reversed edges. If False, the reverse graph is created using a view of the original graph.
2.2.3 MultiGraph—Undirected graphs with self loops and parallel edges

Overview

class MultiGraph(incoming_graph_data=None, **attr)
    An undirected graph class that can store multiedges.
    Multiedges are multiple edges between two nodes. Each edge can hold optional data or attributes.
    A MultiGraph holds undirected edges. Self loops are allowed.
    Nodes can be arbitrary (hashable) Python objects with optional key/value attributes. By convention None is not used as a node.
    Edges are represented as links between nodes with optional key/value attributes.

Parameters

    • incoming_graph_data (input graph (optional, default: None)) – Data to initialize graph. If None (default) an empty graph is created. The data can be any format that is supported by the to_networkx_graph() function, currently including edge list, dict of dicts, dict of lists, NetworkX graph, NumPy matrix or 2d ndarray, SciPy sparse matrix, or PyGraphviz graph.

    • attr (keyword arguments, optional (default= no attributes)) – Attributes to add to graph as key=value pairs.

See also:

Graph, DiGraph, MultiDiGraph, OrderedMultiGraph

Examples

Create an empty graph structure (a “null graph”) with no nodes and no edges.

```python
>>> G = nx.MultiGraph()
```

G can be grown in several ways.

Nodes:
Add one node at a time:

```python
>>> G.add_node(1)
```

Add the nodes from any container (a list, dict, set or even the lines from a file or the nodes from another graph).

```python
>>> G.add_nodes_from([2, 3])
>>> G.add_nodes_from(range(100, 110))
>>> H = nx.path_graph(10)
>>> G.add_nodes_from(H)
```

In addition to strings and integers any hashable Python object (except None) can represent a node, e.g. a customized node object, or even another Graph.

```python
>>> G.add_node(H)
```

Edges:
G can also be grown by adding edges.
Add one edge,
>>> key = G.add_edge(1, 2)

a list of edges,

>>> keys = G.add_edges_from(((1, 2), (1, 3)))

or a collection of edges,

>>> keys = G.add_edges_from(H.edges)

If some edges connect nodes not yet in the graph, the nodes are added automatically. If an edge already exists, an additional edge is created and stored using a key to identify the edge. By default the key is the lowest unused integer.

>>> keys = G.add_edges_from(((4,5,{'route':28}), (4,5,{'route':37})))

>>> G[4]
AdjacencyView({3: {0: {}}, 5: {0: {}, 1: {'route': 28}, 2: {'route': 37}}})

Attributes:

Each graph, node, and edge can hold key/value attribute pairs in an associated attribute dictionary (the keys must be hashable). By default these are empty, but can be added or changed using add_edge, add_node or direct manipulation of the attribute dictionaries named graph, node and edge respectively.

>>> G = nx.MultiGraph(day="Friday")
>>> G.graph
{'day': 'Friday'}

Add node attributes using add_node(), add_nodes_from() or G.nodes

>>> G.add_node(1, time='5pm')
>>> G.add_nodes_from([3], time='2pm')

>>> G.nodes[1]
{'time': '5pm'}

>>> del G.nodes[1]["room"]  # remove attribute

>>> list(G.nodes(data=True))
[(1, {'time': '5pm'}), (3, {'time': '2pm'})]

Add edge attributes using add_edge(), add_edges_from(), subscript notation, or G.edges.

>>> key = G.add_edge(1, 2, weight=4.7)

>>> keys = G.add_edges_from(((3, 4), (4, 5), color='red'))

>>> keys = G.add_edges_from(((1,2,{'color':"blue"}), (2,3,{'weight':8})))

>>> G[1][2][0]['weight'] = 4.7

Warning: we protect the graph data structure by making G.edges[1, 2] a read-only dict-like structure. However, you can assign to attributes in e.g. G.edges[1, 2]. Thus, use 2 sets of brackets to add/change data attributes: G.edges[1, 2][\'weight\'] = 4 (For multigraphs: MG.edges[u, v, key][name] = value).

Shortcuts:

Many common graph features allow python syntax to speed reporting.
Often the best way to traverse all edges of a graph is via the neighbors. The neighbors are reported as an adjacency-dict `G.adj` or `G.adjacency()`.

```python
>>> for n, nbrsdict in G.adjacency():
...    for nbr, keydict in nbrsdict.items():
...        for key, eattr in keydict.items():
...            if 'weight' in eattr:
...                # Do something useful with the edges
...                pass
```

But the `edges()` method is often more convenient:

```python
>>> for u, v, keys, weight in G.edges(data='weight', keys=True):
...    if weight is not None:
...        # Do something useful with the edges
...        pass
```

Reporting:

Simple graph information is obtained using methods and object-attributes. Reporting usually provides views instead of containers to reduce memory usage. The views update as the graph is updated similarly to dict-views. The objects `nodes`, `edges` and `adj` provide access to data attributes via lookup (e.g. `nodes[n]`, `edges[u, v]`, `adj[u][v]`) and iteration (e.g. `nodes.items()`, `nodes.data('color')`, `nodes.data('color', default='blue')`) and similarly for `edges` Views exist for `nodes`, `edges`, `neighbors()`/`adj` and `degree`.

For details on these and other miscellaneous methods, see below.

Subclasses (Advanced):

The MultiGraph class uses a dict-of-dict-of-dict-of-dict data structure. The outer dict (`node_dict`) holds adjacency information keyed by node. The next dict (`adjlist_dict`) represents the adjacency information and holds edge_key dicts keyed by neighbor. The edge_key dict holds each edge_attr dict keyed by edge key. The inner dict (`edge_attr_dict`) represents the edge data and holds edge attribute values keyed by attribute names.

Each of these four dicts in the dict-of-dict-of-dict-of-dict structure can be replaced by a user defined dict-like object. In general, the dict-like features should be maintained but extra features can be added. To replace one of the dicts create a new graph class by changing the class(!) variable holding the factory for that dict-like structure. The variable names are `node_dict_factory`, `node_attr_dict_factory`, `adjlist_inner_dict_factory`, `adjlist_outer_dict_factory`, `edge_key_dict_factory`, `edge_attr_dict_factory` and `graph_attr_dict_factory`.

- **node_dict_factory** [function, (default: dict)] Factory function to be used to create the dict containing node attributes, keyed by node id. It should require no arguments and return a dict-like object
- **node_attr_dict_factory**: function, (default: dict) Factory function to be used to create the node attribute dict which holds attribute values keyed by attribute name. It should require no arguments and return a dict-like object
- **adjlist_outer_dict_factory** [function, (default: dict)] Factory function to be used to create the outer-most dict in the data structure that holds adjacency info keyed by node. It should require no arguments and return a
dict-like object.

adjlist_inner_dict_factory  [function, (default: dict)] Factory function to be used to create the adjacency list
dict which holds multiedge key dicts keyed by neighbor. It should require no arguments and return a
dict-like object.

edge_key_dict_factory  [function, (default: dict)] Factory function to be used to create the edge key dict which
holds edge data keyed by edge key. It should require no arguments and return a dict-like object.

edge_attr_dict_factory  [function, (default: dict)] Factory function to be used to create the edge attribute dict
which holds attribute values keyed by attribute name. It should require no arguments and return a dict-like
object.

graph_attr_dict_factory  [function, (default: dict)] Factory function to be used to create the graph attribute
dict which holds attribute values keyed by attribute name. It should require no arguments and return a
dict-like object.

Typically, if your extension doesn’t impact the data structure all methods will inherited without issue except:
to_directed/to_undirected. By default these methods create a DiGraph/Graph class and you probably
want them to create your extension of a DiGraph/Graph. To facilitate this we define two class variables that you
can set in your subclass.

to_directed_class  [callable, (default: DiGraph or MultiDiGraph)] Class to create a new graph structure in the

Examples

Please see ordered for examples of creating graph subclasses by overwriting the base class dict with a
dictionary-like object.

Methods

Adding and removing nodes and edges

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MultiGraph.clear()  Remove all nodes and edges from the graph.

networkx.MultiGraph.__init__

MultiGraph.__init__(incoming_graph_data=None, **attr)
Initialize a graph with edges, name, or graph attributes.

Parameters

- **incoming_graph_data** (input graph) – Data to initialize graph. If incoming_graph_data=None (default) an empty graph is created. The data can be an edge list, or any NetworkX graph object. If the corresponding optional Python packages are installed the data can also be a NumPy matrix or 2d ndarray, a SciPy sparse matrix, or a PyGraphviz graph.

- **attr** (keyword arguments, optional (default= no attributes)) – Attributes to add to graph as key=value pairs.

See also:

convert()

Examples

```python
>>> G = nx.Graph()        # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G = nx.Graph(name='my graph')
>>> e = [(1, 2), (2, 3), (3, 4)]  # list of edges
>>> G = nx.Graph(e)

Arbitrary graph attribute pairs (key=value) may be assigned
```

```python
>>> G = nx.Graph(e, day="Friday")
>>> G.graph
{'day': 'Friday'}
```

networkx.MultiGraph.add_node

MultiGraph.add_node(node_for_adding, **attr)
Add a single node node_for_adding and update node attributes.

Parameters

- **node_for_adding** (node) – A node can be any hashable Python object except None.

- **attr** (keyword arguments, optional) – Set or change node attributes using key=value.

See also:

add_nodes_from()

Examples

```python
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_node(1)
>>> G.add_node('Hello')
>>> K3 = nx.Graph([(0, 1), (1, 2), (2, 0)])
>>> G.add_node(K3)
>>> G.number_of_nodes()
3

Use keywords set/change node attributes:

>>> G.add_node(1, size=10)
>>> G.add_node(3, weight=0.4, UTM=('13S', 382871, 3972649))

Notes

A hashable object is one that can be used as a key in a Python dictionary. This includes strings, numbers, tuples of strings and numbers, etc.

On many platforms hashable items also include mutables such as NetworkX Graphs, though one should be careful that the hash doesn’t change on mutables.

**networkx.MultiGraph.add_nodes_from**

MultiGraph.add_nodes_from(nodes_for_adding, **attr)**

Add multiple nodes.

Parameters

- **nodes_for_adding** (iterable container) – A container of nodes (list, dict, set, etc.). OR A container of (node, attribute dict) tuples. Node attributes are updated using the attribute dict.
- **attr** (keyword arguments, optional (default= no attributes)) – Update attributes for all nodes in nodes. Node attributes specified in nodes as a tuple take precedence over attributes specified via keyword arguments.

See also:

- add_node()

Examples

>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_nodes_from('Hello')
>>> K3 = nx.Graph([(0, 1), (1, 2), (2, 0)])
>>> G.add_nodes_from(K3)
>>> sorted(G.nodes(), key=str)
[0, 1, 2, 'H', 'e', 'l', 'o']

Use keywords to update specific node attributes for every node.

>>> G.add_nodes_from([(1, 2), size=10])
>>> G.add_nodes_from([(3, 4), weight=0.4])

Use (node, attrdict) tuples to update attributes for specific nodes.
```python
>>> G.add_nodes_from(((1, dict(size=11)), (2, {'color': 'blue'})))
>>> G.nodes[1]['size']
11
>>> H = nx.Graph()
>>> H.add_nodes_from(G.nodes(data=True))
>>> H.nodes[1]['size']
11
```

```text
networkx.MultiGraph.remove_node

MultiGraph.remove_node(n)
Remove node n.
Removes the node n and all adjacent edges. Attempting to remove a non-existent node will raise an exception.

Parameters n (node) – A node in the graph
Raises NetworkXError – If n is not in the graph.
See also: remove_nodes_from()

Examples
```n
```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> list(G.edges)
[(0, 1), (1, 2)]
>>> G.remove_node(1)
>>> list(G.edges)
[]
```

```text
networkx.MultiGraph.remove_nodes_from

MultiGraph.remove_nodes_from(nodes)
Remove multiple nodes.

Parameters nodes (iterable container) – A container of nodes (list, dict, set, etc.). If a node in the container is not in the graph it is silently ignored.

See also: remove_node()

Examples
```n
```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> e = list(G.nodes)
>>> e
[0, 1, 2]
>>> G.remove_nodes_from(e)
>>> list(G.nodes)
[]
```
networkx.MultiGraph.add_edge

MultiGraph.add_edge(u_for_edge, v_for_edge, key=None, **attr)

Add an edge between u and v.

The nodes u and v will be automatically added if they are not already in the graph.

Edge attributes can be specified with keywords or by directly accessing the edge’s attribute dictionary. See examples below.

Parameters

• u_for_edge, v_for_edge (nodes) – Nodes can be, for example, strings or numbers. Nodes must be hashable (and not None) Python objects.

• key (hashable identifier, optional (default=lowest unused integer)) – Used to distinguish multiedges between a pair of nodes.

• attr (keyword arguments, optional) – Edge data (or labels or objects) can be assigned using keyword arguments.

Returns

Return type The edge key assigned to the edge.

See also:

add_edges_from() add a collection of edges

Notes

To replace/update edge data, use the optional key argument to identify a unique edge. Otherwise a new edge will be created.

NetworkX algorithms designed for weighted graphs cannot use multigraphs directly because it is not clear how to handle multiedge weights. Convert to Graph using edge attribute ‘weight’ to enable weighted graph algorithms.

Default keys are generated using the method new_edge_key(). This method can be overridden by subclassing the base class and providing a custom new_edge_key() method.

Examples

The following all add the edge e=(1, 2) to graph G:

```python
>>> G = nx.MultiGraph()
>>> e = (1, 2)
>>> ekey = G.add_edge(1, 2)  # explicit two-node form
>>> G.add_edge(*e)  # single edge as tuple of two nodes
1
>>> G.add_edges_from([(1, 2)])  # add edges from iterable container
[2]
```

Associate data to edges using keywords:

```python
>>> ekey = G.add_edge(1, 2, weight=3)
>>> ekey = G.add_edge(1, 2, key=0, weight=4)  # update data for key=0
>>> ekey = G.add_edge(1, 3, weight=7, capacity=15, length=342.7)
```
For non-string attribute keys, use subscript notation.

```python
>>> ekey = G.add_edge(1, 2)
>>> G[1][2][0].update({0: 5})
>>> G.edges[1, 2, 0].update({0: 5})
```

**networkx.MultiGraph.add_edges_from**

`MultiGraph.add_edges_from(ebunch_to_add, **attr)`

Add all the edges in `ebunch_to_add`.

**Parameters**

- **ebunch_to_add** *(container of edges)* – Each edge given in the container will be added to the graph. The edges can be:
  - 2-tuples `(u, v)` or
  - 3-tuples `(u, v, d)` for an edge data dict `d`, or
  - 3-tuples `(u, v, k)` for not iterable key `k`, or
  - 4-tuples `(u, v, k, d)` for an edge with data and key `k`
- **attr** *(keyword arguments, optional)* – Edge data (or labels or objects) can be assigned using keyword arguments.

**Returns**

A list of edge keys assigned to the edges in `ebunch`.

**See also:**

- `add_edge()` add a single edge
- `add_weighted_edges_from()` convenient way to add weighted edges

**Notes**

Adding the same edge twice has no effect but any edge data will be updated when each duplicate edge is added. Edge attributes specified in an `ebunch` take precedence over attributes specified via keyword arguments.

Default keys are generated using the method `new_edge_key()`. This method can be overridden by subclassing the base class and providing a custom `new_edge_key()` method.

**Examples**

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edges_from([(0, 1), (1, 2)])  # using a list of edge tuples
>>> e = zip(range(0, 3), range(1, 4))
>>> G.add_edges_from(e)  # Add the path graph 0-1-2-3

Associate data to edges

```
networkx.MultiGraph.add_weighted_edges_from

MultiGraph.add_weighted_edges_from(ebunch_to_add, weight='weight', **attr)
Add weighted edges in ebunch_to_add with specified weight attr

Parameters

- ebunch_to_add (container of edges) – Each edge given in the list or container will be added to the graph. The edges must be given as 3-tuples (u, v, w) where w is a number.
- weight (string, optional (default= 'weight')) – The attribute name for the edge weights to be added.
- attr (keyword arguments, optional (default= no attributes)) – Edge attributes to add/update for all edges.

See also:

add_edge() add a single edge
add_edges() add multiple edges

Notes

Adding the same edge twice for Graph/DiGraph simply updates the edge data. For MultiGraph/MultiDiGraph, duplicate edges are stored.

Examples

```python
>>> G = nx.Graph() # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_weighted_edges_from([(0, 1, 3.0), (1, 2, 7.5)])
```

networkx.MultiGraph.new_edge_key

MultiGraph.new_edge_key(u, v)
Returns an unused key for edges between nodes u and v.
The nodes u and v do not need to be already in the graph.

Notes

In the standard MultiGraph class the new key is the number of existing edges between u and v (increased if necessary to ensure unused). The first edge will have key 0, then 1, etc. If an edge is removed further new_edge_keys may not be in this order.

Parameters u, v (nodes)

Returns key

Return type int
networkx.MultiGraph.remove_edge

MultiGraph.remove_edge(\(u, v, \text{key}=\text{None}\))

Remove an edge between \(u\) and \(v\).

Parameters

- \(u, v\) (nodes) – Remove an edge between nodes \(u\) and \(v\).
- \(\text{key}\) (hashable identifier, optional (default=\text{None})) – Used to distinguish multiple edges between a pair of nodes. If None remove a single (arbitrary) edge between \(u\) and \(v\).

Raises

- NetworkXError – If there is not an edge between \(u\) and \(v\), or if there is no edge with the specified key.

See also:

- remove_edges_from()  remove a collection of edges

Examples

```python
>>> G = nx.MultiGraph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.remove_edge(0, 1)
>>> e = (1, 2)
>>> G.remove_edge(*e)  # unpacks e from an edge tuple
```

For multiple edges

```python
>>> G = nx.MultiGraph()  # or MultiDiGraph, etc
>>> G.add_edges_from(((1, 2), (1, 2), (1, 2)))  # key_list returned
[0, 1, 2]
>>> G.remove_edge(1, 2)  # remove a single (arbitrary) edge
```

For edges with keys

```python
>>> G = nx.MultiGraph()  # or MultiDiGraph, etc
>>> G.add_edge(1, 2, key='first')
'first'
>>> G.add_edge(1, 2, key='second')
'second'
>>> G.remove_edge(1, 2, key='second')
```

networkx.MultiGraph.remove_edges_from

MultiGraph.remove_edges_from(\(\text{ebunch}\))

Remove all edges specified in \(\text{ebunch}\).

Parameters \(\text{ebunch}\) (list or container of edge tuples) – Each edge given in the list or container will be removed from the graph. The edges can be:

- 2-tuples \((u, v)\) All edges between \(u\) and \(v\) are removed.
- 3-tuples \((u, v, \text{key})\) The edge identified by key is removed.
- 4-tuples \((u, v, \text{key}, \text{data})\) where data is ignored.

See also:
remove_edge()  remove a single edge

Notes
Will fail silently if an edge in ebunch is not in the graph.

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> ebunch=[(1, 2), (2, 3)]
>>> G.remove_edges_from(ebunch)
```

Removing multiple copies of edges

```python
>>> G = nx.MultiGraph()
>>> keys = G.add_edges_from([(1, 2), (1, 2), (1, 2)])
>>> G.remove_edges_from([(1, 2), (1, 2)])
>>> list(G.edges())
[(1, 2)]
>>> G.remove_edges_from([(1, 2), (1, 2)])  # silently ignore extra copy
>>> list(G.edges)  # now empty graph
[]
```

networkx.MultiGraph.update

MultiGraph.update(edges=None, nodes=None)
Update the graph using nodes/edges/graphs as input.

Like dict.update, this method takes a graph as input, adding the graph’s nodes and edges to this graph. It can also take two inputs: edges and nodes. Finally it can take either edges or nodes. To specify only nodes the keyword nodes must be used.

The collections of edges and nodes are treated similarly to the add_edges_from/add_nodes_from methods. When iterated, they should yield 2-tuples (u, v) or 3-tuples (u, v, datadict).

Parameters

- **edges** (Graph object, collection of edges, or None) – The first parameter can be a graph or some edges. If it has attributes nodes and edges, then it is taken to be a Graph-like object and those attributes are used as collections of nodes and edges to be added to the graph. If the first parameter does not have those attributes, it is treated as a collection of edges and added to the graph. If the first argument is None, no edges are added.

- **nodes** (collection of nodes, or None) – The second parameter is treated as a collection of nodes to be added to the graph unless it is None. If edges is None and nodes is None an exception is raised. If the first parameter is a Graph, then nodes is ignored.

Examples

```python
>>> G = nx.path_graph(5)
>>> G.update(nx.complete_graph(range(4,10)))
>>> from itertools import combinations
>>> edges = ((u, v, {'power': u * v})
```

(continues on next page)
... for u, v in combinations(range(10, 20), 2)
... if u * v < 225

>>> nodes = [1000]  # for singleton, use a container

>>> G.update(edges, nodes)

Notes

If you want to update the graph using an adjacency structure it is straightforward to obtain the edges/nodes from adjacency. The following examples provide common cases, your adjacency may be slightly different and require tweaks of these examples.

```python
>>> # dict-of-set/list/tuple
>>> adj = {1: {2, 3}, 2: {1, 3}, 3: {1, 2}}
>>> e = [(u, v) for u, nbrs in adj.items() for v in nbrs]

>>> DG = nx.DiGraph()
>>> # dict-of-dict-of-attribute
>>> adj = {1: {2: 1.3, 3: 0.7}, 2: {1: 1.4}, 3: {1: 0.7}}
>>> e = [(u, v, {'weight': d}) for u, nbrs in adj.items() for v, d in nbrs.items()]

>>> # predecessor adjacency (dict-of-set)
>>> pred = {1: {2, 3}, 2: {3}, 3: {3}}
>>> e = [(v, u) for u, nbrs in pred.items() for v in nbrs]

>>> # MultiGraph dict-of-dict-of-dict-of-attribute
>>> MDG = nx.MultiDiGraph()
>>> adj = {1: {2: {0: {'weight': 1.3}, 1: {'weight': 1.2}}},
... 3: {2: {0: {'weight': 0.7}}}}
>>> e = [(u, v, ekey, d) for u, nbrs in adj.items()
...  for v, keydict in nbrs.items()
...  for ekey, d in keydict.items()]

>>> MDG.update(edges=e)
```

See also:

- `add_edges_from()` add multiple edges to a graph
- `add_nodes_from()` add multiple nodes to a graph

networkx.MultiGraph.clear

MultiGraph.clear()

Remove all nodes and edges from the graph.
This also removes the name, and all graph, node, and edge attributes.

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.clear()
>>> list(G.nodes)
[]
>>> list(G.edges)
[]
```

Reporting nodes edges and neighbors

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<tr>
<td><code>MultiGraph.has_node(n)</code></td>
<td>Returns True if the graph contains the node n.</td>
</tr>
<tr>
<td><code>MultiGraph.__contains__(n)</code></td>
<td>Returns True if n is a node, False otherwise.</td>
</tr>
<tr>
<td><code>MultiGraph.edges</code></td>
<td>Returns an iterator over the edges.</td>
</tr>
<tr>
<td><code>MultiGraph.has_edge(u, v[, key])</code></td>
<td>Returns True if the graph has an edge between nodes u and v.</td>
</tr>
<tr>
<td><code>MultiGraph.get_edge_data(u, v[, key, default])</code></td>
<td>Returns the attribute dictionary associated with edge (u, v).</td>
</tr>
<tr>
<td><code>MultiGraph.neighbors(n)</code></td>
<td>Returns an iterator over all neighbors of node n.</td>
</tr>
<tr>
<td><code>MultiGraph.adj</code></td>
<td>Graph adjacency object holding the neighbors of each node.</td>
</tr>
<tr>
<td><code>MultiGraph.__getitem__(n)</code></td>
<td>Returns a dict of neighbors of node n.</td>
</tr>
<tr>
<td><code>MultiGraph.adjacency()</code></td>
<td>Returns an iterator over (node, adjacency dict) tuples for all nodes.</td>
</tr>
<tr>
<td><code>MultiGraph.nbunch_iter([nbunch])</code></td>
<td>Returns an iterator over nodes contained in nbunch that are also in the graph.</td>
</tr>
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</table>

**networkx.MultiGraph.nodes**

**property** `MultiGraph.nodes`

A NodeView of the Graph as G.nodes or G.nodes().

Can be used as `G.nodes` for data lookup and for set-like operations. Can also be used as `G.nodes(data='color', default=None)` to return a `NodeDataView` which reports specific node data but no set operations. It presents a dict-like interface as well with `G.nodes.items()` iterating over (node, nodedata) 2-tuples and `G.nodes[3]['foo']` providing the value of the foo attribute for node 3. In addition, a view `G.nodes.data('foo')` provides a dict-like interface to the foo attribute of each node. `G.nodes.data('foo', default=1)` provides a default for nodes that do not have attribute foo.

**Parameters**

- **data (string or bool, optional (default=False))** – The node attribute returned in 2-tuple (n, ddict[data]). If True, return entire node attribute dict as (n, ddict). If False, return just the nodes n.
- **default (value, optional (default=None))** – Value used for nodes that don’t have the requested attribute. Only relevant if data is not True or False.

**Returns** 2.2. Basic graph types 79
Allows set-like operations over the nodes as well as node attribute dict lookup and calling to get a NodeDataView. A NodeDataView iterates over \((n, \text{data})\) and has no set operations. A NodeView iterates over \(n\) and includes set operations.

When called, if data is False, an iterator over nodes. Otherwise an iterator of 2-tuples (node, attribute value) where the attribute is specified in data. If data is True then the attribute becomes the entire data dictionary.

**Return type** NodeView

**Notes**

If your node data is not needed, it is simpler and equivalent to use the expression `for n in G` or `list(G)`.

**Examples**

There are two simple ways of getting a list of all nodes in the graph:

```
>>> G = nx.path_graph(3)
>>> list(G.nodes)
[0, 1, 2]
>>> list(G)
[0, 1, 2]
```

To get the node data along with the nodes:

```
>>> G.add_node(1, time='5pm')
>>> G.nodes[0]['foo'] = 'bar'
>>> list(G.nodes(data=True))
[(0, {'foo': 'bar'}), (1, {'time': '5pm'}), (2, {})]
>>> list(G.nodes.data())
[(0, {'foo': 'bar'}), (1, {'time': '5pm'}), (2, {})]
```

```
>>> list(G.nodes(data='foo'))
[(0, 'bar'), (1, None), (2, None)]
>>> list(G.nodes.data('foo'))
[(0, 'bar'), (1, None), (2, None)]
```

```
>>> list(G.nodes(data='time'))
[(0, None), (1, '5pm'), (2, None)]
>>> list(G.nodes.data('time'))
[(0, None), (1, '5pm'), (2, None)]
```

```
>>> list(G.nodes(data='time', default='Not Available'))
[(0, 'Not Available'), (1, '5pm'), (2, 'Not Available')]
```

```
>>> list(G.nodes.data('time', default='Not Available'))
[(0, 'Not Available'), (1, '5pm'), (2, 'Not Available')]
```

If some of your nodes have an attribute and the rest are assumed to have a default attribute value you can create a dictionary from node/attribute pairs using the `default` keyword argument to guarantee the value is never None:

```
>>> G = nx.Graph()
>>> G.add_node(0)
>>> G.add_node(1, weight=2)
(continues on next page)```
networkx.MultiGraph.__iter__

MultiGraph.__iter__()  
Iterate over the nodes. Use: ‘for n in G’.

Returns niter – An iterator over all nodes in the graph.

Return type iterator

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [n for n in G]
[0, 1, 2, 3]
>>> list(G)
[0, 1, 2, 3]
```

networkx.MultiGraph.has_node

MultiGraph.has_node(n)  
Returns True if the graph contains the node n.

Identical to n in G

Parameters n (node)

Examples

```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.has_node(0)
True
```

It is more readable and simpler to use

```python
>>> 0 in G
True
```

networkx.MultiGraph.__contains__

MultiGraph.__contains__(n)  
Returns True if n is a node, False otherwise. Use: ‘n in G’.

2.2. Basic graph types
Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> 1 in G
True
```

**networkx.MultiGraph.edges**

```
property MultiGraph.edges

Returns an iterator over the edges.
```

```
edges(self, nbunch=None, data=False, keys=False, default=None)

The EdgeView provides set-like operations on the edge-tuples as well as edge attribute lookup. When called, it also provides an EdgeDataView object which allows control of access to edge attributes (but does not provide set-like operations). Hence, G.edges[u, v]['color'] provides the value of the color attribute for edge (u, v) while for (u, v, c) in G.edges(data='color', default='red'): iterates through all the edges yielding the color attribute.

Edges are returned as tuples with optional data and keys in the order (node, neighbor, key, data).

**Parameters**

- **nbunch** *(single node, container, or all nodes (default= all nodes)) –* The view will only report edges incident to these nodes.

- **data** *(string or bool, optional (default=False)) –* The edge attribute returned in 3-tuple (u, v, ddict[data]). If True, return edge attribute dict in 3-tuple (u, v, ddict). If False, return 2-tuple (u, v).

- **keys** *(bool, optional (default=False)) –* If True, return edge keys with each edge.

- **default** *(value, optional (default=None)) –* Value used for edges that don’t have the requested attribute. Only relevant if data is not True or False.

**Returns edges** – A view of edge attributes, usually it iterates over (u, v) (u, v, k) or (u, v, k, d) tuples of edges, but can also be used for attribute lookup as edges[u, v, k]['foo'].

```
Return type MultiEdgeView
```

**Notes**

Nodes in nbunch that are not in the graph will be (quietly) ignored. For directed graphs this returns the out-edges.

**Examples**

```python
>>> G = nx.MultiGraph()  # or MultiDiGraph
>>> nx.add_path(G, [0, 1, 2])
>>> key = G.add_edge(2, 3, weight=5)
>>> [e for e in G.edges()]
[(0, 1), (1, 2), (2, 3)]
>>> G.edges.data()  # default data is {} (empty dict)
MultiEdgeDataView([(0, 1, {}), (1, 2, {}), (2, 3, {'weight': 5})])
>>> G.edges.data('weight', default=1)
MultiEdgeDataView([(0, 1, 1), (1, 2, 1), (2, 3, 5)])
>>> G.edges(keys=True)  # default keys are integers
```
MultiEdgeView([(0, 1, 0), (1, 2, 0), (2, 3, 0)])
>>> G.edges.data(keys=True)
MultiEdgeDataView([(0, 1, 0), (1, 2, 0), (2, 3, 0)])
>>> G.edges['weight', default=1, keys=True)
MultiEdgeDataView([(0, 1, 0, {}), (1, 2, 0, {}), (2, 3, 0, {'weight': 5})])
>>> G.edges([0, 3])
MultiEdgeDataView([(0, 1), (3, 2)])
>>> G.edges(0)
MultiEdgeDataView([(0, 1)])

**networkx.MultiGraph.has_edge**

MultiGraph.has_edge(u, v, key=None)

Returns True if the graph has an edge between nodes u and v.

This is the same as v in G[u] or key in G[u][v] without KeyError exceptions.

**Parameters**

- **u, v** *(nodes)* – Nodes can be, for example, strings or numbers.
- **key** *(hashable identifier, optional (default=None))* – If specified return True only if the edge with key is found.

**Returns** edge_ind – True if edge is in the graph, False otherwise.

**Return type** bool

**Examples**

Can be called either using two nodes u, v, an edge tuple (u, v), or an edge tuple (u, v, key).

```python
>>> G = nx.MultiGraph()  # or MultiDiGraph
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.has_edge(0, 1)  # using two nodes
True
>>> e = (0, 1)
>>> G.has_edge(*e)  # e is a 2-tuple (u, v)
True
>>> G.add_edge(0, 1, key='a')
'a'
>>> G.has_edge(0, 1, key='a')  # specify key
True
>>> e=(0, 1, 'a')
>>> G.has_edge(*e)  # e is a 3-tuple (u, v, 'a')
True
```

The following syntax are equivalent:

```python
>>> G.has_edge(0, 1)
True
>>> 1 in G[0]  # though this gives :exc:`KeyError` if 0 not in G
True
```
networkx.MultiGraph.get_edge_data

`MultiGraph.get_edge_data(u, v, key=None, default=None)`

Returns the attribute dictionary associated with edge (u, v).

This is identical to `G[u][v][key]` except the default is returned instead of an exception if the edge doesn’t exist.

Parameters

- `u, v (nodes)`
- `default (any Python object (default=None))` – Value to return if the edge (u, v) is not found.
- `key (hashable identifier, optional (default=None))` – Return data only for the edge with specified key.

Returns `edge_dict` – The edge attribute dictionary.

Return type dictionary

Examples

```python
>>> G = nx.MultiGraph()  # or MultiDiGraph
>>> key = G.add_edge(0, 1, key='a', weight=7)
>>> G[0][1]['a']  # key='a'
{'weight': 7}
>>> G.edges[0, 1, 'a']  # key='a'
{'weight': 7}
```

Warning: we protect the graph data structure by making `G.edges` and `G[1][2]` read-only dict-like structures. However, you can assign values to attributes in e.g. `G.edges[1, 2, 'a']` or `G[1][2]['a']` using an additional bracket as shown next. You need to specify all edge info to assign to the edge data associated with an edge.

```python
>>> G[0][1]['a']['weight'] = 10
>>> G.edges[0, 1, 'a']['weight'] = 10
>>> G[0][1]['a']['weight']
10
>>> G.edges[1, 0, 'a']['weight']
10
```

```python
>>> G = nx.MultiGraph()  # or MultiDiGraph
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.get_edge_data(0, 1)
{0: {}}
>>> e = (0, 1)
>>> G.get_edge_data(e)  # tuple form
{0: {}}
>>> G.get_edge_data('a', 'b', default=0)  # edge not in graph, return 0
0
```

networkx.MultiGraph.neighbors

`MultiGraph.neighbors(n)`

Returns an iterator over all neighbors of node n.
This is identical to \texttt{iter(G[n])}

\textbf{Parameters} \texttt{n (node)} – A node in the graph
\textbf{Returns} \texttt{neighbors} – An iterator over all neighbors of node \texttt{n}
\textbf{Return type} \texttt{iterator}
\textbf{Raises} \texttt{NetworkXError} – If the node \texttt{n} is not in the graph.

\section*{Examples}

\begin{verbatim}
>>> G = nx.path_graph(4)   # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [n for n in G.neighbors(0)]
[1]
\end{verbatim}

\section*{Notes}

Alternate ways to access the neighbors are \texttt{G.adj[n]} or \texttt{G[n]}:

\begin{verbatim}
>>> G = nx.Graph()   # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge('a', 'b', weight=7)
>>> G['a']
AtlasView({'b': {'weight': 7}})
>>> G = nx.path_graph(4)
>>> [n for n in G[0]]
[1]
\end{verbatim}

\texttt{networkx.MultiGraph.adj}

\textbf{property} \texttt{MultiGraph.adj}

Graph adjacency object holding the neighbors of each node.

This object is a read-only dict-like structure with node keys and neighbor-dict values. The neighbor-dict is keyed by neighbor to the edgekey-data-dict. So \texttt{G.adj[3][2][0]['color']} = 'blue' sets the color of the edge (3, 2, 0) to "blue".

Iterating over \texttt{G.adj} behaves like a dict. Useful idioms include \texttt{for nbr, nbrdict in G.adj\[n\].items()}.

The neighbor information is also provided by subscripting the graph. So \texttt{for nbr, foovalue in G[\texttt{node}].data('foo', default=1):} works.

For directed graphs, \texttt{G.adj} holds outgoing (successor) info.

\texttt{networkx.MultiGraph.__getitem__}

\texttt{MultiGraph.__getitem__\,(n)}

Returns a dict of neighbors of node \texttt{n}. Use: ‘\texttt{G[n]’}.

\textbf{Parameters} \texttt{n (node)} – A node in the graph.
\textbf{Returns} \texttt{adj\_dict} – The adjacency dictionary for nodes connected to \texttt{n}.
\textbf{Return type} \texttt{dictionary}

2.2. Basic graph types
Notes

G[n] is the same as G.adj[n] and similar to G.neighbors(n) (which is an iterator over G.adj[n])

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G[0]
AtlasView({1: {}})
```

networkx.MultiGraph.adjacency

MultiGraph.adjacency()

Returns an iterator over (node, adjacency dict) tuples for all nodes.

For directed graphs, only outgoing neighbors/adjacencies are included.

**Returns** adj_iter – An iterator over (node, adjacency dictionary) for all nodes in the graph.

**Return type** iterator

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [(n, nbrdict) for n, nbrdict in G.adjacency()]
[(0, {1: {}}), (1, {0: {}, 2: {}}), (2, {1: {}, 3: {}}), (3, {2: {}})]
```

networkx.MultiGraph.nbunch_iter

MultiGraph.nbunch_iter(nbunch=None)

Returns an iterator over nodes contained in nbunch that are also in the graph.

The nodes in nbunch are checked for membership in the graph and if not are silently ignored.

**Parameters** nbunch (single node, container, or all nodes (default= all nodes)) – The view will only report edges incident to these nodes.

**Returns** niter – An iterator over nodes in nbunch that are also in the graph. If nbunch is None, iterate over all nodes in the graph.

**Return type** iterator

**Raises** NetworkXError – If nbunch is not a node or or sequence of nodes. If a node in nbunch is not hashable.

See also:

Graph.__iter__()
To test whether nbunch is a single node, one can use “if nbunch in self:”, even after processing with this routine. If nbunch is not a node or a (possibly empty) sequence/iterator or None, a `NetworkXError` is raised. Also, if any object in nbunch is not hashable, a `NetworkXError` is raised.

### Counting nodes edges and neighbors

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<td><code>MultiGraph.order()</code></td>
<td>Returns the number of nodes in the graph.</td>
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<tr>
<td><code>MultiGraph.number_of_nodes()</code></td>
<td>Returns the number of nodes in the graph.</td>
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<td><code>MultiGraph.__len__()</code></td>
<td>Returns the number of nodes in the graph.</td>
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<tr>
<td><code>MultiGraph.degree()</code></td>
<td>A DegreeView for the Graph as G.degree or G.degree().</td>
</tr>
<tr>
<td><code>MultiGraph.size([weight])</code></td>
<td>Returns the number of edges or total of all edge weights.</td>
</tr>
<tr>
<td><code>MultiGraph.number_of_edges([u, v])</code></td>
<td>Returns the number of edges between two nodes.</td>
</tr>
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### networkx.MultiGraph.order

**Method**

`MultiGraph.order()`

**Returns**

`nnodes` – The number of nodes in the graph.

**Return type**

`int`

**See also:**

`number_of_nodes()`, `__len__()`

**Examples**

```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.order()
3
```

### networkx.MultiGraph.number_of_nodes

**Method**

`MultiGraph.number_of_nodes()`

**Returns**

`nnodes` – The number of nodes in the graph.

**Return type**

`int`

**See also:**

`order()`, `__len__()`

**Examples**

```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.number_of_nodes()
3
```
networkx.MultiGraph.__len__

MultiGraph.__len__()

Returns the number of nodes in the graph. Use: ‘len(G)’.

Returns nnodes – The number of nodes in the graph.

Return type int

See also: 

number_of_nodes(), order()

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> len(G)
4
```

networkx.MultiGraph.degree

property MultiGraph.degree

A DegreeView for the Graph as G.degree or G.degree().

The node degree is the number of edges adjacent to the node. The weighted node degree is the sum of the edge weights for edges incident to that node.

This object provides an iterator for (node, degree) as well as lookup for the degree for a single node.

Parameters

• nbunch (single node, container, or all nodes (default= all nodes)) – The view will only report edges incident to these nodes.

• weight (string or None, optional (default=None)) – The name of an edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1. The degree is the sum of the edge weights adjacent to the node.

Returns

• If a single node is requested

• deg (int) – Degree of the node, if a single node is passed as argument.

• OR if multiple nodes are requested

• nd_iter (iterator) – The iterator returns two-tuples of (node, degree).

Examples

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.degree(0)  # node 0 with degree 1
1
>>> list(G.degree([0, 1]))
[(0, 1), (1, 2)]
```
networkx.MultiGraph.size

MultiGraph.size(weight=None)

Returns the number of edges or total of all edge weights.

**Parameters**

- weight (string or None, optional (default=None)) – The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.

**Returns**

- size – The number of edges or (if weight keyword is provided) the total weight sum.

If weight is None, returns an int. Otherwise a float (or more general numeric if the weights are more general).

**Return type** numeric

**See also:**

count eds

count( self, u, v=None )

**Examples**

```
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.size()
3
```

```
>>> G = nx.Graph()       # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge('a', 'b', weight=2)
>>> G.add_edge('b', 'c', weight=4)
>>> G.size()
2
>>> G.size(weight='weight')
6.0
```

networkx.MultiGraph.number_of_edges

MultiGraph.number_of_edges(u=None, v=None)

Returns the number of edges between two nodes.

**Parameters**

- u, v (nodes, optional (default=all edges)) – If u and v are specified, return the number of edges between u and v. Otherwise return the total number of all edges.

**Returns**

- nedges – The number of edges in the graph. If nodes u and v are specified return the number of edges between those nodes. If the graph is directed, this only returns the number of edges from u to v.

**Return type** int

**See also:**

count eds

count( self, u, v=None )

**Examples**

For undirected multigraphs, this method counts the total number of edges in the graph:
>>> G = nx.MultiGraph()
>>> G.add_edges_from([(0, 1), (0, 1), (1, 2)])
[0, 1, 0]
>>> G.number_of_edges()
3

If you specify two nodes, this counts the total number of edges joining the two nodes:

>>> G.number_of_edges(0, 1)
2

For directed multigraphs, this method can count the total number of directed edges from \( u \) to \( v \):

>>> G = nx.MultiDiGraph()
>>> G.add_edges_from([(0, 1), (0, 1), (1, 0)])
[0, 1, 0]
>>> G.number_of_edges(0, 1)
2
>>> G.number_of_edges(1, 0)
1

Making copies and subgraphs

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<td>MultiGraph.copy([as_view])</td>
<td>Returns a copy of the graph.</td>
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<tr>
<td>MultiGraph.to_undirected([as_view])</td>
<td>Returns an undirected copy of the graph.</td>
</tr>
<tr>
<td>MultiGraph.to_directed([as_view])</td>
<td>Returns a directed representation of the graph.</td>
</tr>
<tr>
<td>MultiGraph.subgraph(nodes)</td>
<td>Returns a SubGraph view of the subgraph induced on nodes.</td>
</tr>
<tr>
<td>MultiGraph.edge_subgraph(edges)</td>
<td>Returns the subgraph induced by the specified edges.</td>
</tr>
</tbody>
</table>

networkx.MultiGraph.copy

MultiGraph.copy (as_view=False)
Returns a copy of the graph.

The copy method by default returns an independent shallow copy of the graph and attributes. That is, if an attribute is a container, that container is shared by the original an the copy. Use Python’s copy.deepcopy for new containers.

If as_view is True then a view is returned instead of a copy.

Notes

All copies reproduce the graph structure, but data attributes may be handled in different ways. There are four types of copies of a graph that people might want.

Deepcopy – A “deepcopy” copies the graph structure as well as all data attributes and any objects they might contain. The entire graph object is new so that changes in the copy do not affect the original object. (see Python’s copy.deepcopy)

Data Reference (Shallow) – For a shallow copy the graph structure is copied but the edge, node and graph attribute dicts are references to those in the original graph. This saves time and memory but could cause confusion
if you change an attribute in one graph and it changes the attribute in the other. NetworkX does not provide this level of shallow copy.

Independent Shallow – This copy creates new independent attribute dicts and then does a shallow copy of the attributes. That is, any attributes that are containers are shared between the new graph and the original. This is exactly what `dict.copy()` provides. You can obtain this style copy using:

```python
>>> G = nx.path_graph(5)
>>> H = G.copy()
>>> H = G.copy(as_view=False)
>>> H = nx.Graph(G)
>>> H = G.__class__(G)
```

Fresh Data – For fresh data, the graph structure is copied while new empty data attribute dicts are created. The resulting graph is independent of the original and it has no edge, node or graph attributes. Fresh copies are not enabled. Instead use:

```python
>>> H = G.__class__()
>>> H.add_nodes_from(G)
>>> H.add_edges_from(G.edges)
```

View – Inspired by dict-views, graph-views act like read-only versions of the original graph, providing a copy of the original structure without requiring any memory for copying the information.

See the Python copy module for more information on shallow and deep copies, [https://docs.python.org/2/library/copy.html](https://docs.python.org/2/library/copy.html).

**Parameters**

- `as_view` *(bool, optional (default=False)) – If True, the returned graph-view provides a read-only view of the original graph without actually copying any data.*

**Returns**

- `G` – A copy of the graph.

**Return type** `Graph`

See also:

- `to_directed()` return a directed copy of the graph.

**Examples**

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> H = G.copy()
```

**networkx.MultiGraph.to_undirected**

`MultiGraph.to_undirected(as_view=False)`

Returns an undirected copy of the graph.

**Returns**

- `G` – A deepcopy of the graph.

**Return type** `Graph/MultiGraph`

See also:

- `copy()`, `add_edge()`, `add_edges_from()`
Notes

This returns a “deepcopy” of the edge, node, and graph attributes which attempts to completely copy all of the data and references.

This is in contrast to the similar \( G = \text{nx.MultiGraph}(D) \) which returns a shallow copy of the data.

See the Python copy module for more information on shallow and deep copies, https://docs.python.org/2/library/copy.html.

Warning: If you have subclassed MultiGraph to use dict-like objects in the data structure, those changes do not transfer to the MultiGraph created by this method.

Examples

```python
>>> G = nx.path_graph(2)  # or MultiGraph, etc
>>> H = G.to_directed()
>>> list(H.edges)
[(0, 1), (1, 0)]
>>> G2 = H.to_undirected()
>>> list(G2.edges)
[(0, 1)]
```

networkx.MultiGraph.to_directed

**MultiGraph.to_directed**(as_view=False)

Returns a directed representation of the graph.

**Returns** \( G \) – A directed graph with the same name, same nodes, and with each edge \((u, v, \text{data})\) replaced by two directed edges \((u, v, \text{data})\) and \((v, u, \text{data})\).

**Return type** MultiDiGraph

Notes

This returns a “deepcopy” of the edge, node, and graph attributes which attempts to completely copy all of the data and references.

This is in contrast to the similar \( D=\text{DiGraph}(G) \) which returns a shallow copy of the data.

See the Python copy module for more information on shallow and deep copies, https://docs.python.org/2/library/copy.html.

Warning: If you have subclassed MultiGraph to use dict-like objects in the data structure, those changes do not transfer to the MultiDiGraph created by this method.

Examples

```python
>>> G = nx.Graph()  # or MultiGraph, etc
>>> G.add_edge(0, 1)
>>> H = G.to_directed()
>>> list(H.edges)
[(0, 1), (1, 0)]
```

If already directed, return a (deep) copy
>>> G = nx.DiGraph()  # or MultiDiGraph, etc
>>> G.add_edge(0, 1)
>>> H = G.to_directed()
>>> list(H.edges)
[(0, 1)]

networkx.MultiGraph.subgraph

MultiGraph.\texttt{subgraph}(\texttt{nodes})

Returns a SubGraph view of the subgraph induced on \texttt{nodes}.

The induced subgraph of the graph contains the nodes in \texttt{nodes} and the edges between those nodes.

\textbf{Parameters} \texttt{nodes} (\texttt{list, iterable}) – A container of nodes which will be iterated through once.

\textbf{Returns} \texttt{G} – A subgraph view of the graph. The graph structure cannot be changed but node/edge attributes can and are shared with the original graph.

\textbf{Return type} SubGraph View

\textbf{Notes}

The graph, edge and node attributes are shared with the original graph. Changes to the graph structure is ruled out by the view, but changes to attributes are reflected in the original graph.

To create a subgraph with its own copy of the edge/node attributes use: G.subgraph(nodes).copy()

For an inplace reduction of a graph to a subgraph you can remove nodes: G.remove_nodes_from({\texttt{n} for \texttt{n} in \texttt{G} if \texttt{n} not in set(nodes))}

Subgraph views are sometimes NOT what you want. In most cases where you want to do more than simply look at the induced edges, it makes more sense to just create the subgraph as its own graph with code like:

\begin{verbatim}
# Create a subgraph SG based on a (possibly multigraph) G
SG = G.__class__()
SG.add_nodes_from((n, G.nodes[n]) for n in largest_wcc)
if SG.is_multigraph():
    SG.add_edges_from((n, nbr, key, d)
    for n, nbs in G.adj.items() if n in largest_wcc
    for nbr, keydict in nbs.items() if nbr in largest_wcc
    for key, d in keydict.items())
else:
    SG.add_edges_from((n, nbr, d)
    for n, nbs in G.adj.items() if n in largest_wcc
    for nbr, d in nbs.items() if nbr in largest_wcc)
SG.graph.update(G.graph)
\end{verbatim}

\textbf{Examples}

>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> H = G.subgraph({0, 1, 2})
>>> list(H.edges)
[(0, 1), (1, 2)]
**networkx.MultiGraph.edge_subgraph**

`MultiGraph.edge_subgraph(edges)`

Returns the subgraph induced by the specified edges.

The induced subgraph contains each edge in `edges` and each node incident to any one of those edges.

**Parameters**

- **edges** *(iterable)* – An iterable of edges in this graph.

**Returns**

- **G** – An edge-induced subgraph of this graph with the same edge attributes.

**Return type**

`Graph`

**Notes**

The graph, edge, and node attributes in the returned subgraph view are references to the corresponding attributes in the original graph. The view is read-only.

To create a full graph version of the subgraph with its own copy of the edge or node attributes, use:

```python
>>> G.edge_subgraph(edges).copy()
```

**Examples**

```python
>>> G = nx.path_graph(5)
>>> H = G.edge_subgraph([(0, 1), (3, 4)])
>>> list(H.nodes)
[0, 1, 3, 4]
>>> list(H.edges)
[(0, 1), (3, 4)]
```

---

### 2.2.4 MultiDiGraph—Directed graphs with self loops and parallel edges

**Overview**

**class MultiDiGraph** *(incoming_graph_data=None, **attr)*

A directed graph class that can store multiedges.

Multiedges are multiple edges between two nodes. Each edge can hold optional data or attributes.

A MultiDiGraph holds directed edges. Self loops are allowed.

Nodes can be arbitrary (hashable) Python objects with optional key/value attributes. By convention `None` is not used as a node.

Edges are represented as links between nodes with optional key/value attributes.

**Parameters**

- **incoming_graph_data** *(input graph (optional, default: None)* – Data to initialize graph. If None (default) an empty graph is created. The data can be any format that is supported by the `to_networkx_graph()` function, currently including edge list, dict of dicts, dict of lists, NetworkX graph, NumPy matrix or 2d ndarray, SciPy sparse matrix, or PyGraphviz graph.

- **attr** *(keyword arguments, optional (default= no attributes)) – Attributes to add to graph as key=value pairs.*
See also:

*Graph, DiGraph, MultiGraph, OrderedMultiDiGraph*

### Examples

Create an empty graph structure (a “null graph”) with no nodes and no edges.

```python
>>> G = nx.MultiDiGraph()
```

G can be grown in several ways.

#### Nodes:

Add one node at a time:

```python
>>> G.add_node(1)
```

Add the nodes from any container (a list, dict, set or even the lines from a file or the nodes from another graph).

```python
>>> G.add_nodes_from([2, 3])
>>> G.add_nodes_from(range(100, 110))
>>> H = nx.path_graph(10)
>>> G.add_nodes_from(H)
```

In addition to strings and integers any hashable Python object (except None) can represent a node, e.g. a customized node object, or even another Graph.

```python
>>> G.add_node(H)
```

#### Edges:

G can also be grown by adding edges.

Add one edge,

```python
>>> key = G.add_edge(1, 2)
```

a list of edges,

```python
>>> keys = G.add_edges_from([(1, 2), (1, 3)])
```

or a collection of edges,

```python
>>> keys = G.add_edges_from(H.edges)
```

If some edges connect nodes not yet in the graph, the nodes are added automatically. If an edge already exists, an additional edge is created and stored using a key to identify the edge. By default the key is the lowest unused integer.

```python
>>> keys = G.add_edges_from([(4, 5, dict(route=282)), (4, 5, dict(route=37))])
>>> G[4]  # default key
AdjacencyView({5: {0: {}, 1: {'route': 282}, 2: {'route': 37}}})
```

#### Attributes:

Each graph, node, and edge can hold key/value attribute pairs in an associated attribute dictionary (the keys must be hashable). By default these are empty, but can be added or changed using add_edge, add_node or direct manipulation of the attribute dictionaries named graph, node and edge respectively.

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Add node attributes using add_node(), add_nodes_from() or G.nodes

```python
>>> G.add_node(1, time='5pm')
>>> G.add_nodes_from([3], time='2pm')
>>> G.nodes[1]
{'time': '5pm'}
>>> G.nodes[1]['room'] = 714
>>> del G.nodes[1]['room']  # remove attribute
>>> list(G.nodes(data=True))
[(1, {'time': '5pm'}), (3, {'time': '2pm'})]
```

Add edge attributes using add_edge(), add_edges_from(), subscript notation, or G.edges.

```python
>>> key = G.add_edge(1, 2, weight=4.7)
>>> keys = G.add_edges_from([(3, 4), (4, 5)], color='red')
>>> keys = G.add_edges_from([(1, 2, {'color': 'blue'}), (2, 3, {'weight': 8})])
>>> G[1][2][0]['weight'] = 4.7
>>> G.edges[1, 2, 0]['weight'] = 4
```

Warning: we protect the graph data structure by making G.edges[1, 2] a read-only dict-like structure. However, you can assign to attributes in e.g. G.edges[1, 2]. Thus, use 2 sets of brackets to add/change data attributes: G.edges[1, 2]['weight'] = 4 (For multigraphs: MG.edges[u, v, key][name] = value).

Shortcuts:

Many common graph features allow python syntax to speed reporting.

```python
>>> 1 in G    # check if node in graph
True
>>> [n for n in G if n<3]    # iterate through nodes
[1, 2]
>>> len(G)    # number of nodes in graph
5
>>> G[1]    # adjacency dict-like view keyed by neighbor to edge attributes
AdjacencyView({2: {0: {'weight': 4}, 1: {'color': 'blue'}}})
```

Often the best way to traverse all edges of a graph is via the neighbors. The neighbors are available as an adjacency-view G.adj object or via the method G.adjacency().

```python
>>> for n, nbrdict in G.adjacency():
...     for nbr, keydict in nbrdict.items():
...         for key, eattr in keydict.items():
...             if 'weight' in eattr:
...                 # Do something useful with the edges
...                 pass
```

But the edges() method is often more convenient:

```python
>>> for u, v, keys, weight in G.edges(data='weight', keys=True):
...     if weight is not None:
...         # Do something useful with the edges
...         pass
```
Simple graph information is obtained using methods and object-attributes. Reporting usually provides views instead of containers to reduce memory usage. The views update as the graph is updated similarly to dict-views. The objects `nodes`, `edges` and `adj` provide access to data attributes via lookup (e.g. `nodes[n]`, `edges[u, v]`, `adj[u][v]`) and iteration (e.g. `nodes.items()`, `nodes.data('color')`, `nodes.data('color', default='blue')`) and similarly for `edges` Views exist for `nodes`, `edges`, `neighbors()`/`adj` and `degree`.

For details on these and other miscellaneous methods, see below.

**Subclasses (Advanced):**

The MultiDiGraph class uses a dict-of-dict-of-dict-of-dict structure. The outer dict (node_dict) holds adjacency information keyed by node. The next dict (adjlist_dict) represents the adjacency information and holds edge_key dicts keyed by neighbor. The edge_key dict holds each edge_attr dict keyed by edge key. The inner dict (edge_attr_dict) represents the edge data and holds edge attribute values keyed by attribute names.

Each of these four dicts in the dict-of-dict-of-dict-of-dict structure can be replaced by a user defined dict-like object. In general, the dict-like features should be maintained but extra features can be added. To replace one of the dicts create a new graph class by changing the class(!) variable holding the factory for that dict-like structure. The variable names are node_dict_factory, node_attr_dict_factory, adjlist_outer_dict_factory, adjlist_inner_dict_factory, edge_key_dict_factory, edge_attr_dict_factory and graph_attr_dict_factory.

- `node_dict_factory` [function, (default: dict)] Factory function to be used to create the dict containing node attributes, keyed by node id. It should require no arguments and return a dict-like object.
- `node_attr_dict_factory` [function, (default: dict)] Factory function to be used to create the node attribute dict which holds attribute values keyed by attribute name. It should require no arguments and return a dict-like object.
- `adjlist_outer_dict_factory` [function, (default: dict)] Factory function to be used to create the outer-most dict in the data structure that holds adjacency info keyed by node. It should require no arguments and return a dict-like object.
- `adjlist_inner_dict_factory` [function, (default: dict)] Factory function to be used to create the adjacency list dict which holds multiedge key dicts keyed by neighbor. It should require no arguments and return a dict-like object.
- `edge_key_dict_factory` [function, (default: dict)] Factory function to be used to create the edge key dict which holds edge data keyed by edge key. It should require no arguments and return a dict-like object.
- `edge_attr_dict_factory` [function, (default: dict)] Factory function to be used to create the edge attribute dict which holds attribute values keyed by attribute name. It should require no arguments and return a dict-like object.
- `graph_attr_dict_factory` [function, (default: dict)] Factory function to be used to create the graph attribute dict which holds attribute values keyed by attribute name. It should require no arguments and return a dict-like object.

Typically, if your extension doesn’t impact the data structure all methods will inherited without issue except: `to_directed/to_undirected`. By default these methods create a DiGraph/Graph class and you probably want them to create your extension of a DiGraph/Graph. To facilitate this we define two class variables that you can set in your subclass:

- `to_directed_class` [callable, (default: DiGraph or MultiDiGraph)] Class to create a new graph structure in the `to_directed` method. If `None`, a NetworkX class (DiGraph or MultiDiGraph) is used.
- `to_undirected_class` [callable, (default: Graph or MultiGraph)] Class to create a new graph structure in the `to_undirected` method. If `None`, a NetworkX class (Graph or MultiGraph) is used.
Examples

Please see ordered for examples of creating graph subclasses by overwriting the base class dict with a dictionary-like object.

Methods

Adding and Removing Nodes and Edges

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<th>Description</th>
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<td>MultiDiGraph.clear()</td>
<td>Remove all nodes and edges from the graph.</td>
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networkx.MultiDiGraph.__init__

MultiDiGraph.__init__(incoming_graph_data=None, **attr)

Initialize a graph with edges, name, or graph attributes.

Parameters

- **incoming_graph_data** *(input graph)* – Data to initialize graph. If incoming_graph_data=None (default) an empty graph is created. The data can be an edge list, or any NetworkX graph object. If the corresponding optional Python packages are installed the data can also be a NumPy matrix or 2d ndarray, a SciPy sparse matrix, or a PyGraphviz graph.
- **attr** *(keyword arguments, optional (default= no attributes)) – Attributes to add to graph as key=value pairs.

See also:

convert()

Examples

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G = nx.Graph(name='my graph')
```
>>> e = [(1, 2), (2, 3), (3, 4)]  # list of edges
>>> G = nx.Graph(e)

Arbitrary graph attribute pairs (key=value) may be assigned

>>> G = nx.Graph(e, day="Friday")
>>> G.graph
{'day': 'Friday'}

networkx.MultiDiGraph.add_node

MultiDiGraph.add_node(node_for_adding, **attr)
Add a single node node_for_adding and update node attributes.

Parameters
- node_for_adding (node) – A node can be any hashable Python object except None.
- attr (keyword arguments, optional) – Set or change node attributes using key=value.

See also:
add_nodes_from()

Examples

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_node(1)
>>> G.add_node('Hello')
>>> K3 = nx.Graph([(0, 1), (1, 2), (2, 0)])
>>> G.add_node(K3)
>>> G.number_of_nodes()
3
```

Use keywords set/change node attributes:

```python
>>> G.add_node(1, size=10)
>>> G.add_node(3, weight=0.4, UTM=('13S', 382871, 3972649))
```

Notes

A hashable object is one that can be used as a key in a Python dictionary. This includes strings, numbers, tuples of strings and numbers, etc.

On many platforms hashable items also include mutables such as NetworkX Graphs, though one should be careful that the hash doesn’t change on mutables.

networkx.MultiDiGraph.add_nodes_from

MultiDiGraph.add_nodes_from(nodes_for_adding, **attr)
Add multiple nodes.

Parameters
• **nodes_for_adding** (*iterable container*) – A container of nodes (list, dict, set, etc.). OR A container of (node, attribute dict) tuples. Node attributes are updated using the attribute dict.

• **attr** (*keyword arguments, optional (default= no attributes]*) – Update attributes for all nodes in nodes. Node attributes specified in nodes as a tuple take precedence over attributes specified via keyword arguments.

**See also:**

*add_node()*

**Examples**

```python
>>> G = nx.Graph()       # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_nodes_from('Hello')
>>> K3 = nx.Graph([(0, 1), (1, 2), (2, 0)])
>>> G.add_nodes_from(K3)
>>> sorted(G.nodes(), key=str)
[0, 1, 2, 'H', 'e', 'l', 'o']

Use keywords to update specific node attributes for every node.

```python
>>> G.add_nodes_from([(1, 2), size=10])
```  
```python
>>> G.add_nodes_from([(3, 4), weight=0.4])
```  
Use (node, attrdict) tuples to update attributes for specific nodes.

```python
>>> G.add_nodes_from([(1, dict(size=11)), (2, {'color':'blue'})])
```  
```python
>>> G.nodes[1]['size']
11
```  
```python
>>> H = nx.Graph()
>>> H.add_nodes_from(G.nodes(data=True))
```  
```python
>>> H.nodes[1]['size']
11
```

**networkx.MultiDiGraph.remove_node**

*MultiDiGraph.remove_node(n)*

Remove node n.

Removes the node n and all adjacent edges. Attempting to remove a non-existent node will raise an exception.

**Parameters**

- **n** (*node*) – A node in the graph

**Raises**

- **NetworkXError** – If n is not in the graph.

**See also:**

*remove_nodes_from()*

**Examples**

```python
>>> G = nx.path_graph(3)       # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> list(G.edges)
[(0, 1), (1, 2)]
>>> G.remove_node(1)
```
>>> list(G.edges)  # or DiGraph, MultiGraph, MultiDiGraph, etc
[]

networkx.MultiDiGraph.remove_nodes_from

MultiDiGraph.remove_nodes_from((nodes))
Remove multiple nodes.

Parameters nodes (iterable container) – A container of nodes (list, dict, set, etc.). If a node in the container is not in the graph it is silently ignored.

See also:
remove_node()

Examples

>>> G = nx.path_graph(3)
>>> e = list(G.nodes)
>>> e
[0, 1, 2]
>>> G.remove_nodes_from(e)
>>> list(G.nodes)
[]

networkx.MultiDiGraph.add_edge

MultiDiGraph.add_edge(u_for_edge, v_for_edge, key=None, **attr)
Add an edge between u and v.

The nodes u and v will be automatically added if they are not already in the graph.

Edge attributes can be specified with keywords or by directly accessing the edge’s attribute dictionary. See examples below.

Parameters

- u_for_edge, v_for_edge (nodes) – Nodes can be, for example, strings or numbers. Nodes must be hashable (and not None) Python objects.
- key (hashable identifier, optional (default=lowest unused integer)) – Used to distinguish multiedges between a pair of nodes.
- attr_dict (dictionary, optional (default= no attributes)) – Dictionary of edge attributes. Key/value pairs will update existing data associated with the edge.
- attr (keyword arguments, optional) – Edge data (or labels or objects) can be assigned using keyword arguments.

Returns

Return type The edge key assigned to the edge.

See also:

add_edges_from() add a collection of edges
Notes

To replace/update edge data, use the optional key argument to identify a unique edge. Otherwise a new edge will be created.

NetworkX algorithms designed for weighted graphs cannot use multigraphs directly because it is not clear how to handle multiedge weights. Convert to Graph using edge attribute ‘weight’ to enable weighted graph algorithms.

Default keys are generated using the method new_edge_key(). This method can be overridden by subclassing the base class and providing a custom new_edge_key() method.

Examples

The following all add the edge e=(1, 2) to graph G:

```python
>>> G = nx.MultiDiGraph()
>>> e = (1, 2)
>>> key = G.add_edge(1, 2)  # explicit two-node form
>>> G.add_edge(*e)  # single edge as tuple of two nodes
1
>>> G.add_edges_from([(1, 2)])  # add edges from iterable container
[2]
```

Associate data to edges using keywords:

```python
>>> key = G.add_edge(1, 2, weight=3)
>>> key = G.add_edge(1, 2, key=0, weight=4)  # update data for key=0
>>> key = G.add_edge(1, 3, weight=7, capacity=15, length=342.7)
```

For non-string attribute keys, use subscript notation.

```python
>>> ekey = G.add_edge(1, 2)
>>> G[1][2][0].update({0: 5})
>>> G.edges[1, 2, 0].update({0: 5})
```

networkx.MultiDiGraph.add_edges_from

MultiDiGraph.add_edges_from(ebunch_to_add, **attr)

Add all the edges in ebunch_to_add.

Parameters

- ebunch_to_add (container of edges) – Each edge given in the container will be added to the graph. The edges can be:
  - 2-tuples (u, v) or
  - 3-tuples (u, v, d) for an edge data dict d, or
  - 3-tuples (u, v, k) for not iterable key k, or
  - 4-tuples (u, v, k, d) for an edge with data and key k

- attr (keyword arguments, optional) – Edge data (or labels or objects) can be assigned using keyword arguments.

Returns
Return type  A list of edge keys assigned to the edges in ebunch.

See also:

add_edge()  add a single edge
add_weighted_edges_from()  convenient way to add weighted edges

Notes

Adding the same edge twice has no effect but any edge data will be updated when each duplicate edge is added. Edge attributes specified in an ebunch take precedence over attributes specified via keyword arguments. Default keys are generated using the method new_edge_key(). This method can be overridden by subclassing the base class and providing a custom new_edge_key() method.

Examples

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edges_from([(0, 1), (1, 2)])  # using a list of edge tuples
>>> e = zip(range(0, 3), range(1, 4))
>>> G.add_edges_from(e)  # Add the path graph 0-1-2-3

Associate data to edges

```python
>>> G.add_edges_from([(1, 2), (2, 3)], weight=3)
>>> G.add_edges_from([(3, 4), (1, 4)], label='WN2898')
```

networkx.MultiDiGraph.add_weighted_edges_from

MultiDiGraph.add_weighted_edges_from(ebunch_to_add, weight='weight', **attr)

Add weighted edges in ebunch_to_add with specified weight attr

Parameters

- **ebunch_to_add (container of edges)** – Each edge given in the list or container will be added to the graph. The edges must be given as 3-tuples (u, v, w) where w is a number.
- **weight (string, optional (default= 'weight'))** – The attribute name for the edge weights to be added.
- **attr (keyword arguments, optional (default= no attributes))** – Edge attributes to add/update for all edges.

See also:

add_edge()  add a single edge
add_edges_from()  add multiple edges

Notes

Adding the same edge twice for Graph/DiGraph simply updates the edge data. For MultiGraph/MultiDiGraph, duplicate edges are stored.
Examples

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_weighted_edges_from([(0, 1, 3.0), (1, 2, 7.5)])
```

networkx.MultiDiGraph.new_edge_key

`MultiDiGraph.new_edge_key(u, v)`

Returns an unused key for edges between nodes `u` and `v`.

The nodes `u` and `v` do not need to be already in the graph.

Notes

In the standard MultiGraph class the new key is the number of existing edges between `u` and `v` (increased if necessary to ensure unused). The first edge will have key 0, then 1, etc. If an edge is removed further new_edge_keys may not be in this order.

Parameters `u, v (nodes)`

Returns key

Return type int

networkx.MultiDiGraph.remove_edge

`MultiDiGraph.remove_edge(u, v, key=None)`

Remove an edge between `u` and `v`.

Parameters

- `u, v (nodes)` – Remove an edge between nodes `u` and `v`.
- `key (hashable identifier, optional (default=None))` – Used to distinguish multiple edges between a pair of nodes. If None remove a single (arbitrary) edge between `u` and `v`.

Raises `NetworkXError` – If there is not an edge between `u` and `v`, or if there is no edge with the specified key.

See also:

- `remove_edges_from()` remove a collection of edges

Examples

```python
>>> G = nx.MultiDiGraph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.remove_edge(0, 1)
>>> e = (1, 2)
>>> G.remove_edge(*e)  # unpacks e from an edge tuple
```

For multiple edges
```python
>>> G = nx.MultiDiGraph()
>>> G.add_edges_from([(1, 2), (1, 2), (1, 2)])  # key_list returned
[0, 1, 2]
>>> G.remove_edge(1, 2)  # remove a single (arbitrary) edge

For edges with keys
```}

```python
>>> G = nx.MultiDiGraph()
>>> G.add_edge(1, 2, key='first')
'first'
>>> G.add_edge(1, 2, key='second')
'second'
>>> G.remove_edge(1, 2, key='second')

networkx.MultiDiGraph.remove_edges_from

MultiDiGraph.remove_edges_from(ebunch)
Remove all edges specified in ebunch.

Parameters

**ebunch** *(list or container of edge tuples)* – Each edge given in the list or container will be removed from the graph. The edges can be:

- 2-tuples *(u, v)* All edges between u and v are removed.
- 3-tuples *(u, v, key)* The edge identified by key is removed.
- 4-tuples *(u, v, key, data)* where data is ignored.

See also:

*remove_edge()* remove a single edge

Notes

Will fail silently if an edge in ebunch is not in the graph.

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> ebunch=[(1, 2), (2, 3)]
>>> G.remove_edges_from(ebunch)

Removing multiple copies of edges
```}

```python
>>> G = nx.MultiGraph()
>>> keys = G.add_edges_from([(1, 2), (1, 2), (1, 2)])
>>> G.remove_edges_from([(1, 2), (1, 2)])
>>> list(G.edges())
[(1, 2)]
>>> G.remove_edges_from([(1, 2), (1, 2)])  # silently ignore extra copy
>>> list(G.edges())  # now empty graph
[]
```
networkx.MultiDiGraph.update

MultiDiGraph.update(edges=None, nodes=None)

Update the graph using nodes/edges/graphs as input.

Like dict.update, this method takes a graph as input, adding the graph’s nodes and edges to this graph. It can also take two inputs: edges and nodes. Finally it can take either edges or nodes. To specify only nodes the keyword nodes must be used.

The collections of edges and nodes are treated similarly to the add_edges_from/add_nodes_from methods. When iterated, they should yield 2-tuples (u, v) or 3-tuples (u, v, datadict).

Parameters

- **edges** (Graph object, collection of edges, or None) – The first parameter can be a graph or some edges. If it has attributes nodes and edges, then it is taken to be a Graph-like object and those attributes are used as collections of nodes and edges to be added to the graph. If the first parameter does not have those attributes, it is treated as a collection of edges and added to the graph. If the first argument is None, no edges are added.

- **nodes** (collection of nodes, or None) – The second parameter is treated as a collection of nodes to be added to the graph unless it is None. If edges is None and nodes is None an exception is raised. If the first parameter is a Graph, then nodes is ignored.

Examples

```python
>>> G = nx.path_graph(5)
>>> G.update(nx.complete_graph(range(4,10)))
>>> from itertools import combinations
>>> edges = ((u, v, {'power': u * v})
...     for u, v in combinations(range(10, 20), 2)
...     if u * v < 225)
>>> nodes = [1000]  # for singleton, use a container
>>> G.update(edges, nodes)
```

Notes

It you want to update the graph using an adjacency structure it is straightforward to obtain the edges/nodes from adjacency. The following examples provide common cases, your adjacency may be slightly different and require tweaks of these examples.

```python
>>> # dict-of-set/list/tuple
>>> adj = {1: {2, 3}, 2: {1, 3}, 3: {1, 2}}
>>> e = [(u, v) for u, nbrs in adj.items() for v in nbrs]
>>> G.update(edges=e, nodes=adj)
```

```python
>>> DG = nx.DiGraph()
>>> # dict-of-dict-of-attribute
>>> adj = {1: {2: 1.3, 3: 0.7}, 2: {1: 1.4}, 3: {1: 0.7}}
>>> e = [(u, v, {'weight': d}) for u, nbrs in adj.items() for v, d in nbrs.items()]
>>> DG.update(edges=e, nodes=adj)
```
NetworkX Reference, Release 2.5rc1.dev20191214022633

```python
>>> # dict-of-dict-of-dict
>>> adj = {1: {2: {'weight': 1.3}, 3: {'color': 0.7, 'weight':1.2}}, ...
...     for v, d in nbrs.items()}
>>> DG.update(edges=e, nodes=adj)

>>> # predecessor adjacency (dict-of-set)
>>> pred = {1: {2, 3}, 2: {3}, 3: {3}}
>>> e = [(v, u) for u, nbrs in pred.items() for v in nbrs]

>>> # MultiGraph dict-of-dict-of-dict-of-attribute
>>> MDG = nx.MultiDiGraph()
>>> adj = {1: {2: {0: {'weight': 1.3}, 1: {'weight': 1.2}}}, ...
...     3: {2: {0: {'weight': 0.7}}}}
>>> e = [(u, v, ekey, d) for u, nbrs in adj.items() ...
...     for v, keydict in nbrs.items() ...
...     for ekey, d in keydict.items()]
>>> MDG.update(edges=e)
```

See also:

```python
add_edges_from()  add multiple edges to a graph
add_nodes_from()  add multiple nodes to a graph
```

networkx.MultiDiGraph.clear

MultiDiGraph.clear()

Remove all nodes and edges from the graph.
This also removes the name, and all graph, node, and edge attributes.

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.clear()
>>> list(G.nodes)
[]
>>> list(G.edges)
[]
```

Reporting nodes edges and neighbors

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**networkx.MultiDiGraph.nodes**

**property MultiDiGraph.nodes**

A NodeView of the Graph as G.nodes or G.nodes(). Can be used as G.nodes for data lookup and for set-like operations. Can also be used as G.nodes(data='color', default=None) to return a NodeDataView which reports specific node data but no set operations. It presents a dict-like interface as well with G.nodes.items() iterating over (node, node data) 2-tuples and G.nodes[3]['foo'] providing the value of the foo attribute for node 3. In addition, a view G.nodes.data('foo') provides a dict-like interface to the foo attribute of each node. G.nodes.data('foo', default=1) provides a default for nodes that do not have attribute foo.

**Parameters**

- **data** *(string or bool, optional (default=False))* – The node attribute returned in 2-tuple (n, ddict[data]). If True, return entire node attribute dict as (n, ddict). If False, return just the nodes n.

- **default** *(value, optional (default=None))* – Value used for nodes that don’t have the requested attribute. Only relevant if data is not True or False.

**Returns**

Allows set-like operations over the nodes as well as node attribute dict lookup and calling to get a NodeDataView. A NodeDataView iterates over (n, data) and has no set operations. A NodeView iterates over n and includes set operations.

When called, if data is False, an iterator over nodes. Otherwise an iterator of 2-tuples (node, attribute value) where the attribute is specified in data. If data is True then the attribute becomes the entire data dictionary.

**Return type** NodeView
Notes

If your node data is not needed, it is simpler and equivalent to use the expression `for n in G, or list(G).

Examples

There are two simple ways of getting a list of all nodes in the graph:

```python
>>> G = nx.path_graph(3)
>>> list(G.nodes)
[0, 1, 2]
>>> list(G)
[0, 1, 2]
```

To get the node data along with the nodes:

```python
>>> G.add_node(1, time='5pm')
>>> G.nodes[0]['foo'] = 'bar'
>>> list(G.nodes(data=True))
[(0, {'foo': 'bar'}), (1, {'time': '5pm'}), (2, {})]
>>> list(G.nodes.data())
[(0, {'foo': 'bar'}), (1, {'time': '5pm'}), (2, {})]
```

```python
>>> list(G.nodes(data='foo'))
[(0, 'bar'), (1, None), (2, None)]
>>> list(G.nodes.data('foo'))
[(0, 'bar'), (1, None), (2, None)]
>>> list(G.nodes(data='time'))
[(0, None), (1, '5pm'), (2, None)]
>>> list(G.nodes.data('time'))
[(0, None), (1, '5pm'), (2, None)]
>>> list(G.nodes.data('time', default='Not Available'))
[(0, 'Not Available'), (1, '5pm'), (2, 'Not Available')]
>>> list(G.nodes.data('time', default='Not Available'))
[(0, 'Not Available'), (1, '5pm'), (2, 'Not Available')]
```

If some of your nodes have an attribute and the rest are assumed to have a default attribute value you can create a dictionary from node/attribute pairs using the `default` keyword argument to guarantee the value is never `None`:

```python
>>> G = nx.Graph()
>>> G.add_node(0)
>>> G.add_node(1, weight=2)
>>> G.add_node(2, weight=3)
>>> dict(G.nodes(data='weight', default=1))
{0: 1, 1: 2, 2: 3}
```

`networkx.MultiDiGraph.__iter__`

`MultiDiGraph.__iter__()`

Iterate over the nodes. Use: ‘for n in G’.
**networkx.MultiDiGraph.nodes**

**Returns** `niter` – An iterator over all nodes in the graph.

**Return type** iterator

**Examples**

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [n for n in G]
[0, 1, 2, 3]
>>> list(G)
[0, 1, 2, 3]
```

**networkx.MultiDiGraph.has_node**

`MultiDiGraph.has_node(n)`

Returns True if the graph contains the node `n`.

Identical to `n in G`

**Parameters** `n (node)`

**Examples**

```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.has_node(0)
True
```

It is more readable and simpler to use

```python
>>> 0 in G
True
```

**networkx.MultiDiGraph.__contains__**

`MultiDiGraph.__contains__(n)`

Returns True if `n` is a node, False otherwise. Use: ‘`n in G`’.

**Examples**

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> 1 in G
True
```

**networkx.MultiDiGraph.edges**

**property** `MultiDiGraph.edges`

An OutMultiEdgeView of the Graph as `G.edges` or `G.edges()`.

```python
edges(self, nbunch=None, data=False, keys=False, default=None)
```
The OutMultiEdgeView provides set-like operations on the edge-tuples as well as edge attribute lookup. When called, it also provides an EdgeDataView object which allows control of access to edge attributes (but does not provide set-like operations). Hence, G.edges[u, v]['color'] provides the value of the color attribute for edge (u, v) while for (u, v, c) in G.edges(data='color', default='red'): iterates through all the edges yielding the color attribute with default 'red' if no color attribute exists.

Edges are returned as tuples with optional data and keys in the order (node, neighbor, key, data).

**Parameters**

- **nbunch** *(single node, container, or all nodes (default=all nodes)) –* The view will only report edges incident to these nodes.

- **data** *(string or bool, optional (default=False)) –* The edge attribute returned in 3-tuple (u, v, ddict[data]). If True, return edge attribute dict in 3-tuple (u, v, ddict). If False, return 2-tuple (u, v).

- **keys** *(bool, optional (default=False)) –* If True, return edge keys with each edge.

- **default** *(value, optional (default=None)) –* Value used for edges that don’t have the requested attribute. Only relevant if data is not True or False.

**Returns** edges – A view of edge attributes, usually it iterates over (u, v) (u, v, k) or (u, v, k, d) tuples of edges, but can also be used for attribute lookup as edges[u, v, k]['foo'].

**Return type** EdgeView

**Notes**

Nodes in nbunch that are not in the graph will be (quietly) ignored. For directed graphs this returns the out-edges.

**Examples**

```python
>>> G = nx.MultiDiGraph()
>>> nx.add_path(G, [0, 1, 2])
>>> key = G.add_edge(2, 3, weight=5)
>>> [e for e in G.edges()]
[(0, 1), (1, 2), (2, 3)]
>>> list(G.edges(data=True))  # default data is {} (empty dict)
[(0, 1, {}), (1, 2, {}), (2, 3, {'weight': 5})]
>>> list(G.edges(data='weight', default=1))
[(0, 1, 1), (1, 2, 1), (2, 3, 5)]
>>> list(G.edges(keys=True))  # default keys are integers
[(0, 1, 0), (1, 2, 0), (2, 3, 0)]
>>> list(G.edges(data=True, keys=True))
[(0, 1, 0, {}), (1, 2, 0, {}), (2, 3, 0, {'weight': 5})]
>>> list(G.edges('weight', default=1, keys=True))
[(0, 1, 0, 1), (1, 2, 0, 1), (2, 3, 0, 5)]
>>> list(G.edges([0, 2]))
[(0, 1), (2, 3)]
>>> list(G.edges(0))
[(0, 1)]
```

**See also:**

`in_edges, out_edges`

---

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networkx.MultiDiGraph.out_edges

**property MultiDiGraph.out_edges**

An OutMultiEdgeView of the Graph as G.edges or G.edges().

edges(self, nbunch=None, data=False, keys=False, default=None)

The OutMultiEdgeView provides set-like operations on the edge-tuples as well as edge attribute lookup. When called, it also provides an EdgeDataView object which allows control of access to edge attributes (but does not provide set-like operations). Hence, G.edges[u, v]['color'] provides the value of the color attribute for edge (u, v) while for (u, v, c) in G.edges(data='color', default='red'): iterates through all the edges yielding the color attribute with default 'red' if no color attribute exists.

Edges are returned as tuples with optional data and keys in the order (node, neighbor, key, data).

**Parameters**

- **nbunch** *(single node, container, or all nodes (default= all nodes)) – The view will only report edges incident to these nodes.*
- **data** *(string or bool, optional (default=False)) – The edge attribute returned in 3-tuple (u, v, ddict[data]). If True, return edge attribute dict in 3-tuple (u, v, ddict). If False, return 2-tuple (u, v).*
- **keys** *(bool, optional (default=False)) – If True, return edge keys with each edge.*
- **default** *(value, optional (default=None)) – Value used for edges that don’t have the requested attribute. Only relevant if data is not True or False.*

**Returns** edges – A view of edge attributes, usually it iterates over (u, v) (u, v, k) or (u, v, k, d) tuples of edges, but can also be used for attribute lookup as edges[u, v, k]['foo'].

**Return type** EdgeView

**Notes**

Nodes in nbunch that are not in the graph will be (quietly) ignored. For directed graphs this returns the out-edges.

**Examples**

```python
>>> G = nx.MultiDiGraph()
>>> nx.add_path(G, [0, 1, 2])
>>> key = G.add_edge(2, 3, weight=5)
>>> [e for e in G.edges()]
[(0, 1), (1, 2), (2, 3)]
>>> list(G.edges(data=True))  # default data is {} (empty dict)
[(0, 1, {}), (1, 2, {}), (2, 3, {'weight': 5})]
>>> list(G.edges(data='weight', default=1))
[(0, 1, 1), (1, 2, 1), (2, 3, 5)]
>>> list(G.edges(keys=True))  # default keys are integers
[(0, 1, 0), (1, 2, 0), (2, 3, 0)]
>>> list(G.edges(data=True, keys=True))
[(0, 1, 0, {}), (1, 2, 0, {}), (2, 3, 0, {'weight': 5})]
>>> list(G.edges(data='weight', default=1, keys=True))
[(0, 1, 0, 1), (1, 2, 0, 1), (2, 3, 0, 5)]
>>> list(G.edges([0, 2]))
[(0, 1), (2, 3)]
```

(continues on next page)
>>> list(G.edges(0))
[(0, 1)]

See also:
in_edges, out_edges

networkx.MultiDiGraph.in_edges

property MultiDiGraph.in_edges
An InMultiEdgeView of the Graph as G.in_edges or G.in_edges().

in_edges(self, nbunch=None, data=False, keys=False, default=None)

Parameters
• nbunch (single node, container, or all nodes (default=all nodes)) – The view will only report edges incident to these nodes.

• data (string or bool, optional (default=False)) – The edge attribute returned in 3-tuple (u, v, ddict[data]). If True, return edge attribute dict in 3-tuple (u, v, ddict). If False, return 2-tuple (u, v).

• keys (bool, optional (default=False)) – If True, return edge keys with each edge.

• default (value, optional (default=None)) – Value used for edges that don’t have the requested attribute. Only relevant if data is not True or False.

Returns in_edges – A view of edge attributes, usually it iterates over (u, v) or (u, v, k) or (u, v, k, d) tuples of edges, but can also be used for attribute lookup as edges[u, v, k]['foo'].

Return type InMultiEdgeView

See also: edges

networkx.MultiDiGraph.has_edge

MultiDiGraph.has_edge (u, v, key=None)

Returns True if the graph has an edge between nodes u and v.

This is the same as v in G[u] or key in G[u][v] without KeyError exceptions.

Parameters
• u, v (nodes) – Nodes can be, for example, strings or numbers.

• key (hashable identifier, optional (default=None)) – If specified return True only if the edge with key is found.

Returns edge_ind – True if edge is in the graph, False otherwise.

Return type bool

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Examples

Can be called either using two nodes u, v, an edge tuple (u, v), or an edge tuple (u, v, key).

```python
>>> G = nx.MultiGraph()  # or MultiDiGraph
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.has_edge(0, 1)  # using two nodes
True
>>> e = (0, 1)
>>> G.has_edge(*e)  # e is a 2-tuple (u, v)
True
>>> G.add_edge(0, 1, key='a')
'a'
>>> G.has_edge(0, 1, key='a')  # specify key
True
>>> e=(0, 1, 'a')
>>> G.has_edge(*e)  # e is a 3-tuple (u, v, 'a')
True
```

The following syntax are equivalent:

```python
>>> G.has_edge(0, 1)
True
>>> 1 in G[0]  # though this gives :exc:`KeyError` if 0 not in G
True
```

`networkx.MultiDiGraph.get_edge_data`

`MultiDiGraph.get_edge_data(u, v, key=None, default=None)`

Returns the attribute dictionary associated with edge (u, v).

This is identical to `G[u][v][key]` except the default is returned instead of an exception if the edge doesn’t exist.

Parameters

- `u, v (nodes)`
- `default (any Python object (default=None))` – Value to return if the edge (u, v) is not found.
- `key (hashable identifier, optional (default=None))` – Return data only for the edge with specified key.

Returns `edge_dict` – The edge attribute dictionary.

Return type `dictionary`

Examples

```python
>>> G = nx.MultiGraph()  # or MultiDiGraph
>>> key = G.add_edge(0, 1, key='a', weight=7)
>>> G[0][1]['a']  # key='a'
{'weight': 7}
>>> G.edges[0, 1, 'a']  # key='a'
{'weight': 7}
```
Warning: we protect the graph data structure by making `G.edges` and `G[1][2]` read-only dict-like structures. However, you can assign values to attributes in e.g. `G.edges[1, 2, 'a']` or `G[1][2]['a']` using an additional bracket as shown next. You need to specify all edge info to assign to the edge data associated with an edge.

```python
>>> G[0][1]['a']['weight'] = 10
>>> G.edges[0, 1, 'a']['weight'] = 10
>>> G[0][1]['a']['weight']
10
>>> G.edges[1, 0, 'a']['weight']
10
```

```python
>>> G = nx.MultiGraph()  # or MultiDiGraph
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.get_edge_data(0, 1)
{0: {}}
>>> e = (0, 1)
>>> G.get_edge_data(*e)  # tuple form
{0: {}}
>>> G.get_edge_data('a', 'b', default=0)  # edge not in graph, return 0
0
```

### networkx.MultiDiGraph.neighbors

**MultiDiGraph.neighbors(n)**

Returns an iterator over successor nodes of n.

A successor of n is a node m such that there exists a directed edge from n to m.

**Parameters**

- `n` (*node*) – A node in the graph

**Raises**

- `NetworkXError` – If n is not in the graph.

**See also:**

- `predecessors()`

**Notes**

`neighbors()` and `successors()` are the same.

### networkx.MultiDiGraph.adj

**property** `MultiDiGraph.adj`

Graph adjacency object holding the neighbors of each node.

This object is a read-only dict-like structure with node keys and neighbor-dict values. The neighbor-dict is keyed by neighbor to the edgekey-dict. So `G.adj[3][2][0]['color'] = 'blue'` sets the color of the edge `(3, 2, 0)` to "blue".

Iterating over `G.adj` behaves like a dict. Useful idioms include `for nbr, datadict in G.adj[n].items():`

The neighbor information is also provided by subscripting the graph. So for `nbr, foovalue in G[node].data('foo', default=1):` works.

For directed graphs, `G.adj` holds outgoing (successor) info.
networkx.MultiDiGraph.__getitem__

MultiDiGraph.__getitem__(n)
Returns a dict of neighbors of node n. Use: ‘G[n]’.

Parameters n (node) – A node in the graph.

Returns adj_dict – The adjacency dictionary for nodes connected to n.

Return type dictionary

Notes

G[n] is the same as G.adj[n] and similar to G.neighbors(n) (which is an iterator over G.adj[n])

Examples

```python
>>> G = nx.path_graph(4) # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G[0]
AtlasView({1: {}})
```

networkx.MultiDiGraph.successors

MultiDiGraph.successors(n)
Returns an iterator over successor nodes of n.

A successor of n is a node m such that there exists a directed edge from n to m.

Parameters n (node) – A node in the graph

Raises NetworkXError – If n is not in the graph.

See also:

predecessors()

Notes

neighbors() and successors() are the same.

networkx.MultiDiGraph.succ

property MultiDiGraph.succ
Graph adjacency object holding the successors of each node.

This object is a read-only dict-like structure with node keys and neighbor-dict values. The neighbor-dict is keyed by neighbor to the edgekey-dict. So G.adj[3][2][0]['color'] = 'blue' sets the color of the edge (3, 2, 0) to "blue".

Iterating over G.adj behaves like a dict. Useful idioms include for nbr, datadict in G.adj[n].items():

The neighbor information is also provided by subscripting the graph. So for nbr, foovalue in G[node].data('foo', default=1): works.
For directed graphs, \(G\).succ is identical to \(G\).adj.

**networkx.MultiDiGraph.predecessors**

```python
MultiDiGraph.predecessors(n)
```

Returns an iterator over predecessor nodes of \(n\).

A predecessor of \(n\) is a node \(m\) such that there exists a directed edge from \(m\) to \(n\).

**Parameters**

- \(n\) (node) – A node in the graph

**Raises**

- `NetworkXError` – If \(n\) is not in the graph.

**See also:**

- `successors()`

**networkx.MultiDiGraph.adjacency**

```python
MultiDiGraph.adjacency()
```

Returns an iterator over (node, adjacency dict) tuples for all nodes.

For directed graphs, only outgoing neighbors/adjacencies are included.

**Returns**

- `adj_iter` – An iterator over (node, adjacency dictionary) for all nodes in the graph.

**Return type**

- iterator

**Examples**

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> [(n, nbrdict) for n, nbrdict in G.adjacency()]

[(0, {1: {}}), (1, {0: {}, 2: {}}), (2, {1: {}, 3: {}}), (3, {2: {}})]
```

**networkx.MultiDiGraph.nbunch_iter**

```python
MultiDiGraph.nbunch_iter(nbunch=None)
```

Returns an iterator over nodes contained in \(nbunch\) that are also in the graph.

The nodes in \(nbunch\) are checked for membership in the graph and if not are silently ignored.

**Parameters**

- \(nbunch\) (single node, container, or all nodes (default= all nodes)) – The view will only report edges incident to these nodes.

**Returns**

- `niter` – An iterator over nodes in \(nbunch\) that are also in the graph. If \(nbunch\) is None, iterate over all nodes in the graph.

**Return type**

- iterator

**Raises**

- `NetworkXError` – If \(nbunch\) is not a node or or sequence of nodes. If a node in \(nbunch\) is not hashable.

**See also:**

- `Graph.__iter__()`
Notes

When nbunch is an iterator, the returned iterator yields values directly from nbunch, becoming exhausted when nbunch is exhausted.

To test whether nbunch is a single node, one can use “if nbunch in self:”, even after processing with this routine.

If nbunch is not a node or a (possibly empty) sequence/iterator or None, a :networkx:Error is raised. Also, if any object in nbunch is not hashable, a :networkx:Error is raised.

Counting nodes edges and neighbors

<table>
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<td>MultiDiGraph.order()</td>
<td>Returns the number of nodes in the graph.</td>
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<tr>
<td>MultiDiGraph.degree</td>
<td>A DegreeView for the Graph as G.degree or G.degree().</td>
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<tr>
<td>MultiDiGraph.in_degree</td>
<td>A DegreeView for (node, in_degree) or in_degree for single node.</td>
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</tr>
<tr>
<td>MultiDiGraph.size([weight])</td>
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</tr>
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<td>Returns the number of edges between two nodes.</td>
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</table>

networkx.MultiDiGraph.order

\[ MultiDiGraph.order() \]

Returns the number of nodes in the graph.

**Returns**

- `nnodes` – The number of nodes in the graph.

**Return type**

- `int`

**See also:**

- `number_of_nodes()`, `__len__()`

**Examples**

```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.order()
3
```

networkx.MultiDiGraph.number_of_nodes

\[ MultiDiGraph.number_of_nodes() \]

Returns the number of nodes in the graph.

**Returns**

- `nnodes` – The number of nodes in the graph.

**Return type**

- `int`

**See also:**

- `order()`, `__len__()`
Examples

```python
>>> G = nx.path_graph(3)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.number_of_nodes()
3
```

networkx.MultiDiGraph.__len__

MultiDiGraph.__len__()  
Returns the number of nodes in the graph. Use: `len(G)`.

**Returns**

- **nnodes** – The number of nodes in the graph.

**Return type**

- **int**

**See also:**

- `number_of_nodes()`, `order()`

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> len(G)
4
```

networkx.MultiDiGraph.degree

**property** MultiDiGraph.degree  
A DegreeView for the Graph as G.degree or G.degree().

The node degree is the number of edges adjacent to the node. The weighted node degree is the sum of the edge weights for edges incident to that node.

This object provides an iterator for (node, degree) as well as lookup for the degree for a single node.

**Parameters**

- **nbunch** (*single node, container, or all nodes* (default= all nodes)) – The view will only report edges incident to these nodes.

- **weight** (*string or None*, optional (default=None)) – The name of an edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1. The degree is the sum of the edge weights adjacent to the node.

**Returns**

- If a single nodes is requested

- **deg** (*int*) – Degree of the node

- OR if multiple nodes are requested

- **nd_iter** (*iterator*) – The iterator returns two-tuples of (node, degree).

**See also:**

- `out_degree`, `in_degree`
Examples

```python
>>> G = nx.MultiDiGraph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.degree(0)  # node 0 with degree 1
1
>>> list(G.degree([0, 1, 2]))
[(0, 1), (1, 2), (2, 2)]
```

networkx.MultiDiGraph.in_degree

**property** `MultiDiGraph.in_degree`

A DegreeView for (node, in_degree) or in_degree for single node.

The node in-degree is the number of edges pointing in to the node. The weighted node degree is the sum of the edge weights for edges incident to that node.

This object provides an iterator for (node, degree) as well as lookup for the degree for a single node.

**Parameters**

- `nbunch` *(single node, container, or all nodes (default=all nodes))* – The view will only report edges incident to these nodes.
- `weight` *(string or None, optional (default=None))* – The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1. The degree is the sum of the edge weights adjacent to the node.

**Returns**

- **If a single node is requested**
- `deg` *(int)* – Degree of the node
- **OR if multiple nodes are requested**
- `nd_iter` *(iterator)* – The iterator returns two-tuples of (node, in-degree).

**See also:**

`degree`, `out_degree`

Examples

```python
>>> G = nx.MultiDiGraph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> G.in_degree(0)  # node 0 with degree 0
0
>>> list(G.in_degree([0, 1, 2]))
[(0, 0), (1, 1), (2, 1)]
```

networkx.MultiDiGraph.out_degree

**property** `MultiDiGraph.out_degree`

Returns an iterator for (node, out-degree) or out-degree for single node.

`out_degree(self, nbunch=None, weight=None)`
The node out-degree is the number of edges pointing out of the node. This function returns the out-degree for a
single node or an iterator for a bunch of nodes or if nothing is passed as argument.

Parameters

- **nbunch** *(single node, container, or all nodes (default=all nodes)) – The view will only
report edges incident to these nodes.*

- **weight** *(string or None, optional (default=None)) – The edge attribute that holds the numerical
value used as a weight. If None, then each edge has weight 1. The degree is the sum of
the edge weights.*

Returns

- *If a single node is requested*

  - **deg** *(int) – Degree of the node*

- *OR if multiple nodes are requested*

  - **nd_iter** *(iterator) – The iterator returns two-tuples of (node, out-degree).*

See also:

*degree, in_degree*

---

networkx.MultiDiGraph.size

**MultiDiGraph.size** *(weight=None)*

Returns the number of edges or total of all edge weights.

Parameters **weight** *(string or None, optional (default=None)) – The edge attribute that holds the numerical
value used as a weight. If None, then each edge has weight 1.*

Returns

- **size** *(int) – The number of edges or (if weight keyword is provided) the total weight sum.*

  If weight is None, returns an int. Otherwise a float (or more general numeric if the weights are
more general).

Return type numeric

See also:

*number_of_edges()
Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.size()
3

>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge('a', 'b', weight=2)
>>> G.add_edge('b', 'c', weight=4)
>>> G.size()
2
>>> G.size(weight='weight')
6.0
```

networkx.MultiDiGraph.number_of_edges

MultDiGraph.number_of_edges(u=None, v=None)  
Returns the number of edges between two nodes.

**Parameters** u, v (nodes, optional (Default=all edges)) – If u and v are specified, return the number of edges between u and v. Otherwise return the total number of all edges.

**Returns** nedges – The number of edges in the graph. If nodes u and v are specified return the number of edges between those nodes. If the graph is directed, this only returns the number of edges from u to v.

**Return type** int

See also:

size() size()

Examples

For undirected multigraphs, this method counts the total number of edges in the graph:

```python
>>> G = nx.MultiGraph()
>>> G.add_edges_from([(0, 1), (0, 1), (1, 2)])
[0, 1, 0]
>>> G.number_of_edges()
3
```

If you specify two nodes, this counts the total number of edges joining the two nodes:

```python
>>> G.number_of_edges(0, 1)
2
```

For directed multigraphs, this method can count the total number of directed edges from u to v:

```python
>>> G = nx.MultiDiGraph()
>>> G.add_edges_from([(0, 1), (0, 1), (1, 0)])
[0, 1, 0]
>>> G.number_of_edges(0, 1)
2
>>> G.number_of_edges(1, 0)
1
```
Making copies and subgraphs

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<td>Returns a copy of the graph.</td>
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<tr>
<td><code>MultiDiGraph.to_undirected([reciprocal, as_view])</code></td>
<td>Returns an undirected representation of the digraph.</td>
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<tr>
<td><code>MultiDiGraph.to_directed([as_view])</code></td>
<td>Returns a directed representation of the graph.</td>
</tr>
<tr>
<td><code>MultiDiGraph.subgraph(nodes)</code></td>
<td>Returns a SubGraph view of the subgraph induced on nodes.</td>
</tr>
<tr>
<td><code>MultiDiGraph.edge_subgraph(edges)</code></td>
<td>Returns the subgraph induced by the specified edges.</td>
</tr>
<tr>
<td><code>MultiDiGraph.reverse([copy])</code></td>
<td>Returns the reverse of the graph.</td>
</tr>
</tbody>
</table>

**networkx.MultiDiGraph.copy**

`MultiDiGraph.copy(as_view=False)`

Returns a copy of the graph.

The copy method by default returns an independent shallow copy of the graph and attributes. That is, if an attribute is a container, that container is shared by the original an the copy. Use Python’s `copy.deepcopy` for new containers.

If `as_view` is True then a view is returned instead of a copy.

**Notes**

All copies reproduce the graph structure, but data attributes may be handled in different ways. There are four types of copies of a graph that people might want.

Democopy – A “deepcopy” copies the graph structure as well as all data attributes and any objects they might contain. The entire graph object is new so that changes in the copy do not affect the original object. (see Python’s `copy.deepcopy`)

Data Reference (Shallow) – For a shallow copy the graph structure is copied but the edge, node and graph attribute dicts are references to those in the original graph. This saves time and memory but could cause confusion if you change an attribute in one graph and it changes the attribute in the other. NetworkX does not provide this level of shallow copy.

Independent Shallow – This copy creates new independent attribute dicts and then does a shallow copy of the attributes. That is, any attributes that are containers are shared between the new graph and the original. This is exactly what `dict.copy()` provides. You can obtain this style copy using:

```python
>>> G = nx.path_graph(5)
>>> H = G.copy()
>>> H = G.copy(as_view=False)
>>> H = nx.Graph(G)
>>> H = G.__class__(G)
```

Fresh Data – For fresh data, the graph structure is copied while new empty data attribute dicts are created. The resulting graph is independent of the original and it has no edge, node or graph attributes. Fresh copies are not enabled. Instead use:

```python
>>> H = G.__class__()
>>> H.add_nodes_from(G)
>>> H.add_edges_from(G.edges)
```
View – Inspired by dict-views, graph-views act like read-only versions of the original graph, providing a copy of the original structure without requiring any memory for copying the information.

See the Python copy module for more information on shallow and deep copies, https://docs.python.org/2/library/copy.html.

Parameters as_view (bool, optional (default=False)) – If True, the returned graph-view provides a read-only view of the original graph without actually copying any data.

Returns G – A copy of the graph.

Return type Graph

See also:

to_directed() return a directed copy of the graph.

Examples

```python
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> H = G.copy()
```

networkx.MultiDiGraph.to_undirected

MultiDiGraph.to_undirected (reciprocal=False, as_view=False)

Returns an undirected representation of the digraph.

Parameters

- reciprocal (bool (optional)) – If True only keep edges that appear in both directions in the original digraph.
- as_view (bool (optional, default=False)) – If True return an undirected view of the original directed graph.

Returns G – An undirected graph with the same name and nodes and with edge (u, v, data) if either (u, v, data) or (v, u, data) is in the digraph. If both edges exist in digraph and their edge data is different, only one edge is created with an arbitrary choice of which edge data to use. You must check and correct for this manually if desired.

Return type MultiGraph

See also:

MultiGraph(), copy(), add_edge(), add_edges_from()

Notes

This returns a “deepcopy” of the edge, node, and graph attributes which attempts to completely copy all of the data and references.

This is in contrast to the similar D=MultiGraph(G) which returns a shallow copy of the data.

See the Python copy module for more information on shallow and deep copies, https://docs.python.org/2/library/copy.html.

Warning: If you have subclassed MultiDiGraph to use dict-like objects in the data structure, those changes do not transfer to the MultiGraph created by this method.
Examples

```python
>>> G = nx.path_graph(2)  # or MultiGraph, etc
>>> H = G.to_directed()
>>> list(H.edges)
[(0, 1), (1, 0)]
>>> G2 = H.to_undirected()
>>> list(G2.edges)
[(0, 1)]
```

`networkx.MultiDiGraph.to_directed`

`MultiDiGraph.to_directed(as_view=False)`

Returns a directed representation of the graph.

Returns `G` – A directed graph with the same name, same nodes, and with each edge (u, v, data) replaced by two directed edges (u, v, data) and (v, u, data).

Return type: `MultiDiGraph`

Notes

This returns a “deepcopy” of the edge, node, and graph attributes which attempts to completely copy all of the data and references.

This is in contrast to the similar D=DiGraph(G) which returns a shallow copy of the data.

See the Python copy module for more information on shallow and deep copies, `https://docs.python.org/2/library/copy.html`.

Warning: If you have subclassed MultiGraph to use dict-like objects in the data structure, those changes do not transfer to the MultiDiGraph created by this method.

Examples

```python
>>> G = nx.Graph()  # or MultiGraph, etc
>>> G.add_edge(0, 1)
>>> H = G.to_directed()
>>> list(H.edges)
[(0, 1), (1, 0)]
If already directed, return a (deep) copy
```  

`networkx.MultiDiGraph.subgraph`

`MultiDiGraph.subgraph(nodes)`

Returns a SubGraph view of the subgraph induced on `nodes`.  

2.2. Basic graph types 125
The induced subgraph of the graph contains the nodes in \texttt{nodes} and the edges between those nodes.

**Parameters** \texttt{nodes} (*list, iterable*) – A container of nodes which will be iterated through once.

**Returns** \texttt{G} – A subgraph view of the graph. The graph structure cannot be changed but node/edge attributes can and are shared with the original graph.

**Return type** SubGraph View

**Notes**

The graph, edge and node attributes are shared with the original graph. Changes to the graph structure is ruled out by the view, but changes to attributes are reflected in the original graph.

To create a subgraph with its own copy of the edge/node attributes use: \texttt{G.subgraph(nodes).copy()}

For an inplace reduction of a graph to a subgraph you can remove nodes: \texttt{G.remove_nodes_from([n for n in G if n not in set(nodes)])}

Subgraph views are sometimes NOT what you want. In most cases where you want to do more than simply look at the induced edges, it makes more sense to just create the subgraph as its own graph with code like:

```python
# Create a subgraph SG based on a (possibly multigraph) G
SG = G.__class__()
SG.add_nodes_from((n, G.nodes[n]) for n in largest_wcc)
if SG.is_multigraph():
    SG.add_edges_from((n, nbr, key, d)
                        for n, nbrs in G.adj.items() if n in largest_wcc
                        for nbr, keydict in nbrs.items() if nbr in largest_wcc
                        for key, d in keydict.items())
else:
    SG.add_edges_from((n, nbr, d)
                        for n, nbrs in G.adj.items() if n in largest_wcc
                        for nbr, d in nbrs.items() if nbr in largest_wcc)
SG.graph.update(G.graph)
```

**Examples**

```python
>>> G = nx.path_graph(4)  # DiGraph, MultiGraph, MultiDiGraph, etc
>>> H = G.subgraph([0, 1, 2])
>>> list(H.edges)
[(0, 1), (1, 2)]
```

**networkx.MultiDiGraph.edge_subgraph**

\texttt{MultiDiGraph.edge_subgraph(edges)}

Returns the subgraph induced by the specified edges.

The induced subgraph contains each edge in \texttt{edges} and each node incident to any one of those edges.

**Parameters** \texttt{edges} (*iterable*) – An iterable of edges in this graph.

**Returns** \texttt{G} – An edge-induced subgraph of this graph with the same edge attributes.

**Return type** \texttt{Graph}
Notes

The graph, edge, and node attributes in the returned subgraph view are references to the corresponding attributes in the original graph. The view is read-only.

To create a full graph version of the subgraph with its own copy of the edge or node attributes, use:

```python
>>> G.edge_subgraph(edges).copy()
```

Examples

```python
>>> G = nx.path_graph(5)
>>> H = G.edge_subgraph([(0, 1), (3, 4)])
>>> list(H.nodes)
[0, 1, 3, 4]
>>> list(H.edges)
[(0, 1), (3, 4)]
```

`networkx.MultiDiGraph.reverse`

`MultiDiGraph.reverse(copy=True)`

Returns the reverse of the graph.

The reverse is a graph with the same nodes and edges but with the directions of the edges reversed.

Parameters

- `copy` (bool optional (default=True)) – If True, return a new DiGraph holding the reversed edges. If False, the reverse graph is created using a view of the original graph.

2.2.5 Ordered Graphs—Consistently ordered graphs

Consistently ordered variants of the default base classes. Note that if you are using Python 3.6+, you shouldn’t need these classes because the dicts in Python 3.6+ are ordered. Note also that there are many differing expectations for the word “ordered” and that these classes may not provide the order you expect. The intent here is to give a consistent order not a particular order.

The Ordered (Di/Multi/MultiDi) Graphs give a consistent order for reporting of nodes and edges. The order of node reporting agrees with node adding, but for edges, the order is not necessarily the order that the edges were added.

In general, you should use the default (i.e., unordered) graph classes. However, there are times (e.g., when testing) when you may need the order preserved.

Special care is required when using subgraphs of the Ordered classes. The order of nodes in the subclass is not necessarily the same order as the original class. In general it is probably better to avoid using subgraphs and replace with code similar to:

```python
# instead of SG = G.subgraph(ordered_nodes)
SG=nx.OrderedGraph()
SG.add_nodes_from(ordered_nodes)
SG.add_edges_from((u, v) for (u, v) in G.edges() if u in SG if v in SG)
```

`class OrderedGraph` (incoming_graph_data=None, **attr)

Consistently ordered variant of `Graph`.

`class OrderedDiGraph` (incoming_graph_data=None, **attr)

Consistently ordered variant of `DiGraph`. 127
class OrderedMultiGraph(incoming_graph_data=None, **attr)
    Consistently ordered variant of MultiGraph.

class OrderedMultiDiGraph(incoming_graph_data=None, **attr)
    Consistently ordered variant of MultiDiGraph.

Note: NetworkX uses dicts to store the nodes and neighbors in a graph. So the reporting of nodes and edges for the base graph classes will not necessarily be consistent across versions and platforms. If you need the order of nodes and edges to be consistent (e.g., when writing automated tests), please see OrderedGraph, OrderedDiGraph, OrderedMultiGraph, or OrderedMultiDiGraph, which behave like the base graph classes but give a consistent order for reporting of nodes and edges.

2.3 Graph Views

View of Graphs as SubGraph, Reverse, Directed, Undirected.

In some algorithms it is convenient to temporarily morph a graph to exclude some nodes or edges. It should be better to do that via a view than to remove and then re-add. In other algorithms it is convenient to temporarily morph a graph to reverse directed edges, or treat a directed graph as undirected, etc. This module provides those graph views.

The resulting views are essentially read-only graphs that report data from the original graph object. We provide an attribute G._graph which points to the underlying graph object.

Note: Since graphviews look like graphs, one can end up with view-of-view-of-view chains. Be careful with chains because they become very slow with about 15 nested views. For the common simple case of node induced subgraphs created from the graph class, we short-cut the chain by returning a subgraph of the original graph directly rather than a subgraph of a subgraph. We are careful not to disrupt any edge filter in the middle subgraph. In general, determining how to short-cut the chain is tricky and much harder with restricted_views than with induced subgraphs. Often it is easiest to use .copy() to avoid chains.

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<tr>
<th>Function</th>
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<tr>
<td>generic_graph_view(G[, create_using])</td>
<td>View of G applying a filter on nodes and edges.</td>
</tr>
<tr>
<td>subgraph_view(G[, filter_node, filter_edge])</td>
<td>View of G with edge directions reversed</td>
</tr>
</tbody>
</table>

2.3.1 networkx.classes.graphviews.generic_graph_view

generic_graph_view(G, create_using=None)

2.3.2 networkx.classes.graphviews.subgraph_view

subgraph_view(G[, filter_node=<function no_filter>, filter_edge=<function no_filter>])
    View of G applying a filter on nodes and edges.

    subgraph_view provides a read-only view of the input graph that excludes nodes and edges based on the outcome of two filter functions filter_node and filter_edge.

    The filter_node function takes one argument — the node — and returns True if the node should be included in the subgraph, and False if it should not be included.

    The filter_edge function takes two arguments — the nodes describing an edge — and returns True if the edge should be included in the subgraph, and False if it should not be included.
Both node and edge filter functions are called on graph elements as they are queried, meaning there is no up-front cost to creating the view.

**Parameters**

- **G** *(networkx.Graph)* – A directed/undirected graph/multigraph
- **filter_node** *(callable, optional)* – A function taking a node as input, which returns True if the node should appear in the view.
- **filter_edge** *(callable, optional)* – A function taking as input the two nodes describing an edge, which returns True if the edge should appear in the view.

**Returns** graph – A read-only graph view of the input graph.

**Return type** networkx.Graph

**Examples**

```python
>>> import networkx as nx
>>> G = nx.path_graph(6)

Filter functions operate on the node, and return True if the node should appear in the view:

```python
>>> def filter_node(n1):
...     return n1 != 5
...

>>> view = nx.subgraph_view(G, filter_node=filter_node)

>>> view.nodes()
NodeView((0, 1, 2, 3, 4))
```

We can use a closure pattern to filter graph elements based on additional data — for example, filtering on edge data attached to the graph:

```python
>>> G[3][4]['cross_me'] = False

>>> def filter_edge(n1, n2):
...     return G[n1][n2].get('cross_me', True)
...

>>> view = nx.subgraph_view(G, filter_edge=filter_edge)

>>> view.edges()
EdgeView([(0, 1), (1, 2), (2, 3), (4, 5)])
```

```python
>>> view = nx.subgraph_view(G, filter_node=filter_node, filter_edge=filter_edge)

>>> view.nodes()
NodeView((0, 1, 2, 3))

>>> view.edges()
EdgeView([(0, 1), (1, 2), (2, 3)])
```
2.3.3 networkx.classes.graphviews.reverse_view

reverse_view(G)

View of G with edge directions reversed

reverse_view returns a read-only view of the input graph where edge directions are reversed.

Identical to digraph.reverse(copy=False)

Parameters  

- **G** (*networkx.DiGraph*)

Returns  

- **graph** (*networkx.DiGraph*)

Examples

```python
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_edge(1, 2)
>>> G.add_edge(2, 3)
>>> G.edges()
OutEdgeView([(1, 2), (2, 3)])

>>> view = nx.reverse_view(G)
>>> view.edges()
OutEdgeView([(2, 1), (3, 2)])
```
3.1 Approximations and Heuristics

Approximations of graph properties and Heuristic functions for optimization problems.

**Warning:** The approximation submodule is not imported in the top-level networkx.

These functions can be imported with `from networkx.algorithms import approximation`.

### 3.1.1 Connectivity

Fast approximation for node connectivity

```python
all_pairs_node_connectivity(G[, nbunch, cutoff])
```

Compute node connectivity between all pairs of nodes.

```python
local_node_connectivity(G, source, target[, ...])
```

Compute node connectivity between source and target.

```python
node_connectivity(G[, s, t])
```

Returns an approximation for node connectivity for a graph or digraph G.

*networkx.algorithms.approximation.connectivity.all_pairs_node_connectivity*

**all_pairs_node_connectivity** *(G, nbunch=None, cutoff=None)*

Compute node connectivity between all pairs of nodes.

Pairwise or local node connectivity between two distinct and nonadjacent nodes is the minimum number of nodes that must be removed (minimum separating cutset) to disconnect them. By Menger’s theorem, this is equal to the number of node independent paths (paths that share no nodes other than source and target). Which is what we compute in this function.

This algorithm is a fast approximation that gives an strict lower bound on the actual number of node independent paths between two nodes. It works for both directed and undirected graphs.

**Parameters**

- `G` *(NetworkX graph)*

---

• **nbunch** *(container)* – Container of nodes. If provided node connectivity will be computed only over pairs of nodes in nbunch.

• **cutoff** *(integer)* – Maximum node connectivity to consider. If None, the minimum degree of source or target is used as a cutoff in each pair of nodes. Default value None.

**Returns** K – Dictionary, keyed by source and target, of pairwise node connectivity

**Return type** dictionary

**See also:**

`local_node_connectivity()`, `node_connectivity()`

**References**

`networkx.algorithms.approximation.connectivity.local_node_connectivity`

`local_node_connectivity(G, source, target, cutoff=None)`

Compute node connectivity between source and target.

Pairwise or local node connectivity between two distinct and nonadjacent nodes is the minimum number of nodes that must be removed (minimum separating cutset) to disconnect them. By Menger’s theorem, this is equal to the number of node independent paths (paths that share no nodes other than source and target). Which is what we compute in this function.

This algorithm is a fast approximation that gives an strict lower bound on the actual number of node independent paths between two nodes\(^1\). It works for both directed and undirected graphs.

**Parameters**

• **G** *(NetworkX graph)*

• **source** *(node)* – Starting node for node connectivity

• **target** *(node)* – Ending node for node connectivity

• **cutoff** *(integer)* – Maximum node connectivity to consider. If None, the minimum degree of source or target is used as a cutoff. Default value None.

**Returns** k – pairwise node connectivity

**Return type** integer

**Examples**

```python
>>> # Platonic octahedral graph has node connectivity 4
>>> # for each non adjacent node pair
>>> from networkx.algorithms import approximation as approx
>>> G = nx.octahedral_graph()
>>> approx.local_node_connectivity(G, 0, 5)
4
```

---


[http://eclectic.ss.uci.edu/~drwhite/working.pdf](http://eclectic.ss.uci.edu/~drwhite/working.pdf)
Notes

This algorithm\(^1\) finds node independents paths between two nodes by computing their shortest path using BFS, marking the nodes of the path found as ‘used’ and then searching other shortest paths excluding the nodes marked as used until no more paths exist. It is not exact because a shortest path could use nodes that, if the path were longer, may belong to two different node independent paths. Thus it only guarantees an strict lower bound on node connectivity.

Note that the authors propose a further refinement, losing accuracy and gaining speed, which is not implemented yet.

See also:

all_pairs_node_connectivity(), node_connectivity()

References

networkx.algorithms.approximation.connectivity.node_connectivity

def node_connectivity(G, s=None, t=None)
    Returns an approximation for node connectivity for a graph or digraph G.

    Node connectivity is equal to the minimum number of nodes that must be removed to disconnect G or render it trivial. By Menger’s theorem, this is equal to the number of node independent paths (paths that share no nodes other than source and target).

    If source and target nodes are provided, this function returns the local node connectivity: the minimum number of nodes that must be removed to break all paths from source to target in G.

    This algorithm is based on a fast approximation that gives an strict lower bound on the actual number of node independent paths between two nodes\(^1\). It works for both directed and undirected graphs.

Parameters

* G (NetworkX graph) – Undirected graph
* s (node) – Source node. Optional. Default value: None.
* t (node) – Target node. Optional. Default value: None.

Returns K – Node connectivity of G, or local node connectivity if source and target are provided.

Return type integer

Examples

```python
>>> # Platonic octahedral graph is 4-node-connected
>>> from networkx.algorithms import approximation as approx
>>> G = nx.octahedral_graph()
>>> approx.node_connectivity(G)
4
```


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Notes

This algorithm\(^1\) finds node independents paths between two nodes by computing their shortest path using BFS, marking the nodes of the path found as ‘used’ and then searching other shortest paths excluding the nodes marked as used until no more paths exist. It is not exact because a shortest path could use nodes that, if the path were longer, may belong to two different node independent paths. Thus it only guarantees an strict lower bound on node connectivity.

See also:

- `all_pairs_node_connectivity()`, `local_node_connectivity()`

References

3.1.2 K-components

Fast approximation for k-component structure

```
_k_components(G[, min_density])
```

Returns the approximate k-component structure of a graph G.

```
networkx.algorithms.approximation.kcomponents.k_components
```

\(k\)_components \((G, min\_density=0.95)\)

Returns the approximate k-component structure of a graph G.

A \(k\)-component is a maximal subgraph of a graph \(G\) that has, at least, node connectivity \(k\): we need to remove at least \(k\) nodes to break it into more components. \(k\)-components have an inherent hierarchical structure because they are nested in terms of connectivity: a connected graph can contain several 2-components, each of which can contain one or more 3-components, and so forth.

This implementation is based on the fast heuristics to approximate the \(k\)-component structure of a graph\(^1\). Which, in turn, it is based on a fast approximation algorithm for finding good lower bounds of the number of node independent paths between two nodes\(^2\).

Parameters

- \(G\) (NetworkX graph) – Undirected graph
- \(\text{min}\_\text{density}\) (Float) – Density relaxation threshold. Default value 0.95

Returns \(k\)_components – Dictionary with connectivity level \(k\) as key and a list of sets of nodes that form a \(k\)-component of level \(k\) as values.

Return type dict

Examples

```python
>>> # Petersen graph has 10 nodes and it is triconnected, thus all
>>> # nodes are in a single component on all three connectivity levels
>>> from networkx.algorithms import approximation as apxa
```


NetworkX Reference, Release 2.5rc1.dev20191214022633

(continued from previous page)

```python
>>> G = nx.petersen_graph()
>>> k_components = apxa.k_components(G)
```

Notes

The logic of the approximation algorithm for computing the k-component structure\(^1\) is based on repeatedly applying simple and fast algorithms for k-cores and biconnected components in order to narrow down the number of pairs of nodes over which we have to compute White and Newman’s approximation algorithm for finding node independent paths\(^2\). More formally, this algorithm is based on Whitney’s theorem, which states an inclusion relation among node connectivity, edge connectivity, and minimum degree for any graph G. This theorem implies that every k-component is nested inside a k-edge-component, which in turn, is contained in a k-core. Thus, this algorithm computes node independent paths among pairs of nodes in each biconnected part of each k-core, and repeats this procedure for each k from 3 to the maximal core number of a node in the input graph.

Because, in practice, many nodes of the core of level k inside a bicomponent actually are part of a component of level k, the auxiliary graph needed for the algorithm is likely to be very dense. Thus, we use a complement graph data structure (see AntiGraph) to save memory. AntiGraph only stores information of the edges that are not present in the actual auxiliary graph. When applying algorithms to this complement graph data structure, it behaves as if it were the dense version.

See also:

- `k_components()`

References

3.1.3 Clique

Functions for computing large cliques.

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<td>Find the Maximum Clique</td>
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<td><code>clique_removal(G)</code></td>
<td>Repeatedly remove cliques from the graph.</td>
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<td><code>large_clique_size(G)</code></td>
<td>Find the size of a large clique in a graph.</td>
</tr>
</tbody>
</table>

networkx.algorithms.approximation.clique.max_clique

`max_clique(G)`

Find the Maximum Clique

Finds the \(O(|V|/(\log |V|)^2)\) apx of maximum clique/independent set in the worst case.

Parameters: G (NetworkX graph) – Undirected graph

Returns: clique – The apx-maximum clique of the graph

Return type: set

Notes

A clique in an undirected graph \(G = (V, E)\) is a subset of the vertex set \(C \subseteq V\) such that for every two vertices in \(C\) there exists an edge connecting the two. This is equivalent to saying that the subgraph induced by

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C is complete (in some cases, the term clique may also refer to the subgraph).

A maximum clique is a clique of the largest possible size in a given graph. The clique number $\omega(G)$ of a graph $G$ is the number of vertices in a maximum clique in $G$. The intersection number of $G$ is the smallest number of cliques that together cover all edges of $G$.

https://en.wikipedia.org/wiki/Maximum_clique

References

networkx.algorithms.approximation.clique.clique_removal

clique_removal $(G)$
Repeatedly remove cliques from the graph.

Results in a $O(|V|/(\log |V|)^2)$ approximation of maximum clique and independent set. Returns the largest independent set found, along with found maximal cliques.

Parameters $G$ (NetworkX graph) – Undirected graph

Returns max_ind_cliques – 2-tuple of Maximal Independent Set and list of maximal cliques (sets).

Return type (set, list) tuple

References

networkx.algorithms.approximation.clique.large_clique_size

large_clique_size $(G)$
Find the size of a large clique in a graph.

A clique is a subset of nodes in which each pair of nodes is adjacent. This function is a heuristic for finding the size of a large clique in the graph.

Parameters $G$ (NetworkX graph)

Returns The size of a large clique in the graph.

Return type int

Notes

This implementation is from$^1$. Its worst case time complexity is $O(nd^2)$, where $n$ is the number of nodes in the graph and $d$ is the maximum degree.

This function is a heuristic, which means it may work well in practice, but there is no rigorous mathematical guarantee on the ratio between the returned number and the actual largest clique size in the graph.

References

See also:

networkx.algorithms.approximation.clique.max_clique() A function that returns an approximate maximum clique with a guarantee on the approximation ratio.

networkx.algorithms.clique Functions for finding the exact maximum clique in a graph.

3.1.4 Clustering

average_clustering(G[, trials, seed]) Estimates the average clustering coefficient of G.

networkx.algorithms.approximation.clustering_coefficient.average_clustering

average_clustering (G, trials=1000, seed=None) Estimates the average clustering coefficient of G.

The local clustering of each node in G is the fraction of triangles that actually exist over all possible triangles in its neighborhood. The average clustering coefficient of a graph G is the mean of local clusterings. This function finds an approximate average clustering coefficient for G by repeating n times (defined in trials) the following experiment: choose a node at random, choose two of its neighbors at random, and check if they are connected. The approximate coefficient is the fraction of triangles found over the number of trials¹.

Parameters

- G (NetworkX graph)
- trials (integer) – Number of trials to perform (default 1000).
- seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns c – Approximated average clustering coefficient.

Return type float

References

3.1.5 Dominating Set

Functions for finding node and edge dominating sets.

A dominating set for an undirected graph G with vertex set V and edge set E is a subset D of V such that every vertex not in D is adjacent to at least one member of D. An edge dominating set is a subset F of E such that every edge not in F is incident to an endpoint of at least one edge in F.

min_weighted_dominating_set(G[, weight]) Returns a dominating set that approximates the minimum weight node dominating set.

min_edge_dominating_set(G) Returns minimum cardinality edge dominating set.

References

• **G** (*NetworkX* graph) – Undirected graph.
• **weight** (*string*) – The node attribute storing the weight of an node. If provided, the node attribute with this key must be a number for each node. If not provided, each node is assumed to have weight one.

**Returns** _min_weight_dominating_set_ – A set of nodes, the sum of whose weights is no more than \((\log w(V)) w(V^*)\), where \(w(V)\) denotes the sum of the weights of each node in the graph and \(w(V^*)\) denotes the sum of the weights of each node in the minimum weight dominating set.

**Return type** *set*

**Notes**

This algorithm computes an approximate minimum weighted dominating set for the graph \(G\). The returned solution has weight \((\log w(V)) w(V^*)\), where \(w(V)\) denotes the sum of the weights of each node in the graph and \(w(V^*)\) denotes the sum of the weights of each node in the minimum weight dominating set for the graph.

This implementation of the algorithm runs in \(O(m)\) time, where \(m\) is the number of edges in the graph.

**References**

*networkx.algorithms.approximation.dominating_set.min_edge_dominating_set*

**min_edge_dominating_set** \((G)\)

Returns minimum cardinality edge dominating set.

**Parameters** **G** (*NetworkX* graph) – Undirected graph

**Returns** _min_edge_dominating_set_ – Returns a set of dominating edges whose size is no more than 2 * OPT.

**Return type** *set*

**Notes**

The algorithm computes an approximate solution to the edge dominating set problem. The result is no more than 2 * OPT in terms of size of the set. Runtime of the algorithm is \(O(|E|)\).

### 3.1.6 Independent Set

**Independent Set**

Independent set or stable set is a set of vertices in a graph, no two of which are adjacent. That is, it is a set \(I\) of vertices such that for every two vertices in \(I\), there is no edge connecting the two. Equivalently, each edge in the graph has at most one endpoint in \(I\). The size of an independent set is the number of vertices it contains.

A maximum independent set is a largest independent set for a given graph \(G\) and its size is denoted \(\alpha(G)\). The problem of finding such a set is called the maximum independent set problem and is an NP-hard optimization problem. As such, it is unlikely that there exists an efficient algorithm for finding a maximum independent set of a graph.

**Wikipedia:** Independent set

Independent set algorithm is based on the following paper:
$O(|V|/(\log|V|)^2)$ apx of maximum clique/independent set.


```
maximum_independent_set(G) Returns an approximate maximum independent set.
```

```
maximum_independent_set(G)
Returns an approximate maximum independent set.

Parameters  G (NetworkX graph) – Undirected graph

Returns  iset – The apx-maximum independent set

Return type  Set
```

**Notes**

Finds the $O(|V|/(\log|V|)^2)$ apx of independent set in the worst case.

**References**

### 3.1.7 Matching

**Graph Matching**

Given a graph $G = (V,E)$, a matching $M$ in $G$ is a set of pairwise non-adjacent edges; that is, no two edges share a common vertex.

Wikipedia: Matching

```
min_maximal_matching(G) Returns the minimum maximal matching of $G$.
```

```
min_maximal_matching(G)
Returns the minimum maximal matching of $G$. That is, out of all maximal matchings of the graph $G$, the smallest is returned.

Parameters  G (NetworkX graph) – Undirected graph

Returns  min_maximal_matching – Returns a set of edges such that no two edges share a common endpoint and every edge not in the set shares some common endpoint in the set. Cardinality will be $2*OPT$ in the worst case.

Return type  set
```

**Notes**

The algorithm computes an approximate solution fo the minimum maximal cardinality matching problem. The solution is no more than $2 * OPT$ in size. Runtime is $O(|E|)$.  

3.1. Approximations and Heuristics
3.1.8 Ramsey

Ramsey numbers.

\[ \text{ramsey}_R^2(G) \] Approximately computes the Ramsey number \( R(2; s, t) \) for graph.

**networkx.algorithms.approximation.ramsey.ramsey_R2**

\text{ramsey}_R^2 (G) \]

Approximately computes the Ramsey number \( R(2; s, t) \) for graph.

**Parameters**

\- \( G \) (*NetworkX graph*) – Undirected graph

**Returns**

\- \text{max\_pair} – Maximum clique, Maximum independent set.

**Return type**

(set, set) tuple

3.1.9 Steiner Tree

**metric_closure**(*G*, \( \text{weight} \))

Return the metric closure of a graph.

**networkx.algorithms.approximation.steiner_tree.metric_closure**

metric_closure \( (G, \text{weight}= 'weight') \) \]

Return the metric closure of a graph.

The metric closure of a graph \( G \) is the complete graph in which each edge is weighted by the shortest path distance between the nodes in \( G \).

**Parameters**

\- \( G \) (*NetworkX graph*)

**Returns**

Metric closure of the graph \( G \).

**Return type**

NetworkX graph

**steiner_tree**(*G*, \( \text{terminal\_nodes} \), \( \text{weight} \))

Return an approximation to the minimum Steiner tree of a graph.

**networkx.algorithms.approximation.steiner_tree.steiner_tree**

steiner_tree \( (G, \text{terminal\_nodes}, \text{weight}= 'weight') \) \]

Return an approximation to the minimum Steiner tree of a graph.

**Parameters**

\- \( G \) (*NetworkX graph*)

\- \text{terminal\_nodes} (*list*) – A list of terminal nodes for which minimum steiner tree is to be found.

**Returns**

Approximation to the minimum steiner tree of \( G \) induced by \text{terminal\_nodes}.

**Return type**

NetworkX graph
Notes

Steiner tree can be approximated by computing the minimum spanning tree of the subgraph of the metric closure of the graph induced by the terminal nodes, where the metric closure of $G$ is the complete graph in which each edge is weighted by the shortest path distance between the nodes in $G$. This algorithm produces a tree whose weight is within a $(2 - (2 / t))$ factor of the weight of the optimal Steiner tree where $t$ is number of terminal nodes.

3.1.10 Treewidth

Functions for computing treewidth decomposition.

Treewidth of an undirected graph is a number associated with the graph. It can be defined as the size of the largest vertex set (bag) in a tree decomposition of the graph minus one.

Wikipedia: Treewidth

The notions of treewidth and tree decomposition have gained their attractiveness partly because many graph and network problems that are intractable (e.g., NP-hard) on arbitrary graphs become efficiently solvable (e.g., with a linear time algorithm) when the treewidth of the input graphs is bounded by a constant\textsuperscript{12}.

There are two different functions for computing a tree decomposition: \texttt{treewidth_min_degree()} and \texttt{treewidth_min_fill_in()}.

<table>
<thead>
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<th>Function</th>
<th>Description</th>
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<tbody>
<tr>
<td>\texttt{treewidth_min_degree()}</td>
<td>Returns a treewidth decomposition using the Minimum Degree heuristic.</td>
</tr>
<tr>
<td>\texttt{treewidth_min_fill_in()}</td>
<td>Returns a treewidth decomposition using the Minimum Fill-in heuristic.</td>
</tr>
</tbody>
</table>

\texttt{networkx.algorithms.approximation.treewidth.treewidth_min_degree}

\texttt{treewidth_min_degree}(G)  
Returns a treewidth decomposition using the Minimum Degree heuristic.

The heuristic chooses the nodes according to their degree, i.e., first the node with the lowest degree is chosen, then the graph is updated and the corresponding node is removed. Next, a new node with the lowest degree is chosen, and so on.

Parameters  
\texttt{G (NetworkX graph)}

Returns  
Treewidth decomposition – 2-tuple with treewidth and the corresponding decomposed tree.

Return type  
(int, Graph) tuple

\texttt{networkx.algorithms.approximation.treewidth.treewidth_min_fill_in}

\texttt{treewidth_min_fill_in}(G)  
Returns a treewidth decomposition using the Minimum Fill-in heuristic.

The heuristic chooses a node from the graph, where the number of edges added turning the neighbourhood of the chosen node into clique is as small as possible.

\begin{itemize}
  \url{http://dx.doi.org/10.1016/j.ic.2009.03.008}
\end{itemize}
**Parameters**  
$G$ (*NetworkX graph*)

**Returns**  
Tree width decomposition – 2-tuple with treewidth and the corresponding decomposed tree.

**Return type**  
(int, *Graph*) tuple

### 3.1.11 Vertex Cover

Functions for computing an approximate minimum weight vertex cover.  
A vertex cover is a subset of nodes such that each edge in the graph is incident to at least one node in the subset.

```python
min_weighted_vertex_cover(G[, weight])  
Returns an approximate minimum weighted vertex cover.
```

#### `networkx.algorithms.approximation.vertex_cover.min_weighted_vertex_cover`

**min_weighted_vertex_cover**  
$G$, $weight=None$  
Returns an approximate minimum weighted vertex cover.  
The set of nodes returned by this function is guaranteed to be a vertex cover, and the total weight of the set is guaranteed to be at most twice the total weight of the minimum weight vertex cover. In other words,

$$w(S) \leq 2 \times w(S^*),$$

where $S$ is the vertex cover returned by this function, $S^*$ is the vertex cover of minimum weight out of all vertex covers of the graph, and $w$ is the function that computes the sum of the weights of each node in that given set.

**Parameters**

- $G$ (*NetworkX graph*)
- $weight$ (*string*, optional (default = None)) – If None, every node has weight 1. If a string, use this node attribute as the node weight. A node without this attribute is assumed to have weight 1.

**Returns**  
`min_weighted_cover` – Returns a set of nodes whose weight sum is no more than twice the weight sum of the minimum weight vertex cover.

**Return type**  
set

**Notes**

For a directed graph, a vertex cover has the same definition: a set of nodes such that each edge in the graph is incident to at least one node in the set. Whether the node is the head or tail of the directed edge is ignored.

This is the local-ratio algorithm for computing an approximate vertex cover. The algorithm greedily reduces the costs over edges, iteratively building a cover. The worst-case runtime of this implementation is $O(m \log n)$, where $n$ is the number of nodes and $m$ the number of edges in the graph.

**References**

### 3.2 Assortativity
3.2.1 Assortativity

**degree_assortativity_coefficient** (G[, x, y,...])  
Compute degree assortativity of graph.

**attribute_assortativity_coefficient** (G, attribute)  
Compute assortativity for node attributes.

**numeric_assortativity_coefficient** (G, attribute)  
Compute assortativity for numerical node attributes.

**degree_pearson_correlation_coefficient** (G,...)  
Compute degree assortativity of graph.

**networkx.algorithms.assortativity.degree_assortativity_coefficient**

**degree_assortativity_coefficient** (G, x='out', y='in', weight=None, nodes=None)  
Compute degree assortativity of graph.

Assortativity measures the similarity of connections in the graph with respect to the node degree.

**Parameters**

- **G** *(NetworkX graph)*
- **x** *(string ('in','out'))* – The degree type for source node (directed graphs only).
- **y** *(string ('in','out'))* – The degree type for target node (directed graphs only).
- **weight** *(string or None, optional (default=None))* – The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1. The degree is the sum of the edge weights adjacent to the node.
- **nodes** *(list or iterable (optional))* – Compute degree assortativity only for nodes in container. The default is all nodes.

**Returns**  
**r** – Assortativity of graph by degree.

**Return type**  
**float**

**Examples**

```python
>>> G = nx.path_graph(4)
>>> r = nx.degree_assortativity_coefficient(G)
>>> print("%.3f"%r)
-0.5
```

**See also:**

- **attribute_assortativity_coefficient()**,  
- **numeric_assortativity_coefficient()**,  
- **neighbor_connectivity()**,  
- **degree_mixing_dict()**,  
- **degree_mixing_matrix()**

**Notes**

This computes Eq. (21) in Ref.\(^1\), where \(e\) is the joint probability distribution (mixing matrix) of the degrees. If \(G\) is directed then the matrix \(e\) is the joint probability of the user-specified degree type for the source and target.

References

networkx.algorithms.assortativity.attribute_assortativity_coefficient

attribute_assortativity_coefficient \( (G, \text{attribute}, \text{nodes}=\text{None}) \)

Compute assortativity for node attributes.

Assortativity measures the similarity of connections in the graph with respect to the given attribute.

Parameters

- \( G \) (\text{NetworkX graph})
- \( \text{attribute} \) (\text{string}) – Node attribute key
- \( \text{nodes} \) (\text{list or iterable (optional)}) – Compute attribute assortativity for nodes in container. The default is all nodes.

Returns \( r \) – Assortativity of graph for given attribute

Return type \( \text{float} \)

Examples

```python
>>> G = nx.Graph()
>>> G.add_nodes_from([0,1], color='red')
>>> G.add_nodes_from([2,3], color='blue')
>>> G.add_edges_from([(0,1), (2,3)])
>>> print(nx.attribute_assortativity_coefficient(G, 'color'))
1.0
```

Notes

This computes Eq. (2) in Ref.\(^1\), \((\text{trace}(M)-\text{sum}(M^2))/(1-\text{sum}(M^2))\), where \( M \) is the joint probability distribution (mixing matrix) of the specified attribute.

References

networkx.algorithms.assortativity.numeric_assortativity_coefficient

numeric_assortativity_coefficient \( (G, \text{attribute}, \text{nodes}=\text{None}) \)

Compute assortativity for numerical node attributes.

Assortativity measures the similarity of connections in the graph with respect to the given numeric attribute. The numeric attribute must be an integer.

Parameters

- \( G \) (\text{NetworkX graph})
- \( \text{attribute} \) (\text{string}) – Node attribute key. The corresponding attribute value must be an integer.
- \( \text{nodes} \) (\text{list or iterable (optional)}) – Compute numeric assortativity only for attributes of nodes in container. The default is all nodes.

Returns \( r \) – Assortativity of graph for given attribute

References

networkx.algorithms.assortativity.numeric_assortativity_coefficient

numeric_assortativity_coefficient \( (G, \text{attribute}, \text{nodes}=\text{None}) \)

Compute assortativity for numerical node attributes.

Assortativity measures the similarity of connections in the graph with respect to the given numeric attribute. The numeric attribute must be an integer.

Parameters

- \( G \) (\text{NetworkX graph})
- \( \text{attribute} \) (\text{string}) – Node attribute key. The corresponding attribute value must be an integer.
- \( \text{nodes} \) (\text{list or iterable (optional)}) – Compute numeric assortativity only for attributes of nodes in container. The default is all nodes.

Returns \( r \) – Assortativity of graph for given attribute

Return type  float

Examples

```python
>>> G = nx.Graph()
>>> G.add_nodes_from([0,1],size=2)
>>> G.add_nodes_from([2,3],size=3)
>>> G.add_edges_from([(0,1),(2,3)])
>>> print(nx.numeric_assortativity_coefficient(G,'size'))
1.0
```

Notes

This computes Eq. (21) in Ref.¹, for the mixing matrix of of the specified attribute.

References

networkx.algorithms.assortativity.degree_pearson_correlation_coefficient

degree_pearson_correlation_coefficient (G, x='out', y='in', weight=None, nodes=None)
Compute degree assortativity of graph.

Assortativity measures the similarity of connections in the graph with respect to the node degree.

This is the same as degree_assortativity_coefficient but uses the potentially faster scipy.stats.pearsonr function.

Parameters

• `G` *(NetworkX graph)*
• `x` *(string ('in','out'))* – The degree type for source node (directed graphs only).
• `y` *(string ('in','out'))* – The degree type for target node (directed graphs only).
• `weight` *(string or None, optional (default=None))* – The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1. The degree is the sum of the edge weights adjacent to the node.
• `nodes` *(list or iterable (optional))* – Compute pearson correlation of degrees only for specified nodes. The default is all nodes.

Returns  `r` – Assortativity of graph by degree.

Return type  float

Examples

```python
>>> G = nx.path_graph(4)
>>> r = nx.degree_pearson_correlation_coefficient(G)
>>> print("%3.1f"%r)
-0.5
```

Notes
This calls scipy.stats.pearsonr.

References

3.2.2 Average neighbor degree

\[ k_{nn,i} = \frac{1}{|N(i)|} \sum_{j \in N(i)} k_j \]

where \( N(i) \) are the neighbors of node \( i \) and \( k_j \) is the degree of node \( j \) which belongs to \( N(i) \). For weighted graphs, an analogous measure can be defined\(^1\),

\[ k_{nn,i}^w = \frac{1}{s_i} \sum_{j \in N(i)} w_{ij}k_j \]

where \( s_i \) is the weighted degree of node \( i \), \( w_{ij} \) is the weight of the edge that links \( i \) and \( j \) and \( N(i) \) are the neighbors of node \( i \).

Parameters
- \( G \) (NetworkX graph)
- source (string (“in”|“out”)) – Directed graphs only. Use “in”- or “out”-degree for source node.
- target (string (“in”|“out”)) – Directed graphs only. Use “in”- or “out”-degree for target node.
- nodes (list or iterable, optional) – Compute neighbor degree for specified nodes. The default is all nodes in the graph.
- weight (string or None, optional (default=None)) – The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.

Returns \( d \) – A dictionary keyed by node with average neighbors degree value.

Return type dict

Examples

```python
>>> G = nx.path_graph(4)
>>> G.edges[0, 1]['weight'] = 5
>>> G.edges[2, 3]['weight'] = 3

>>> nx.average_neighbor_degree(G)
{0: 2.0, 1: 1.5, 2: 1.5, 3: 2.0}

>>> nx.average_neighbor_degree(G, weight='weight')
{0: 2.0, 1: 1.1666666666666667, 2: 1.25, 3: 2.0}

>>> G = nx.DiGraph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> nx.average_neighbor_degree(G, source='in', target='in')
{0: 1.0, 1: 1.0, 2: 1.0, 3: 0.0}

>>> nx.average_neighbor_degree(G, source='out', target='out')
{0: 1.0, 1: 1.0, 2: 0.0, 3: 0.0}
```

Notes

For directed graphs you can also specify in-degree or out-degree by passing keyword arguments.

See also:

average_degree_connectivity()

References

3.2.3 Average degree connectivity

```python
average_degree_connectivity(G, ...)

k_nearest_neighbors(G, ...)
```

networkx.algorithms.assortativity.average_degree_connectivity

average_degree_connectivity (G, source='in+out', target='in+out', nodes=None, weight=None)

Compute the average degree connectivity of graph.

The average degree connectivity is the average nearest neighbor degree of nodes with degree k. For weighted graphs, an analogous measure can be computed using the weighted average neighbors degree defined in', for a node i, as

\[ k_{nn,i}^w = \frac{1}{s_i} \sum_{j \in N(i)} w_{ij} k_j \]

where \( s_i \) is the weighted degree of node i, \( w_{ij} \) is the weight of the edge that links i and j, and \( N(i) \) are the neighbors of node i.

Parameters

- G (NetworkX graph)

References

3.2. Assortativity

NetworkX Reference, Release 2.5rc1.dev20191214022633

- **source** ("in"|"out"|"in+out" (default:"in+out")) – Directed graphs only. Use “in”- or “out”-degree for source node.
- **target** ("in"|"out"|"in+out" (default:"in+out")) – Directed graphs only. Use “in”- or “out”-degree for target node.
- **nodes** (list or iterable (optional)) – Compute neighbor connectivity for these nodes. The default is all nodes.
- **weight** (string or None, optional (default=None)) – The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.

Returns d – A dictionary keyed by degree k with the value of average connectivity.

Return type dict

Raises ValueError – If either source or target are not one of ‘in’, ‘out’, or ‘in+out’.

**Examples**

```python
g=nx.path_graph(4)
g.edges[1, 2][‘weight’] = 3
>>> nx.k_nearest_neighbors(G)
{1: 2.0, 2: 1.5}
>>> nx.k_nearest_neighbors(G, weight=’weight’)
{1: 2.0, 2: 1.75}
```

See also:

neighbors_average_degree()

**Notes**

This algorithm is sometimes called “k nearest neighbors” and is also available as k_nearest_neighbors.

**References**

networkx.algorithms.assortativity.k_nearest_neighbors

k_nearest_neighbors(G, source='in-out', target='in-out', nodes=None, weight=None)

Compute the average degree connectivity of graph.

The average degree connectivity is the average nearest neighbor degree of nodes with degree k. For weighted graphs, an analogous measure can be computed using the weighted average neighbors degree defined in¹, for a node \( i \), as

\[
k_{w_{nn,i}} = \frac{1}{s_i} \sum_{j \in N(i)} w_{ij}k_j
\]

where \( s_i \) is the weighted degree of node \( i \), \( w_{ij} \) is the weight of the edge that links \( i \) and \( j \), and \( N(i) \) are the neighbors of node \( i \).

**Parameters**

- **G** (NetworkX graph)

• source ("in"|"out"|"in+out" (default:"in+out")) – Directed graphs only. Use “in”- or “out”-degree for source node.
• target ("in"|"out"|"in+out" (default:"in+out")) – Directed graphs only. Use “in”- or “out”-degree for target node.
• nodes (list or iterable (optional)) – Compute neighbor connectivity for these nodes. The default is all nodes.
• weight (string or None, optional (default=None)) – The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.

Returns d – A dictionary keyed by degree k with the value of average connectivity.

Return type dict

Raises ValueError – If either source or target are not one of ‘in’, ‘out’, or ‘in+out’.

Examples

```python
>>> G=nx.path_graph(4)
>>> G.edges[1, 2]['weight'] = 3
>>> nx.k_nearest_neighbors(G)
{1: 2.0, 2: 1.5}
>>> nx.k_nearest_neighbors(G, weight='weight')
{1: 2.0, 2: 1.75}
```

See also:

neighbors_average_degree()

Notes

This algorithm is sometimes called “k nearest neighbors” and is also available as k_nearest_neighbors.

References

3.2.4 Mixing

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<tr>
<td>mixing_dict</td>
<td>Returns a dictionary representation of mixing matrix.</td>
</tr>
</tbody>
</table>

networkx.algorithms.assortativity.attribute_mixing_matrix

attribute_mixing_matrix (G, attribute[,...])

Returns mixing matrix for attribute.

Parameters
• **G** *(graph)* – NetworkX graph object.
• **attribute** *(string)* – Node attribute key.
• **nodes** *(list or iterable (optional))* – Use only nodes in container to build the matrix. The default is all nodes.
• **mapping** *(dictionary, optional)* – Mapping from node attribute to integer index in matrix. If not specified, an arbitrary ordering will be used.
• **normalized** *(bool (default=True))* – Return counts if False or probabilities if True.

**Returns**  
- **m** – Counts or joint probability of occurrence of attribute pairs.

**Return type**  
- numpy array

---

**networkx.algorithms.assortativity.degree_mixing_matrix**  

**degree_mixing_matrix** *(G, x='out', y='in', weight=None, nodes=None, normalized=True)*  
Returns mixing matrix for attribute.

**Parameters**

- **G** *(graph)* – NetworkX graph object.
- **x** *(string)* – The degree type for source node (directed graphs only).
- **y** *(string)* – The degree type for target node (directed graphs only).
- **nodes** *(list or iterable (optional))* – Build the matrix using only nodes in container. The default is all nodes.
- **weight** *(string or None, optional (default=None))* – The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1. The degree is the sum of the edge weights adjacent to the node.
- **normalized** *(bool (default=True))* – Return counts if False or probabilities if True.

**Returns**  
- **m** – Counts, or joint probability, of occurrence of node degree.

**Return type**  
- numpy array

---

**networkx.algorithms.assortativity.numeric_mixing_matrix**  

**numeric_mixing_matrix** *(G, attribute, nodes=None, normalized=True)*  
Returns numeric mixing matrix for attribute.

The attribute must be an integer.

**Parameters**

- **G** *(graph)* – NetworkX graph object.
- **attribute** *(string)* – Node attribute key. The corresponding attribute must be an integer.
- **nodes** *(list or iterable (optional))* – Build the matrix only with nodes in container. The default is all nodes.
- **normalized** *(bool (default=True))* – Return counts if False or probabilities if True.

**Returns**  
- **m** – Counts, or joint probability, of occurrence of node attribute pairs.

**Return type**  
- numpy array
networkx.algorithms.assortativity.attribute_mixing_dict

attribute_mixing_dict (G, attribute, nodes=None, normalized=False)

Returns dictionary representation of mixing matrix for attribute.

Parameters

- **G** (graph) – NetworkX graph object.
- **attribute** (string) – Node attribute key.
- **nodes** (list or iterable (optional)) – Unse nodes in container to build the dict. The default is all nodes.
- **normalized** (bool (default=False)) – Return counts if False or probabilities if True.

Examples

```python
>>> G=nx.Graph()
>>> G.add_nodes_from([0,1],color='red')
>>> G.add_nodes_from([2,3],color='blue')
>>> G.add_edge(1,3)
>>> d=nx.attribute_mixing_dict(G,'color')
>>> print(d['red']['blue'])
1
>>> print(d['blue']['red']) # d symmetric for undirected graphs
1
```

Returns d – Counts or joint probability of occurrence of attribute pairs.

Return type dictionary

networkx.algorithms.assortativity.degree_mixing_dict

degree_mixing_dict (G, x='out', y='in', weight=None, nodes=None, normalized=False)

Returns dictionary representation of mixing matrix for degree.

Parameters

- **G** (graph) – NetworkX graph object.
- **x** (string (‘in’,’out’)) – The degree type for source node (directed graphs only).
- **y** (string (‘in’,’out’)) – The degree type for target node (directed graphs only).
- **weight** (string or None, optional (default=None)) – The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1. The degree is the sum of the edge weights adjacent to the node.
- **normalized** (bool (default=False)) – Return counts if False or probabilities if True.

Returns d – Counts or joint probability of occurrence of degree pairs.

Return type dictionary

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networkx.algorithms.assortativity.mixing_dict

**mixing_dict** *(xy, normalized=False)*

Returns a dictionary representation of mixing matrix.

**Parameters**

- **xy** *(list or container of two-tuples)* – Pairs of (x,y) items.
- **attribute** *(string)* – Node attribute key
- **normalized** *(bool, default=False)* – Return counts if False or probabilities if True.

**Returns**

- **d** – Counts or Joint probability of occurrence of values in xy.

**Return type** dictionary

### 3.3 Asteroidal

Algorithms for asteroidal triples and asteroidal numbers in graphs.

An asteroidal triple in a graph G is a set of three non-adjacent vertices u, v and w such that there exist a path between any two of them that avoids closed neighborhood of the third. More formally, v_j, v_k belongs to the same connected component of G - N[v_i], where N[v_i] denotes the closed neighborhood of v_i. A graph which does not contain any asteroidal triples is called an AT-free graph. The class of AT-free graphs is a graph class for which many NP-complete problems are solvable in polynomial time. Amongst them, independent set and coloring.

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<td>Check if a graph is AT-free.</td>
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<tr>
<td>find_asteroidal_triple</td>
<td>Find an asteroidal triple in the given graph.</td>
</tr>
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</table>

#### 3.3.1 networkx.algorithms.asteroidal.is_at_free

**is_at_free** *(G)*

Check if a graph is AT-free.

The method uses the find_asteroidal_triple method to recognize an AT-free graph. If no asteroidal triple is found the graph is AT-free and True is returned. If at least one asteroidal triple is found the graph is not AT-free and False is returned.

**Parameters**

- **G** *(NetworkX Graph)* – The graph to check whether is AT-free or not.

**Returns**

- **True** if G is AT-free and **False** otherwise.

**Return type** bool

**Examples**

```python
>>> G = nx.Graph([(0, 1), (0, 2), (1, 2), (1, 3), (1, 4), (4, 5)])
>>> nx.is_at_free(G)
True

>>> G = nx.cycle_graph(6)
>>> nx.is_at_free(G)
False
```
3.3.2 networkx.algorithms.asteroidal.find_asteroidal_triple

find_asteroidal_triple \((G)\)
Find an asteroidal triple in the given graph.

An asteroidal triple is a triple of non-adjacent vertices such that there exists a path between any two of them which avoids the closed neighborhood of the third. It checks all independent triples of vertices and whether they are an asteroidal triple or not. This is done with the help of a data structure called a component structure. A component structure encodes information about which vertices belong to the same connected component when the closed neighborhood of a given vertex is removed from the graph. The algorithm used to check is the trivial one, outlined in\(^1\), which has a runtime of \(O(|V||E| + |V||E|)\), where the second term is the creation of the component structure.

**Parameters**  
\(G\) (NetworkX Graph) – The graph to check whether is AT-free or not

**Returns**  
An asteroidal triple is returned as a list of nodes. If no asteroidal triple exists, i.e. the graph is AT-free, then None is returned. The returned value depends on the certificate parameter. The default option is a bool which is True if the graph is AT-free, i.e. the given graph contains no asteroidal triples, and False otherwise, i.e. if the graph contains at least one asteroidal triple.

**Return type** list or None

**Notes**

The component structure and the algorithm is described in\(^1\). The current implementation implements the trivial algorithm for simple graphs.

**References**

3.4 Bipartite

This module provides functions and operations for bipartite graphs. Bipartite graphs \(B = (U, V, E)\) have two node sets \(U, V\) and edges in \(E\) that only connect nodes from opposite sets. It is common in the literature to use an spatial analogy referring to the two node sets as top and bottom nodes.

The bipartite algorithms are not imported into the networkx namespace at the top level so the easiest way to use them is with:

```
>>> import networkx as nx
>>> from networkx.algorithms import bipartite
```

NetworkX does not have a custom bipartite graph class but the Graph() or DiGraph() classes can be used to represent bipartite graphs. However, you have to keep track of which set each node belongs to, and make sure that there is no edge between nodes of the same set. The convention used in NetworkX is to use a node attribute named `bipartite` with values 0 or 1 to identify the sets each node belongs to. This convention is not enforced in the source code of bipartite functions, it’s only a recommendation.

For example:

```
>>> B = nx.Graph()
>>> # Add nodes with the node attribute "bipartite"
>>> B.add_nodes_from([1, 2, 3, 4], bipartite=0)
```

---

Many algorithms of the bipartite module of NetworkX require, as an argument, a container with all the nodes that belong to one set, in addition to the bipartite graph \( B \). The functions in the bipartite package do not check that the node set is actually correct nor that the input graph is actually bipartite. If \( B \) is connected, you can find the two node sets using a two-coloring algorithm:

```python
>>> nx.is_connected(B)
True
>>> bottom_nodes, top_nodes = bipartite.sets(B)
```

However, if the input graph is not connected, there are more than one possible colorations. This is the reason why we require the user to pass a container with all nodes of one bipartite node set as an argument to most bipartite functions. In the face of ambiguity, we refuse the temptation to guess and raise an `AmbiguousSolution` Exception if the input graph for `bipartite.sets` is disconnected.

Using the `bipartite` node attribute, you can easily get the two node sets:

```python
>>> top_nodes = {n for n, d in B.nodes(data=True) if d['bipartite']==0}
>>> bottom_nodes = set(B) - top_nodes
```

So you can easily use the bipartite algorithms that require, as an argument, a container with all nodes that belong to one node set:

```python
>>> print(round(bipartite.density(B, bottom_nodes), 2))
0.5
>>> G = bipartite.projected_graph(B, top_nodes)
```

All bipartite graph generators in NetworkX build bipartite graphs with the `bipartite` node attribute. Thus, you can use the same approach:

```python
>>> RB = bipartite.random_graph(5, 7, 0.2)
>>> RB_top = {n for n, d in RB.nodes(data=True) if d['bipartite']==0}
>>> RB_bottom = set(RB) - RB_top
>>> list(RB_top)
[0, 1, 2, 3, 4]
>>> list(RB_bottom)
[5, 6, 7, 8, 9, 10, 11]
```

For other bipartite graph generators see `Generators`.

### 3.4.1 Basic functions

#### Bipartite Graph Algorithms

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<thead>
<tr>
<th>Function</th>
<th>Description</th>
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<tr>
<td><code>is_bipartite(G)</code></td>
<td>Returns True if graph ( G ) is bipartite, False if not.</td>
</tr>
<tr>
<td><code>is_bipartite_node_set(G, nodes)</code></td>
<td>Returns True if nodes and ( G/nodes ) are a bipartition of ( G ).</td>
</tr>
<tr>
<td><code>sets(G[, top_nodes])</code></td>
<td>Returns bipartite node sets of graph ( G ).</td>
</tr>
<tr>
<td><code>color(G)</code></td>
<td>Returns a two-coloring of the graph.</td>
</tr>
<tr>
<td><code>density(B, nodes)</code></td>
<td>Returns density of bipartite graph ( B ).</td>
</tr>
</tbody>
</table>
Table 18 – continued from previous page

| degrees(B, nodes[, weight]) | Returns the degrees of the two node sets in the bipartite graph B. |

**networkx.algorithms.bipartite.basic.is_bipartite**

**is_bipartite(G)**

Returns True if graph G is bipartite, False if not.

**Parameters**

- **G** (NetworkX graph)

**Examples**

```python
>>> from networkx.algorithms import bipartite
>>> G = nx.path_graph(4)
>>> print(bipartite.is_bipartite(G))
True
```

**See also:**

color(), is_bipartite_node_set()

**networkx.algorithms.bipartite.basic.is_bipartite_node_set**

**is_bipartite_node_set(G, nodes)**

Returns True if nodes and G/nodes are a bipartition of G.

**Parameters**

- **G** (NetworkX graph)
- **nodes** (list or container) – Check if nodes are a one of a bipartite set.

**Examples**

```python
>>> from networkx.algorithms import bipartite
>>> G = nx.path_graph(4)
>>> X = set([1,3])
>>> bipartite.is_bipartite_node_set(G,X)
True
```

**Notes**

For connected graphs the bipartite sets are unique. This function handles disconnected graphs.

**networkx.algorithms.bipartite.basic.sets**

**sets(G, top_nodes=None)**

Returns bipartite node sets of graph G.

Raises an exception if the graph is not bipartite or if the input graph is disconnected and thus more than one valid solution exists. See **bipartite documentation** for further details on how bipartite graphs are handled in NetworkX.

### 3.4. Bipartite
Parameters

- **G** *(NetworkX graph)*
- **top_nodes** *(container, optional)* – Container with all nodes in one bipartite node set. If not supplied it will be computed. But if more than one solution exists an exception will be raised.

Returns

- **X** *(set)* – Nodes from one side of the bipartite graph.
- **Y** *(set)* – Nodes from the other side.

Raises

- **AmbiguousSolution** – Raised if the input bipartite graph is disconnected and no container with all nodes in one bipartite set is provided. When determining the nodes in each bipartite set more than one valid solution is possible if the input graph is disconnected.
- **NetworkXError** – Raised if the input graph is not bipartite.

Examples

```python
>>> from networkx.algorithms import bipartite
>>> G = nx.path_graph(4)
>>> X, Y = bipartite.sets(G)
>>> list(X)
[0, 2]
>>> list(Y)
[1, 3]
```

See also:

color()
You can use this to set a node attribute indicating the bipartite set:

```python
>>> nx.set_node_attributes(G, c, 'bipartite')
>>> print(G.nodes[0]['bipartite'])
1
>>> print(G.nodes[1]['bipartite'])
0
```

**networkx.algorithms.bipartite.basic.density**

```python
density(B, nodes)
```

Returns density of bipartite graph B.

**Parameters**

- `G` (NetworkX graph)
- `nodes` (list or container) – Nodes in one node set of the bipartite graph.

**Returns**

- `d` – The bipartite density

**Return type**

float

**Examples**

```python
>>> from networkx.algorithms import bipartite
>>> G = nx.complete_bipartite_graph(3,2)
>>> X=set([0,1,2])
>>> bipartite.density(G,X)
1.0
>>> Y=set([3,4])
>>> bipartite.density(G,Y)
1.0
```

**Notes**

The container of nodes passed as argument must contain all nodes in one of the two bipartite node sets to avoid ambiguity in the case of disconnected graphs. See *bipartite documentation* for further details on how bipartite graphs are handled in NetworkX.

**See also:**

`color()`

**networkx.algorithms.bipartite.basic.degrees**

```python
degrees(B, nodes, weight=None)
```

Returns the degrees of the two node sets in the bipartite graph B.

**Parameters**

- `G` (NetworkX graph)
- `nodes` (list or container) – Nodes in one node set of the bipartite graph.
• **weight** *(string or None, optional (default=None)) – The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1. The degree is the sum of the edge weights adjacent to the node.*

**Returns** *(degX,degY) – The degrees of the two bipartite sets as dictionaries key by node.*

**Return type**  tuple of dictionaries

**Examples**

```python
>>> from networkx.algorithms import bipartite
>>> G = nx.complete_bipartite_graph(3,2)
>>> Y=set([3,4])
>>> degX,degY=bipartite.degrees(G,Y)
>>> dict(degX)
{0: 2, 1: 2, 2: 2}
```

**Notes**

The container of nodes passed as argument must contain all nodes in one of the two bipartite node sets to avoid ambiguity in the case of disconnected graphs. See *bipartite documentation* for further details on how bipartite graphs are handled in NetworkX.

**See also:**

color(), density()

**3.4.2 Matching**

Provides functions for computing maximum cardinality matchings and minimum weight full matchings in a bipartite graph.

If you don’t care about the particular implementation of the maximum matching algorithm, simply use the *maximum_matching()* . If you do care, you can import one of the named maximum matching algorithms directly.

For example, to find a maximum matching in the complete bipartite graph with two vertices on the left and three vertices on the right:

```python
>>> import networkx as nx
>>> G = nx.complete_bipartite_graph(2, 3)
>>> left, right = nx.bipartite.sets(G)
>>> list(left)
[0, 1]
>>> list(right)
[2, 3, 4]
>>> nx.bipartite.maximum_matching(G)
{0: 2, 1: 3, 2: 0, 3: 1}
```

The dictionary returned by *maximum_matching()* includes a mapping for vertices in both the left and right vertex sets.

Similarly, *minimum_weight_full_matching()* produces, for a complete weighted bipartite graph, a matching whose cardinality is the cardinality of the smaller of the two partitions, and for which the sum of the weights of the edges included in the matching is minimal.
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<td><code>eppstein_matching(G[, top_nodes])</code></td>
<td>Returns the maximum cardinality matching of the bipartite graph <code>G</code>.</td>
</tr>
<tr>
<td><code>hopcroft_karp_matching(G[, top_nodes])</code></td>
<td>Returns the maximum cardinality matching of the bipartite graph <code>G</code>.</td>
</tr>
<tr>
<td><code>to_vertex_cover(G, matching[, top_nodes])</code></td>
<td>Returns the minimum vertex cover corresponding to the given maximum matching of the bipartite graph <code>G</code>.</td>
</tr>
<tr>
<td><code>minimum_weight_full_matching(G[, top_nodes,...])</code></td>
<td>Returns the minimum weight full matching of the bipartite graph <code>G</code>.</td>
</tr>
</tbody>
</table>

**networkx.algorithms.bipartite.matching.eppstein_matching**

`eppstein_matching(G, top_nodes=None)`

Returns the maximum cardinality matching of the bipartite graph `G`.

**Parameters**

- `G` *(NetworkX graph)* – Undirected bipartite graph
- `top_nodes` *(container)* – Container with all nodes in one bipartite node set. If not supplied it will be computed. But if more than one solution exists an exception will be raised.

**Returns** `matches` – The matching is returned as a dictionary, `matching`, such that \(\text{matching}[v] = w\) if node \(v\) is matched to node \(w\). Unmatched nodes do not occur as a key in `mate`.

**Return type** dictionary

**Raises** `AmbiguousSolution` – Raised if the input bipartite graph is disconnected and no container with all nodes in one bipartite set is provided. When determining the nodes in each bipartite set more than one valid solution is possible if the input graph is disconnected.

**Notes**

This function is implemented with David Eppstein’s version of the algorithm Hopcroft–Karp algorithm (see `hopcroft_karp_matching()`), which originally appeared in the Python Algorithms and Data Structures library (PADS).

See bipartite documentation for further details on how bipartite graphs are handled in NetworkX.

See also:

`hopcroft_karp_matching()`

**networkx.algorithms.bipartite.matching.hopcroft_karp_matching**

`hopcroft_karp_matching(G, top_nodes=None)`

Returns the maximum cardinality matching of the bipartite graph `G`.

**Parameters**

- `G` *(NetworkX graph)* – Undirected bipartite graph
- `top_nodes` *(container)* – Container with all nodes in one bipartite node set. If not supplied it will be computed. But if more than one solution exists an exception will be raised.

**Returns** `matches` – The matching is returned as a dictionary, `matches`, such that \(\text{matches}[v] = w\) if node \(v\) is matched to node \(w\). Unmatched nodes do not occur as a key in `mate`.

**Return type** dictionary
Raises *AmbiguousSolution* – Raised if the input bipartite graph is disconnected and no container with all nodes in one bipartite set is provided. When determining the nodes in each bipartite set more than one valid solution is possible if the input graph is disconnected.

Notes

This function is implemented with the Hopcroft–Karp matching algorithm for bipartite graphs. See *bipartite documentation* for further details on how bipartite graphs are handled in NetworkX. See also: `eppstein_matching()`

References

`networkx.algorithms.bipartite.matching.to_vertex_cover`

`to_vertex_cover(G, matching, top_nodes=None)`

Returns the minimum vertex cover corresponding to the given maximum matching of the bipartite graph `G`.

Parameters

- `G` (*NetworkX graph*) – Undirected bipartite graph
- `matching` (*dictionary*) – A dictionary whose keys are vertices in `G` and whose values are the distinct neighbors comprising the maximum matching for `G`, as returned by, for example, `maximum_matching()`. The dictionary *must* represent the maximum matching.
- `top_nodes` (*container*) – Container with all nodes in one bipartite node set. If not supplied it will be computed. But if more than one solution exists an exception will be raised.

Returns `vertex_cover` – The minimum vertex cover in `G`.

Return type *set*

Raises *AmbiguousSolution* – Raised if the input bipartite graph is disconnected and no container with all nodes in one bipartite set is provided. When determining the nodes in each bipartite set more than one valid solution is possible if the input graph is disconnected.

Notes

This function is implemented using the procedure guaranteed by *Konig's theorem*, which proves an equivalence between a maximum matching and a minimum vertex cover in bipartite graphs.

Since a minimum vertex cover is the complement of a maximum independent set for any graph, one can compute the maximum independent set of a bipartite graph this way:

```python
>>> import networkx as nx
>>> G = nx.complete_bipartite_graph(2, 3)
>>> matching = nx.bipartite.maximum_matching(G)
>>> vertex_cover = nx.bipartite.to_vertex_cover(G, matching)
>>> independent_set = set(G) - vertex_cover
>>> print(list(independent_set))
[2, 3, 4]
```

See *bipartite documentation* for further details on how bipartite graphs are handled in NetworkX.
The problem of determining a minimum weight full matching is also known as the rectangular linear assignment problem. This implementation defers the calculation of the assignment to SciPy.

Notes

The problem of determining a minimum weight full matching is also known as the rectangular linear assignment problem. This implementation defers the calculation of the assignment to SciPy.

References

3.4.3 Matrix

Biadjacency matrices

<table>
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<th>function</th>
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<td>biadjacency_matrix(G, row_order[, ...])</td>
<td>Returns the biadjacency matrix of the bipartite graph G.</td>
</tr>
<tr>
<td>from_biadjacency_matrix(A[, create_using, ...])</td>
<td>Creates a new bipartite graph from a biadjacency matrix given as a SciPy sparse matrix.</td>
</tr>
</tbody>
</table>

**biadjacency_matrix**

```
biadjacency_matrix(G, row_order=None, column_order=None, dtype=None, weight='weight', format='csr')
```

Returns the biadjacency matrix of the bipartite graph G.

Let $G = (U, V, E)$ be a bipartite graph with node sets $U = u_1, \ldots, u_r$ and $V = v_1, \ldots, v_s$. The biadjacency matrix $B$ is the $r \times s$ matrix in which $b_{i,j} = 1$ if, and only if, $(u_i, v_j) \in E$. If the parameter weight is not None and matches the name of an edge attribute, its value is used instead of 1.

**Parameters**

- **G (graph)** – A NetworkX graph
- **row_order** (list of nodes) – The rows of the matrix are ordered according to the list of nodes.
- **column_order** (list, optional) – The columns of the matrix are ordered according to the list of nodes. If column_order is None, then the ordering of columns is arbitrary.
- **dtype** (NumPy data-type, optional) – A valid NumPy dtype used to initialize the array. If None, then the NumPy default is used.
- **weight** (string or None, optional (default='weight')) – The edge data key used to provide each value in the matrix. If None, then each edge has weight 1.
- **format** (str in {'bsr', 'csr', 'csc', 'coo', 'lil', 'dia', 'dok'}) – The type of the matrix to be returned (default ‘csr’). For some algorithms different implementations of sparse matrices can perform better. See¹ for details.

**Returns**

M – Biadjacency matrix representation of the bipartite graph G.

**Return type**

SciPy sparse matrix

**Notes**

No attempt is made to check that the input graph is bipartite.

For directed bipartite graphs only successors are considered as neighbors. To obtain an adjacency matrix with ones (or weight values) for both predecessors and successors you have to generate two biadjacency matrices where the rows of one of them are the columns of the other, and then add one to the transpose of the other.

**See also:**

adjacency_matrix(), from_biadjacency_matrix()

**References**

networkx.algorithms.bipartite.matrix.from_biadjacency_matrix

```
from_biadjacency_matrix(A, create_using=None, edge_attribute='weight')
```

Creates a new bipartite graph from a biadjacency matrix given as a SciPy sparse matrix.

**Parameters**

- **A** (scipy sparse matrix) – A biadjacency matrix representation of a graph
- **create_using** (NetworkX graph) – Use specified graph for result. The default is Graph()

¹ https://en.wikipedia.org/wiki/Adjacency_matrix#Adjacency_matrix_of_a_bipartite_graph
• **edge_attribute** *(string)* – Name of edge attribute to store matrix numeric value. The data will have the same type as the matrix entry (int, float, (real,imag)).

**Notes**

The nodes are labeled with the attribute `bipartite` set to an integer 0 or 1 representing membership in part 0 or part 1 of the bipartite graph.

If `create_using` is an instance of `networkx.MultiGraph` or `networkx.MultiDiGraph` and the entries of `A` are of type `int`, then this function returns a multigraph (of the same type as `create_using`) with parallel edges. In this case, `edge_attribute` will be ignored.

**See also:**

`biadjacency_matrix()`, `from_numpy_matrix()`

**References**


### 3.4.4 Projections

One-mode (unipartite) projections of bipartite graphs.

<table>
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<th>Function</th>
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<tr>
<td><code>projected_graph(B, nodes[, multigraph])</code></td>
<td>Returns the projection of B onto one of its node sets.</td>
</tr>
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<td><code>weighted_projected_graph(B, nodes[, ratio])</code></td>
<td>Returns a weighted projection of B onto one of its node sets.</td>
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<tr>
<td><code>collaboration_weighted_projected_graph(B, nodes)</code></td>
<td>Newman’s weighted projection of B onto one of its node sets.</td>
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<tr>
<td><code>overlap_weighted_projected_graph(B, nodes[, ...])</code></td>
<td>Overlap weighted projection of B onto one of its node sets.</td>
</tr>
<tr>
<td><code>generic_weighted_projected_graph(B, nodes[, ...])</code></td>
<td>Weighted projection of B with a user-specified weight function.</td>
</tr>
</tbody>
</table>

**networkx.algorithms.bipartite.projection.projected_graph**

`projected_graph` *(B, nodes, multigraph=False)*

Returns the projection of B onto one of its node sets.

Returns the graph G that is the projection of the bipartite graph B onto the specified nodes. They retain their attributes and are connected in G if they have a common neighbor in B.

**Parameters**

- `B` *(NetworkX graph)* – The input graph should be bipartite.
- `nodes` *(list or iterable)* – Nodes to project onto (the “bottom” nodes).
- `multigraph` *(bool (default=False))* – If True return a multigraph where the multiple edges represent multiple shared neighbors. They edge key in the multigraph is assigned to the label of the neighbor.

**Returns** *Graph* – A graph that is the projection onto the given nodes.

**Return type** NetworkX graph or multigraph
Examples

```python
>>> from networkx.algorithms import bipartite
>>> B = nx.path_graph(4)
>>> G = bipartite.projected_graph(B, [1, 3])
>>> list(G)
[1, 3]
>>> list(G.edges())
[(1, 3)]
```

If nodes a and b are connected through both nodes 1 and 2 then building a multigraph results in two edges in the projection onto \([a, b]\):

```python
>>> B = nx.Graph()
>>> B.add_edges_from([('a', 1), ('b', 1), ('a', 2), ('b', 2)])
>>> G = bipartite.projected_graph(B, ['a', 'b'], multigraph=True)
>>> print([sorted((u, v)) for u, v in G.edges()])
[['a', 'b'], ['a', 'b']]
```

Notes

No attempt is made to verify that the input graph B is bipartite. Returns a simple graph that is the projection of the bipartite graph B onto the set of nodes given in list nodes. If multigraph=True then a multigraph is returned with an edge for every shared neighbor.

Directed graphs are allowed as input. The output will also then be a directed graph with edges if there is a directed path between the nodes.

The graph and node properties are (shallow) copied to the projected graph.

See `bipartite documentation` for further details on how bipartite graphs are handled in NetworkX.

See also:

- `is_bipartite()`, `is_bipartite_node_set()`, `sets()`, `weighted_projected_graph()`, `collaboration_weighted_projected_graph()`, `overlap_weighted_projected_graph()`, `generic_weighted_projected_graph()`

```
networkx.algorithms.bipartite.projection.weighted_projected_graph
```

`weighted_projected_graph` *(B, nodes, ratio=False)*

Returns a weighted projection of B onto one of its node sets.

The weighted projected graph is the projection of the bipartite network B onto the specified nodes with weights representing the number of shared neighbors or the ratio between actual shared neighbors and possible shared neighbors if ratio is True. The nodes retain their attributes and are connected in the resulting graph if they have an edge to a common node in the original graph.

**Parameters**

- `B` *(NetworkX graph)* – The input graph should be bipartite.
- `nodes` *(list or iterable)* – Nodes to project onto (the “bottom” nodes).

---

• **ratio** *(Bool (default=False))* – If True, edge weight is the ratio between actual shared neighbors and maximum possible shared neighbors (i.e., the size of the other node set). If False, edges weight is the number of shared neighbors.

**Returns**  **Graph** – A graph that is the projection onto the given nodes.

**Return type**  **NetworkX graph**

**Examples**

```python
>>> from networkx.algorithms import bipartite
>>> G = bipartite.weighted_projected_graph(B, [1, 3])
[(1, 3)]

Notes

No attempt is made to verify that the input graph B is bipartite. The graph and node properties are (shallow) copied to the projected graph.

See **bipartite documentation** for further details on how bipartite graphs are handled in NetworkX.

See also:

- `is_bipartite()`, `is_bipartite_node_set()`, `sets()`, `collaboration_weighted_projected_graph()`,
- `overlap_weighted_projected_graph()`, `generic_weighted_projected_graph()`, `projected_graph()`

References

networkx.algorithms.bipartite.projection.collaboration_weighted_projected_graph

`collaboration_weighted_projected_graph (B, nodes)`

Newman’s weighted projection of B onto one of its node sets.

The collaboration weighted projection is the projection of the bipartite network B onto the specified nodes with weights assigned using Newman’s collaboration model:

\[
w_{u,v} = \sum_k \frac{\delta^B_k \delta^T_k}{d_k - 1}
\]

where \(u\) and \(v\) are nodes from the bottom bipartite node set, and \(k\) is a node of the top node set. The value \(d_k\) is the degree of node \(k\) in the bipartite network and \(\delta^B_{\{u\}} \cap \{k\}\) is 1 if node \(u\) is linked to node \(k\) in the original bipartite graph or 0 otherwise.

The nodes retain their attributes and are connected in the resulting graph if have an edge to a common node in the original bipartite graph.

**Parameters**

• B (*NetworkX graph*) – The input graph should be bipartite.
• nodes (*list or iterable*) – Nodes to project onto (the “bottom” nodes).

**Returns** Graph – A graph that is the projection onto the given nodes.

**Return type** NetworkX graph

**Examples**

```python
>>> from networkx.algorithms import bipartite
>>> B = nx.path_graph(5)
>>> B.add_edge(1, 5)
>>> G = bipartite.collaboration_weighted_projected_graph(B, [0, 2, 4, 5])
>>> list(G)
[0, 2, 4, 5]
>>> for edge in sorted(G.edges(data=True)): print(edge)
... (0, 2, {'weight': 0.5})
(0, 5, {'weight': 0.5})
(2, 4, {'weight': 1.0})
(2, 5, {'weight': 0.5})
```

**Notes**

No attempt is made to verify that the input graph B is bipartite. The graph and node properties are (shallow) copied to the projected graph.

See [bipartite documentation](https://networkx.github.io/documentation/stable/algorithms.html#bipartite) for further details on how bipartite graphs are handled in NetworkX.

**See also:**

`is_bipartite()`, `is_bipartite_node_set()`, `sets()`, `weighted_projected_graph()`, `overlap_weighted_projected_graph()`, `generic_weighted_projected_graph()`, `projected_graph()`

**References**

networkx.algorithms.bipartite.projection.overlap_weighted_projected_graph

**overlap_weighted_projected_graph** (*B, nodes, jaccard=True*)

Overlap weighted projection of B onto one of its node sets.

The overlap weighted projection is the projection of the bipartite network B onto the specified nodes with weights representing the Jaccard index between the neighborhoods of the two nodes in the original bipartite network:

\[ w_{v,u} = \frac{|N(u) \cap N(v)|}{|N(u) \cup N(v)|} \]

or if the parameter ‘jaccard’ is False, the fraction of common neighbors by minimum of both nodes degree in the original bipartite graph:

\[ w_{v,u} = \frac{|N(u) \cap N(v)|}{\min(|N(u)|, |N(v)|)} \]

The nodes retain their attributes and are connected in the resulting graph if they have an edge to a common node in the original bipartite graph.

**Parameters**

- **B** (*NetworkX graph*) – The input graph should be bipartite.
- **nodes** (*list or iterable*) – Nodes to project onto (the “bottom” nodes).
- **jaccard** (*bool* (default=True))

**Returns**

- **Graph** – A graph that is the projection onto the given nodes.

**Return type**

- *NetworkX graph*

**Examples**

```python
>>> from networkx.algorithms import bipartite
>>> B = nx.path_graph(5)
>>> nodes = [0, 2, 4]
>>> G = bipartite.overlap_weighted_projected_graph(B, nodes)
>>> list(G)
[0, 2, 4]
>>> list(G.edges(data=True))
[(0, 2, {'weight': 0.5}), (2, 4, {'weight': 0.5})]
>>> G = bipartite.overlap_weighted_projected_graph(B, nodes, jaccard=False)
>>> list(G.edges(data=True))
[(0, 2, {'weight': 1.0}), (2, 4, {'weight': 1.0})]
```

**Notes**

No attempt is made to verify that the input graph B is bipartite. The graph and node properties are (shallow) copied to the projected graph.

See *bipartite documentation* for further details on how bipartite graphs are handled in NetworkX.

See also:

- `is_bipartite()`, `is_bipartite_node_set()`, `sets()`, `weighted_projected_graph()`, `collaboration_weighted_projected_graph()`, `generic_weighted_projected_graph()`, `projected_graph()`

**References**

networkx.algorithms.bipartite.projection.generic_weighted_projected_graph

`generic_weighted_projected_graph(B, nodes, weight_function=None)`

Weighted projection of B with a user-specified weight function.

The bipartite network B is projected on to the specified nodes with weights computed by a user-specified function. This function must accept as a parameter the neighborhood sets of two nodes and return an integer or a float.

The nodes retain their attributes and are connected in the resulting graph if they have an edge to a common node in the original graph.

**Parameters**
- **B** (NetworkX graph) – The input graph should be bipartite.
- **nodes** (list or iterable) – Nodes to project onto (the “bottom” nodes).
- **weight_function** (function) – This function must accept as parameters the same input graph that this function, and two nodes; and return an integer or a float. The default function computes the number of shared neighbors.

**Returns** Graph – A graph that is the projection onto the given nodes.

**Return type** NetworkX graph

### Examples

```python
>>> from networkx.algorithms import bipartite
>>> # Define some custom weight functions
... def jaccard(G, u, v):
...     unbrs = set(G[u])
...     vnbrs = set(G[v])
...     return float(len(unbrs & vnbrs)) / len(unbrs | vnbrs)
... def my_weight(G, u, v, weight='weight'):
...     w = 0
...     for nbr in set(G[u]) & set(G[v]):
...         w += G[u][nbr].get(weight, 1) + G[v][nbr].get(weight, 1)
...     return w

... # A complete bipartite graph with 4 nodes and 4 edges
... B = nx.complete_bipartite_graph(2, 2)
... # Add some arbitrary weight to the edges
... for i, (u, v) in enumerate(B.edges()):
...     B.edges[u, v]['weight'] = i + 1
...     for edge in B.edges(data=True):
...         print(edge)
...
... (0, 2, {'weight': 1})
... (0, 3, {'weight': 2})
... (1, 2, {'weight': 3})
... (1, 3, {'weight': 4})
... # By default, the weight is the number of shared neighbors
... G = bipartite.generic_weighted_projected_graph(B, [0, 1])
... print(list(G.edges(data=True)))
... [(0, 1, {'weight': 2})]
... # To specify a custom weight function use the weight_function parameter
... G = bipartite.generic_weighted_projected_graph(B, [0, 1], weight_function=jaccard)
... print(list(G.edges(data=True)))
... [(0, 1, {'weight': 1.0})]
... G = bipartite.generic_weighted_projected_graph(B, [0, 1], weight_function=my_weight)
... print(list(G.edges(data=True)))
... [(0, 1, {'weight': 10})]
```
Notes

No attempt is made to verify that the input graph B is bipartite. The graph and node properties are (shallow) copied to the projected graph.

See bipartite documentation for further details on how bipartite graphs are handled in NetworkX.

See also:

is_bipartite(), is_bipartite_node_set(), sets(), weighted_projected_graph(), collaboration_weighted_projected_graph(), overlap_weighted_projected_graph(), projected_graph()

3.4.5 Spectral

Spectral bipartivity measure.

\[
\text{spectral_bipartivity}(G[, \text{nodes}, \text{weight}]) \quad \text{Returns the spectral bipartivity.}
\]

networkx.algorithms.bipartite.spectral.spectral_bipartivity

\text{spectral_bipartivity} (G, \text{nodes}=None, \text{weight}=’weight’)

Returns the spectral bipartivity.

Parameters

- \text{G} (NetworkX graph)
- \text{nodes} (list or container optional(default is all nodes)) – Nodes to return value of spectral bipartivity contribution.
- \text{weight} (string or None optional (default = ‘weight’)) – Edge data key to use for edge weights. If None, weights set to 1.

Returns \text{sb} – A single number if the keyword nodes is not specified, or a dictionary keyed by node with the spectral bipartivity contribution of that node as the value.

Return type float or dict

Examples

```python
>>> from networkx.algorithms import bipartite
>>> G = nx.path_graph(4)
>>> bipartite.spectral_bipartivity(G)
1.0
```

Notes

This implementation uses Numpy (dense) matrices which are not efficient for storing large sparse graphs.

See also:
color()
References

3.4.6 Clustering

Functions for computing clustering of pairs

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
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<td><code>networkx.algorithms.bipartite.cluster.clustering</code></td>
<td>Compute a bipartite clustering coefficient for nodes.</td>
</tr>
<tr>
<td><code>average_clustering(G[, nodes, mode])</code></td>
<td>Compute the average bipartite clustering coefficient.</td>
</tr>
<tr>
<td><code>latapy_clustering(G[, nodes, mode])</code></td>
<td>Compute a bipartite clustering coefficient for nodes.</td>
</tr>
<tr>
<td><code>robins_alexander_clustering(G)</code></td>
<td>Compute the bipartite clustering of G.</td>
</tr>
</tbody>
</table>

The bipartite clustering coefficient is a measure of local density of connections defined as:

\[
\hat{c}_u = \frac{\sum_{v \in N(N(u))} c_{uv}}{|N(N(u))|}
\]

where \(N(N(u))\) are the second order neighbors of \(u\) in \(G\) excluding \(u\), and \(c_{\{uv\}}\) is the pairwise clustering coefficient between nodes \(u\) and \(v\).

The mode selects the function for \(c_{\{uv\}}\) which can be:

- dot:
  \[
  c_{uv} = \frac{|N(u) \cap N(v)|}{|N(u) \cup N(v)|}
  \]
- min:
  \[
  c_{uv} = \frac{|N(u) \cap N(v)|}{\min(|N(u)|, |N(v)|)}
  \]
- max:
  \[
  c_{uv} = \frac{|N(u) \cap N(v)|}{\max(|N(u)|, |N(v)|)}
  \]

Parameters

- \(G\) (graph) – A bipartite graph
- \(nodes\) (list or iterable (optional)) – Compute bipartite clustering for these nodes. The default is all nodes in \(G\).
- \(mode\) (string) – The pairwise bipartite clustering method to be used in the computation. It must be “dot”, “max”, or “min”.

Returns clustering – A dictionary keyed by node with the clustering coefficient value.

Return type dictionary

Examples

---

```python
>>> from networkx.algorithms import bipartite
>>> G = nx.path_graph(4)  # path graphs are bipartite
>>> c = bipartite.clustering(G)
0.5
>>> c = bipartite.clustering(G, mode='min')
1.0
```

See also:

`robins_alexander_clustering()`, `square_clustering()`, `average_clustering()`

References

networkx.algorithms.bipartite.cluster.average_clustering

`average_clustering(G, nodes=None, mode='dot')`  
Compute the average bipartite clustering coefficient.

A clustering coefficient for the whole graph is the average,

\[
C = \frac{1}{n} \sum_{v \in G} c_v,
\]

where \( n \) is the number of nodes in \( G \).

Similar measures for the two bipartite sets can be defined\(^1\)

\[
C_X = \frac{1}{|X|} \sum_{v \in X} c_v,
\]

where \( X \) is a bipartite set of \( G \).

Parameters

- \( G \) (graph) – a bipartite graph
- \( nodes \) (list or iterable, optional) – A container of nodes to use in computing the average. The nodes should be either the entire graph (the default) or one of the bipartite sets.
- \( mode \) (string) – The pairwise bipartite clustering method. It must be “dot”, “max”, or “min”

Returns clustering – The average bipartite clustering for the given set of nodes or the entire graph if no nodes are specified.

Return type float

Examples

```python
>>> from networkx.algorithms import bipartite
>>> G=nx.star_graph(3)  # star graphs are bipartite
>>> bipartite.average_clustering(G)
0.75
>>> X,Y=bipartite.sets(G)
(continues on next page)
```

bipartite.average_clustering(G,X)
0.0
bipartite.average_clustering(G,Y)
1.0

See also:
clustering()

Notes

The container of nodes passed to this function must contain all of the nodes in one of the bipartite sets
("top" or "bottom") in order to compute the correct average bipartite clustering coefficients. See bipartite
documentation for further details on how bipartite graphs are handled in NetworkX.

References

networkx.algorithms.bipartite.cluster.latapy_clustering

latapy_clustering(G, nodes=None, mode='dot')
Compute a bipartite clustering coefficient for nodes.

The bipartite clustering coefficient is a measure of local density of connections defined as\(^1\):

\[
c_u = \frac{\sum_{v \in N(N(u))} c_{uv}}{|N(N(u))|}
\]

where \(N(N(u))\) are the second order neighbors of \(u\) in \(G\) excluding \(u\), and \(c_{uv}\) is the pairwise clustering
coefficient between nodes \(u\) and \(v\).

The mode selects the function for \(c_{uv}\) which can be:

dot:

\[
c_{uv} = \frac{|N(u) \cap N(v)|}{|N(u) \cup N(v)|}
\]

min:

\[
c_{uv} = \frac{|N(u) \cap N(v)|}{\min(|N(u)|, |N(v)|)}
\]

max:

\[
c_{uv} = \frac{|N(u) \cap N(v)|}{\max(|N(u)|, |N(v)|)}
\]

Parameters

- \(G\) (graph) – A bipartite graph
- \(nodes\) (list or iterable (optional)) – Compute bipartite clustering for these nodes. The default
  is all nodes in \(G\).
- \(mode\) (string) – The pairwise bipartite clustering method to be used in the computation. It
  must be “dot”, “max”, or “min”.

---

\(^1\) Latapy, Matthieu, Clémence Magnien, and Nathalie Del Vecchio (2008). Basic notions for the analysis of large two-mode networks. Social
Networks 30(1), 31–48.
Returns clustering – A dictionary keyed by node with the clustering coefficient value.

Return type dictionary

Examples

```python
>>> from networkx.algorithms import bipartite
>>> G = nx.path_graph(4)  # path graphs are bipartite
>>> c = bipartite.clustering(G)
>>> c[0]
0.5
>>> c = bipartite.clustering(G, mode='min')
>>> c[0]
1.0
```

See also:

`robins_alexander_clustering()`, `square_clustering()`, `average_clustering()`

References

networkx.algorithms.bipartite.cluster.robins_alexander_clustering

`robins_alexander_clustering(G)`

Compute the bipartite clustering of G.

Robins and Alexander\(^1\) defined bipartite clustering coefficient as four times the number of four cycles \(C_4\) divided by the number of three paths \(L_3\) in a bipartite graph:

\[
CC_4 = \frac{4 \cdot C_4}{L_3}
\]

Parameters G (graph) – a bipartite graph

Returns clustering – The Robins and Alexander bipartite clustering for the input graph.

Return type float

Examples

```python
>>> from networkx.algorithms import bipartite
>>> G = nx.davis_southern_women_graph()
>>> print(round(bipartite.robins_alexander_clustering(G), 3))
0.468
```

See also:

`latapy_clustering()`, `square_clustering()`

---

3.4.7 Redundancy

Node redundancy for bipartite graphs.

\[ node\_redundancy(G[, \text{nodes}]) \]
Computes the node redundancy coefficients for the nodes in the bipartite graph \( G \).

```
networkx.algorithms.bipartite.redundancy.node_redundancy
```

\[ node\_redundancy (G, \text{nodes}=None) \]
Computes the node redundancy coefficients for the nodes in the bipartite graph \( G \).

The redundancy coefficient of a node \( v \) is the fraction of pairs of neighbors of \( v \) that are both linked to other nodes. In a one-mode projection these nodes would be linked together even if \( v \) were not there.

More formally, for any vertex \( v \), the redundancy coefficient of \( v \) is defined by

\[
rc(v) = \frac{|\{\{u, w\} \subseteq N(v), \exists v' \neq v, (v', u) \in E \text{ and } (v', w) \in E\}|}{\frac{|N(v)|(|N(v)|-1)}{2}},
\]

where \( N(v) \) is the set of neighbors of \( v \) in \( G \).

**Parameters**

- \( G \) (graph) – A bipartite graph
- \( \text{nodes} \) (list or iterable (optional)) – Compute redundancy for these nodes. The default is all nodes in \( G \).

**Returns**

- redundancy – A dictionary keyed by node with the node redundancy value.

**Return type**

dictionary

**Examples**

Compute the redundancy coefficient of each node in a graph:

```
>>> import networkx as nx
>>> from networkx.algorithms import bipartite
>>> G = nx.cycle_graph(4)
>>> rc = bipartite.node_redundancy(G)
>>> rc[0]
1.0
```

Compute the average redundancy for the graph:

```
>>> import networkx as nx
>>> from networkx.algorithms import bipartite
>>> G = nx.cycle_graph(4)
>>> rc = bipartite.node_redundancy(G)
>>> sum(rc.values()) / len(G)
1.0
```

Compute the average redundancy for a set of nodes:
```python
>>> import networkx as nx
>>> from networkx.algorithms import bipartite
>>> G = nx.cycle_graph(4)
>>> rc = bipartite.node_redundancy(G)
>>> nodes = [0, 2]
>>> sum(rc[n] for n in nodes) / len(nodes)
1.0
```

**Raises** NetworkXError – If any of the nodes in the graph (or in `nodes`, if specified) has (out-)degree less than two (which would result in division by zero, according to the definition of the redundancy coefficient).

## References

### 3.4.8 Centrality

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#### networkx.algorithms.bipartite.centrality.closeness_centrality

**closeness_centrality** *(G, nodes[, normalized])*  
Compute the closeness centrality for nodes in a bipartite network.

The closeness of a node is the distance to all other nodes in the graph or in the case that the graph is not connected to all other nodes in the connected component containing that node.

**Parameters**

- **G** *(graph)* – A bipartite network
- **nodes** *(list or container)* – Container with all nodes in one bipartite node set.
- **normalized** *(bool, optional)* – If True (default) normalize by connected component size.

**Returns** closeness – Dictionary keyed by node with bipartite closeness centrality as the value.

**Return type** dictionary

**See also:**

`betweenness_centrality()`, `degree_centrality()`, `sets()`, `is_bipartite()`

**Notes**

The nodes input parameter must contain all nodes in one bipartite node set, but the dictionary returned contains all nodes from both node sets. See `bipartite documentation` for further details on how bipartite graphs are handled in NetworkX.

Closeness centrality is normalized by the minimum distance possible. In the bipartite case the minimum distance for a node in one bipartite node set is 1 from all nodes in the other node set and 2 from all other nodes in its own
set\(^1\). Thus the closeness centrality for node \(v\) in the two bipartite sets \(U\) with \(n\) nodes and \(V\) with \(m\) nodes is

\[
c_v = \frac{m + 2(n - 1)}{d}, \text{ for } v \in U,
\]

\[
c_v = \frac{n + 2(m - 1)}{d}, \text{ for } v \in V,
\]

where \(d\) is the sum of the distances from \(v\) to all other nodes.

Higher values of closeness indicate higher centrality.

As in the unipartite case, setting normalized=True causes the values to normalized further to \(n-1 / \text{size}(G)-1\) where \(n\) is the number of nodes in the connected part of graph containing the node. If the graph is not completely connected, this algorithm computes the closeness centrality for each connected part separately.

References

networkx.algorithms.bipartite.centrality.degree_centrality

degree_centrality \((G, \text{nodes})\)

Compute the degree centrality for nodes in a bipartite network.

The degree centrality for a node \(v\) is the fraction of nodes connected to it.

Parameters

- \(G\) (graph) – A bipartite network
- \(\text{nodes}\) (list or container) – Container with all nodes in one bipartite node set.

Returns centrality – Dictionary keyed by node with bipartite degree centrality as the value.

Return type dictionary

See also:

betweenness_centrality(), closeness_centrality(), sets(), is_bipartite()

Notes

The nodes input parameter must contain all nodes in one bipartite node set, but the dictionary returned contains all nodes from both bipartite node sets. See bipartite documentation for further details on how bipartite graphs are handled in NetworkX.

For unipartite networks, the degree centrality values are normalized by dividing by the maximum possible degree (which is \(n-1\) where \(n\) is the number of nodes in \(G\)).

In the bipartite case, the maximum possible degree of a node in a bipartite node set is the number of nodes in the opposite node set\(^1\). The degree centrality for a node \(v\) in the bipartite sets \(U\) with \(n\) nodes and \(V\) with \(m\) nodes is

\[
d_v = \frac{\text{deg}(v)}{m}, \text{ for } v \in U,
\]

\[
d_v = \frac{\text{deg}(v)}{n}, \text{ for } v \in V,
\]

where \(\text{deg}(v)\) is the degree of node \(v\).

---


References

networkx.algorithms.bipartite.centrality.betweenness_centrality

betweenness_centrality \((G, \text{nodes})\)

Compute betweenness centrality for nodes in a bipartite network.

Betweenness centrality of a node \(v\) is the sum of the fraction of all-pairs shortest paths that pass through \(v\).

Values of betweenness are normalized by the maximum possible value which for bipartite graphs is limited by the relative size of the two node sets\(^1\).

Let \(n\) be the number of nodes in the node set \(U\) and \(m\) be the number of nodes in the node set \(V\), then nodes in \(U\) are normalized by dividing by

\[
\frac{1}{2} \left[ m^2 (s + 1)^2 + m(s + 1)(2t - s - 1) - t(2s - t + 3) \right],
\]

where

\[
s = (n - 1) \div m, t = (n - 1) \mod m,
\]

and nodes in \(V\) are normalized by dividing by

\[
\frac{1}{2} \left[ n^2 (p + 1)^2 + n(p + 1)(2r - p - 1) - r(2p - r + 3) \right],
\]

where,

\[
p = (m - 1) \div n, r = (m - 1) \mod n.
\]

Parameters

- \(G\) (graph) – A bipartite graph
- \(\text{nodes}\) (list or container) – Container with all nodes in one bipartite node set.

Returns \(\text{betweenness}\) – Dictionary keyed by node with bipartite betweenness centrality as the value.

Return type dictionary

See also:

\(\text{degree_centrality()}, \text{closeness_centrality()}, \text{sets()}, \text{is_bipartite()}\)

Notes

The nodes input parameter must contain all nodes in one bipartite node set, but the dictionary returned contains all nodes from both node sets. See bipartite documentation for further details on how bipartite graphs are handled in NetworkX.

References

3.4.9 Generators

Generators and functions for bipartite graphs.

**networkx.algorithms.bipartite.generators.complete_bipartite_graph**

**complete_bipartite_graph** *(n1, n2, create_using=None)*  
Returns the complete bipartite graph $K_{n_1, n_2}$.

The graph is composed of two partitions with nodes 0 to (n1 - 1) in the first and nodes n1 to (n1 + n2 - 1) in the second. Each node in the first is connected to each node in the second.

**Parameters**

- **n1** *(integer)* – Number of nodes for node set A.
- **n2** *(integer)* – Number of nodes for node set B.
- **create_using** *(NetworkX graph instance, optional)* – Return graph of this type.

**Notes**

Node labels are the integers 0 to $n_1 + n_2 - 1$.

The nodes are assigned the attribute ‘bipartite’ with the value 0 or 1 to indicate which bipartite set the node belongs to.

This function is not imported in the main namespace. To use it use nx.bipartite.complete_bipartite_graph

**networkx.algorithms.bipartite.generators.configuration_model**

**configuration_model** *(aseq, bseq, create_using=None, seed=None)*  
Returns a random bipartite graph from two given degree sequences.

**Parameters**

- **aseq** *(list)* – Degree sequence for node set A.
- **bseq** *(list)* – Degree sequence for node set B.
- **create_using** *(NetworkX graph instance, optional)* – Return graph of this type.
- **seed** *(integer, random_state, or None (default))* – Indicator of random number generation state. See Randomness.
• The graph is composed of two partitions. Set A has nodes 0 to 
• (len(aseq) - 1) and set B has nodes len(aseq) to (len(bseq) - 1).
• Nodes from set A are connected to nodes in set B by choosing
• randomly from the possible free stubs, one in A and one in B.

Notes
The sum of the two sequences must be equal: \( \text{sum}(a) = \text{sum}(b) \) If no graph type is specified use MultiGraph with parallel edges. If you want a graph with no parallel edges use create_using=Graph() but then the resulting degree sequences might not be exact.

The nodes are assigned the attribute ‘bipartite’ with the value 0 or 1 to indicate which bipartite set the node belongs to.

This function is not imported in the main namespace. To use it use nx.bipartite.configuration_model

\[ \text{networkx.algorithms.bipartite.generators.havel_hakimi_graph} \]

\[ \text{havel_hakimi_graph}(aseq, bseq, create_using=None) \]
Returns a bipartite graph from two given degree sequences using a Havel-Hakimi style construction.

The graph is composed of two partitions. Set A has nodes 0 to (len(aseq) - 1) and set B has nodes len(aseq) to (len(bseq) - 1). Nodes from the set A are connected to nodes in the set B by connecting the highest degree nodes in set A to the highest degree nodes in set B until all stubs are connected.

Parameters
• \( aseq \) (list) – Degree sequence for node set A.
• \( bseq \) (list) – Degree sequence for node set B.
• \( create_using \) (NetworkX graph instance, optional) – Return graph of this type.

Notes
The sum of the two sequences must be equal: \( \text{sum}(a) = \text{sum}(b) \) If no graph type is specified use MultiGraph with parallel edges. If you want a graph with no parallel edges use create_using=Graph() but then the resulting degree sequences might not be exact.

The nodes are assigned the attribute ‘bipartite’ with the value 0 or 1 to indicate which bipartite set the node belongs to.

This function is not imported in the main namespace. To use it use nx.bipartite.havel_hakimi_graph

\[ \text{networkx.algorithms.bipartite.generators.reverse_havel_hakimi_graph} \]

\[ \text{reverse_havel_hakimi_graph}(aseq, bseq, create_using=None) \]
Returns a bipartite graph from two given degree sequences using a Havel-Hakimi style construction.

The graph is composed of two partitions. Set A has nodes 0 to (len(aseq) - 1) and set B has nodes len(aseq) to (len(bseq) - 1). Nodes from set A are connected to nodes in the set B by connecting the highest degree nodes in set A to the lowest degree nodes in set B until all stubs are connected.

Parameters
• \( aseq \) (list) – Degree sequence for node set A.
- **bseq** *(list)* – Degree sequence for node set B.
- **create_using** *(NetworkX graph instance, optional)* – Return graph of this type.

**Notes**

The sum of the two sequences must be equal: `sum(aseq)=sum(bseq)` If no graph type is specified use MultiGraph with parallel edges. If you want a graph with no parallel edges use `create_using=Graph()` but then the resulting degree sequences might not be exact.

The nodes are assigned the attribute ‘bipartite’ with the value 0 or 1 to indicate which bipartite set the node belongs to.

This function is not imported in the main namespace. To use it use `nx.bipartite.reverse_havel_hakimi_graph`

---

**alternating_havel_hakimi_graph** *(aseq, bseq, create_using=None)*

Returns a bipartite graph from two given degree sequences using an alternating Havel-Hakimi style construction.

The graph is composed of two partitions. Set A has nodes 0 to `(len(aseq) - 1)` and set B has nodes `len(aseq)` to `(len(bseq) - 1)`. Nodes from the set A are connected to nodes in the set B by connecting the highest degree nodes in set A to alternatively the highest and the lowest degree nodes in set B until all stubs are connected.

**Parameters**

- **aseq** *(list)* – Degree sequence for node set A.
- **bseq** *(list)* – Degree sequence for node set B.
- **create_using** *(NetworkX graph instance, optional)* – Return graph of this type.

**Notes**

The sum of the two sequences must be equal: `sum(aseq)=sum(bseq)` If no graph type is specified use MultiGraph with parallel edges. If you want a graph with no parallel edges use `create_using=Graph()` but then the resulting degree sequences might not be exact.

The nodes are assigned the attribute ‘bipartite’ with the value 0 or 1 to indicate which bipartite set the node belongs to.

This function is not imported in the main namespace. To use it use `nx.bipartite.alternating_havel_hakimi_graph`

---

**preferential_attachment_graph** *(aseq, p, create_using=None, seed=None)*

Create a bipartite graph with a preferential attachment model from a given single degree sequence.

The graph is composed of two partitions. Set A has nodes 0 to `(len(aseq) - 1)` and set B has nodes starting with node `len(aseq)`. The number of nodes in set B is random.

**Parameters**

- **aseq** *(list)* – Degree sequence for node set A.
- **p** *(float)* – Probability that a new bottom node is added.
- **create_using** *(NetworkX graph instance, optional)* – Return graph of this type.
• **seed** *(integer, random_state, or None (default)) –* Indicator of random number generation state. See *Randomness.*

**References**

**Notes**

The nodes are assigned the attribute ‘bipartite’ with the value 0 or 1 to indicate which bipartite set the node belongs to.

This function is not imported in the main namespace. To use it use `nx.bipartite.preferential_attachment_graph`

**networkx.algorithms.bipartite.generators.random_graph**

`random_graph(n, m, p, seed=None, directed=False)`

Returns a bipartite random graph.

This is a bipartite version of the binomial (Erdős-Rényi) graph. The graph is composed of two partitions. Set A has nodes 0 to \((n - 1)\) and set B has nodes \(n\) to \((n + m - 1)\).

**Parameters**

- **n** *(int)* – The number of nodes in the first bipartite set.
- **m** *(int)* – The number of nodes in the second bipartite set.
- **p** *(float)* – Probability for edge creation.
- **seed** *(integer, random_state, or None (default)) –* Indicator of random number generation state. See *Randomness.*
- **directed** *(bool, optional (default=False)) –* If True return a directed graph

**Notes**

The bipartite random graph algorithm chooses each of the \(n^*m\) (undirected) or \(2^*nm\) (directed) possible edges with probability \(p\).

This algorithm is \(O(n + m)\) where \(m\) is the expected number of edges.

The nodes are assigned the attribute ‘bipartite’ with the value 0 or 1 to indicate which bipartite set the node belongs to.

This function is not imported in the main namespace. To use it use `nx.bipartite.random_graph`

See also:

`gnp_random_graph()`, `configuration_model()`

**References**

**networkx.algorithms.bipartite.generators.gnmk_random_graph**

`gnmk_random_graph(n, m, k, seed=None, directed=False)`

Returns a random bipartite graph \(G\}_{n,m,k}\).
Produces a bipartite graph chosen randomly out of the set of all graphs with \( n \) top nodes, \( m \) bottom nodes, and \( k \) edges. The graph is composed of two sets of nodes. Set A has nodes 0 to \((n - 1)\) and set B has nodes \( n \) to \((n + m - 1)\).

**Parameters**

- \( n \) (int) – The number of nodes in the first bipartite set.
- \( m \) (int) – The number of nodes in the second bipartite set.
- \( k \) (int) – The number of edges
- \( \text{seed} \) (integer, random_state, or None (default)) – Indicator of random number generation state. See *Randomness*.
- \( \text{directed} \) (bool, optional (default=False)) – If True return a directed graph

**Examples**

```python
from nx.algorithms import bipartite
G = bipartite.gnmk_random_graph(10, 20, 50)
```

**See also:**

`gnm_random_graph()`

**Notes**

If \( k > m \times n \) then a complete bipartite graph is returned.

This graph is a bipartite version of the \( G_{nm} \) random graph model.

The nodes are assigned the attribute ‘bipartite’ with the value 0 or 1 to indicate which bipartite set the node belongs to.

This function is not imported in the main namespace. To use it use `nx.bipartite.gnmk_random_graph`
will be used. Other possibilities include `eppstein_matching()`.

**Returns** A set of the edges in a minimum edge cover of the graph, given as pairs of nodes. It contains both the edges \((u, v)\) and \((v, u)\) for given nodes \(u\) and \(v\) among the edges of minimum edge cover.

**Return type** `set`

**Notes**

An edge cover of a graph is a set of edges such that every node of the graph is incident to at least one edge of the set. A minimum edge cover is an edge covering of smallest cardinality.

Due to its implementation, the worst-case running time of this algorithm is bounded by the worst-case running time of the function `matching_algorithm`.

### 3.5 Boundary

Routines to find the boundary of a set of nodes.

An edge boundary is a set of edges, each of which has exactly one endpoint in a given set of nodes (or, in the case of directed graphs, the set of edges whose source node is in the set).

A node boundary of a set \(S\) of nodes is the set of (out-)neighbors of nodes in \(S\) that are outside \(S\).

**edge_boundary** \((G, nbunch1[, nbunch2, data, ...])\) Returns the edge boundary of \(nbunch1\).

**node_boundary** \((G, nbunch1[, nbunch2])\) Returns the node boundary of \(nbunch1\).

#### 3.5.1 networkx.algorithms.boundary.edge_boundary

**edge_boundary** \((G, nbunch1, nbunch2=None, data=False, keys=False, default=None)\) Returns the edge boundary of \(nbunch1\).

The *edge boundary* of a set \(S\) with respect to a set \(T\) is the set of edges \((u, v)\) such that \(u\) is in \(S\) and \(v\) is in \(T\). If \(T\) is not specified, it is assumed to be the set of all nodes not in \(S\).

**Parameters**

- \(G\) (*NetworkX graph*)
- \(nbunch1\) (*iterable*) – Iterable of nodes in the graph representing the set of nodes whose edge boundary will be returned. (This is the set \(S\) from the definition above.)
- \(nbunch2\) (*iterable*) – Iterable of nodes representing the target (or “exterior”) set of nodes. (This is the set \(T\) from the definition above.) If not specified, this is assumed to be the set of all nodes in \(G\) not in \(nbunch1\).
- \(keys\) (*bool*) – This parameter has the same meaning as in `MultiGraph.edges()`.
- \(data\) (*bool or object*) – This parameter has the same meaning as in `MultiGraph.edges()`.
- \(default\) (*object*) – This parameter has the same meaning as in `MultiGraph.edges()`.

**Returns** An iterator over the edges in the boundary of \(nbunch1\) with respect to \(nbunch2\). If \(keys, data,\) or \(default\) are specified and \(G\) is a multigraph, then edges are returned with \(keys\) and/or \(data\), as in `MultiGraph.edges()`.

---

**3.5. Boundary**

---

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Return type  iterator

Notes

Any element of nbunch that is not in the graph G will be ignored.

nbunch1 and nbunch2 are usually meant to be disjoint, but in the interest of speed and generality, that is not required here.

3.5.2 networkx.algorithms.boundary.node_boundary

node_boundary (G, nbunch1, nbunch2=None)

Returns the node boundary of nbunch1.

The node boundary of a set S with respect to a set T is the set of nodes v in T such that for some u in S, there is an edge joining u to v. If T is not specified, it is assumed to be the set of all nodes not in S.

Parameters

• G (NetworkX graph)

• nbunch1 (iterable) – Iterable of nodes in the graph representing the set of nodes whose node boundary will be returned. (This is the set S from the definition above.)

• nbunch2 (iterable) – Iterable of nodes representing the target (or “exterior”) set of nodes. (This is the set T from the definition above.) If not specified, this is assumed to be the set of all nodes in G not in nbunch1.

Returns  The node boundary of nbunch1 with respect to nbunch2.

Return type  set

Notes

Any element of nbunch that is not in the graph G will be ignored.

nbunch1 and nbunch2 are usually meant to be disjoint, but in the interest of speed and generality, that is not required here.

3.6 Bridges

Bridge-finding algorithms.

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<td>Generate all bridges in a graph.</td>
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<td>Iterate over local bridges of G optionally computing the span</td>
</tr>
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</table>

3.6.1 networkx.algorithms.bridges.bridges

bridges (G, root=None)

Generate all bridges in a graph.
A bridge in a graph is an edge whose removal causes the number of connected components of the graph to increase. Equivalently, a bridge is an edge that does not belong to any cycle.

**Parameters**

- **G** *(undirected graph)*
- **root** *(node (optional)) – A node in the graph G.* If specified, only the bridges in the connected component containing this node will be returned.

**Yields** *(edge)* – An edge in the graph whose removal disconnects the graph (or causes the number of connected components to increase).

**Raises** *NodeNotFound – If root is not in the graph G.*

**Examples**

The barbell graph with parameter zero has a single bridge:

```python
>>> G = nx.barbell_graph(10, 0)
>>> list(nx.bridges(G))
[(9, 10)]
```

**Notes**

This is an implementation of the algorithm described in [1]. An edge is a bridge if and only if it is not contained in any chain. Chains are found using the `networkx.chain_decomposition()` function.

Ignoring polylogarithmic factors, the worst-case time complexity is the same as the `networkx.chain_decomposition()` function, $O(m + n)$, where $n$ is the number of nodes in the graph and $m$ is the number of edges.

**References**

3.6.2 `networkx.algorithms.bridges.has_bridges`

**has_bridges** *(G, root=None)*

Decide whether a graph has any bridges.

A bridge in a graph is an edge whose removal causes the number of connected components of the graph to increase.

**Parameters**

- **G** *(undirected graph)*
- **root** *(node (optional)) – A node in the graph G.* If specified, only the bridges in the connected component containing this node will be considered.

**Returns** Whether the graph (or the connected component containing root) has any bridges.

**Return type** *bool*

**Raises** *NodeNotFound – If root is not in the graph G.*
Examples

The barbell graph with parameter zero has a single bridge:

```python
>>> G = nx.barbell_graph(10, 0)
>>> nx.has_bridges(G)
True
```

On the other hand, the cycle graph has no bridges:

```python
>>> G = nx.cycle_graph(5)
>>> nx.has_bridges(G)
False
```

Notes

This implementation uses the `networkx.bridges()` function, so it shares its worst-case time complexity, \( O(m + n) \), ignoring polylogarithmic factors, where \( n \) is the number of nodes in the graph and \( m \) is the number of edges.

3.6.3 `networkx.algorithms.bridges.local_bridges`

`local_bridges(G, with_span=True, weight=None)`

Iterate over local bridges of \( G \) optionally computing the span

A local bridge is an edge whose endpoints have no common neighbors. That is, the edge is not part of a triangle in the graph.

The span of a local bridge is the shortest path length between the endpoints if the local bridge is removed.

Parameters

- \( G \) (undirected graph)
- `with_span` (bool) – If True, yield a 3-tuple \((u, v, \text{span})\)
- `weight` (function, string or None (default: None)) – If function, used to compute edge weights for the span. If string, the edge data attribute used in calculating span. If None, all edges have weight 1.

Yields \( e \) (edge) – The local bridges as an edge 2-tuple of nodes \((u, v)\) or as a 3-tuple \((u, v, \text{span})\) when `with_span` is True.

Examples

A cycle graph has every edge a local bridge with span N-1.

```python
>>> G = nx.cycle_graph(9)
>>> (0, 8, 8) in set(nx.local_bridges(G))
True
```
3.7 Centrality

3.7.1 Degree

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<th>Function</th>
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<td>Compute the degree centrality for nodes.</td>
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<td>in_degree_centrality(G)</td>
<td>Compute the in-degree centrality for nodes.</td>
</tr>
<tr>
<td>out_degree_centrality(G)</td>
<td>Compute the out-degree centrality for nodes.</td>
</tr>
</tbody>
</table>

**networkx.algorithms.centrality.degree_centrality**

**degree_centrality(G)**

Compute the degree centrality for nodes.

- **Parameters**
  - G (graph) – A networkx graph
- **Returns**
  - nodes – Dictionary of nodes with degree centrality as the value.
- **Return type**
  - dictionary
- **Notes**
  - The degree centrality values are normalized by dividing by the maximum possible degree in a simple graph \(n-1\) where \(n\) is the number of nodes in \(G\).
  - For multigraphs or graphs with self loops the maximum degree might be higher than \(n-1\) and values of degree centrality greater than 1 are possible.

**networkx.algorithms.centrality.in_degree_centrality**

**in_degree_centrality(G)**

Compute the in-degree centrality for nodes.

- **Parameters**
  - G (graph) – A NetworkX graph
- **Returns**
  - nodes – Dictionary of nodes with in-degree centrality as values.
- **Return type**
  - dictionary
- **Raises**
  - NetworkXNotImplemented – If G is undirected.
- **Notes**
  - The degree centrality values are normalized by dividing by the maximum possible degree in a simple graph \(n-1\) where \(n\) is the number of nodes in \(G\).
For multigraphs or graphs with self loops the maximum degree might be higher than n-1 and values of degree centrality greater than 1 are possible.

**networkx.algorithms.centrality.out_degree_centrality**

**out_degree_centrality**(G)
Compute the out-degree centrality for nodes.

The out-degree centrality for a node v is the fraction of nodes its outgoing edges are connected to.

**Parameters**
- G *(graph)* – A NetworkX graph

**Returns**
- nodes – Dictionary of nodes with out-degree centrality as values.

**Return type**
dictionary

**Raises**
- `NetworkXNotImplemented` – If G is undirected.

**See also:**
- `degree_centrality()`, `in_degree_centrality()`

**Notes**

The degree centrality values are normalized by dividing by the maximum possible degree in a simple graph n-1 where n is the number of nodes in G.

For multigraphs or graphs with self loops the maximum degree might be higher than n-1 and values of degree centrality greater than 1 are possible.

### 3.7.2 Eigenvector

**eigenvector_centrality**(G,
max_iter=100,
tol=1e-06,
nstart=None,
weight=None)
Compute the eigenvector centrality for the graph G.

Eigenvector centrality computes the centrality for a node based on the centrality of its neighbors. The eigenvector centrality for node $i$ is the $i$-th element of the vector $x$ defined by the equation

\[ Ax = \lambda x \]

where $A$ is the adjacency matrix of the graph $\mathcal{G}$ with eigenvalue $\lambda$. By virtue of the Perron–Frobenius theorem, there is a unique solution $x$, all of whose entries are positive, if $\lambda$ is the largest eigenvalue of the adjacency matrix $A$.²

**Parameters**

• **G (graph)** – A networkx graph
• **max_iter** (*integer, optional (default=100)*) – Maximum number of iterations in power method.
• **tol** (*float, optional (default=1.0e-6)*) – Error tolerance used to check convergence in power method iteration.
• **nstart** (*dictionary, optional (default=None)*) – Starting value of eigenvector iteration for each node.
• **weight** (*None or string, optional (default=None)*) – If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

**Returns** nodes – Dictionary of nodes with eigenvector centrality as the value.

**Return type** dictionary

**Examples**

```python
g = nx.path_graph(4)
centrality = nx.eigenvector_centrality(g)
sorted((v, '{:.2f}'.format(c)) for v, c in centrality.items())

[(0, '0.37'), (1, '0.60'), (2, '0.60'), (3, '0.37')]
```

**Raises**

• **NetworkXPointlessConcept** – If the graph G is the null graph.
• **NetworkXError** – If each value in nstart is zero.
• **PowerIterationFailedConvergence** – If the algorithm fails to converge to the specified tolerance within the specified number of iterations of the power iteration method.

**See also:**

*eigenvector_centrality_numpy(), pagerank(), hits()*

**Notes**

The measure was introduced by\(^1\) and is discussed in\(^2\).

The power iteration method is used to compute the eigenvector and convergence is not guaranteed. Our method stops after max_iter iterations or when the change in the computed vector between two iterations is smaller than an error tolerance of G.number_of_nodes() * tol. This implementation uses \((A + I)\) rather than the adjacency matrix \(A\) because it shifts the spectrum to enable discerning the correct eigenvector even for networks with multiple dominant eigenvalues.

For directed graphs this is “left” eigenvector centrality which corresponds to the in-edges in the graph. For out-edges eigenvector centrality first reverse the graph with G.reverse().

NetworkX Reference, Release 2.5rc1.dev20191214022633

References

networkx.algorithms.centrality.eigenvector_centrality_numpy

eigenvector_centrality_numpy (G, weight=None, max_iter=50, tol=0)

Compute the eigenvector centrality for the graph G.

Eigenvector centrality computes the centrality for a node based on the centrality of its neighbors. The eigenvector centrality for node \(i\) is

\[ Ax = \lambda x \]

where \(A\) is the adjacency matrix of the graph \(G\) with eigenvalue \(\lambda\). By virtue of the Perron–Frobenius theorem, there is a unique and positive solution if \(\lambda\) is the largest eigenvalue associated with the eigenvector of the adjacency matrix \(A\) (1).

Parameters

- \(G\) (graph) – A networkx graph
- \(weight\) (None or string, optional (default=None)) – The name of the edge attribute used as weight. If None, all edge weights are considered equal.
- \(max_iter\) (integer, optional (default=100)) – Maximum number of iterations in power method.
- \(tol\) (float, optional (default=1.0e-6)) – Relative accuracy for eigenvalues (stopping criterion). The default value of 0 implies machine precision.

Returns

- **nodes** – Dictionary of nodes with eigenvector centrality as the value.

Return type
dictionary

Examples

```python
>>> G = nx.path_graph(4)
>>> centrality = nx.eigenvector_centrality_numpy(G)
>>> print(['{} {:0.2f}'.format(node, centrality[node]) for node in centrality])

['0 0.37', '1 0.60', '2 0.60', '3 0.37']
```

See also:
eigenvector_centrality(), pagerank(), hits()

Notes

The measure was introduced by(1).

This algorithm uses the SciPy sparse eigenvalue solver (ARPACK) to find the largest eigenvalue/eigenvector pair.

For directed graphs this is “left” eigenvector centrality which corresponds to the in-edges in the graph. For out-edges eigenvector centrality first reverse the graph with \(G.reverse()\).

Raises

NetworkXPointlessConcept – If the graph \(G\) is the null graph.

---


References

networkx.algorithms.centrality.katz_centrality

katz_centrality(G, alpha=0.1, beta=1.0, max_iter=1000, tol=1e-06, nstart=None, normalized=True, weight=None)

Compute the Katz centrality for the nodes of the graph G.

Katz centrality computes the centrality for a node based on the centrality of its neighbors. It is a generalization of the eigenvector centrality. The Katz centrality for node $i$ is

$$x_i = \alpha \sum_j A_{ij} x_j + \beta,$$

where $A$ is the adjacency matrix of graph G with eigenvalues $\lambda$.

The parameter $\beta$ controls the initial centrality and

$$\alpha < \frac{1}{\lambda_{\text{max}}}.$$ 

Katz centrality computes the relative influence of a node within a network by measuring the number of the immediate neighbors (first degree nodes) and also all other nodes in the network that connect to the node under consideration through these immediate neighbors.

Extra weight can be provided to immediate neighbors through the parameter $\beta$. Connections made with distant neighbors are, however, penalized by an attenuation factor $\alpha$ which should be strictly less than the inverse largest eigenvalue of the adjacency matrix in order for the Katz centrality to be computed correctly. More information is provided in\(^1\).

Parameters

- **G (graph)** – A NetworkX graph.
- **alpha (float)** – Attenuation factor
- **beta (scalar or dictionary, optional (default=1.0))** – Weight attributed to the immediate neighborhood. If not a scalar, the dictionary must have an value for every node.
- **max_iter (integer, optional (default=1000))** – Maximum number of iterations in power method.
- **tol (float, optional (default=1.0e-6))** – Error tolerance used to check convergence in power method iteration.
- **nstart (dictionary, optional)** – Starting value of Katz iteration for each node.
- **normalized (bool, optional (default=True))** – If True normalize the resulting values.
- **weight (None or string, optional (default=None))** – If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

Returns **nodes** – Dictionary of nodes with Katz centrality as the value.

Return type dictionary

Raises

- **NetworkXError** – If the parameter beta is not a scalar but lacks a value for at least one node
- **PowerIterationFailedConvergence** – If the algorithm fails to converge to the specified tolerance within the specified number of iterations of the power iteration method.

Examples

```python
>>> import math

>>> G = nx.path_graph(4)
>>> phi = (1 + math.sqrt(5)) / 2.0  # largest eigenvalue of adj matrix
>>> centrality = nx.katz_centrality(G, 1/phi - 0.01)
>>> for n, c in sorted(centrality.items()):
...     print("%d %0.2f" % (n, c))
0 0.37
1 0.60
2 0.60
3 0.37
```

See also:

katz_centrality_numpy(), eigenvector_centrality(), eigenvector_centrality_numpy(), pagerank(), hits()

Notes

Katz centrality was introduced by\(^2\).

This algorithm it uses the power method to find the eigenvector corresponding to the largest eigenvalue of the adjacency matrix of G. The parameter \(\alpha\) should be strictly less than the inverse of largest eigenvalue of the adjacency matrix for the algorithm to converge. You can use `max(nx.adjacency_spectrum(G))` to get \(\lambda_{\text{max}}\) the largest eigenvalue of the adjacency matrix. The iteration will stop after \(\text{max}_\text{iter}\) iterations or an error tolerance of \(\text{number}_\text{of}_\text{nodes}(G) \times \text{tol}\) has been reached.

When \(\alpha = 1/\lambda_{\text{max}}\) and \(\beta = 0\), Katz centrality is the same as eigenvector centrality.

For directed graphs this finds “left” eigenvectors which corresponds to the in-edges in the graph. For out-edges Katz centrality first reverse the graph with `G.reverse()`.

References

networkx.algorithms.centrality.katz_centrality_numpy

katz_centrality_numpy (G, alpha=0.1, beta=1.0, normalized=True, weight=None)

Compute the Katz centrality for the graph G.

Katz centrality computes the centrality for a node based on the centrality of its neighbors. It is a generalization of the eigenvector centrality. The Katz centrality for node \(i\) is

\[
x_i = \alpha \sum_j A_{ij} x_j + \beta,
\]

where \(A\) is the adjacency matrix of graph G with eigenvalues \(\lambda\).

The parameter \(\beta\) controls the initial centrality and

\[
\alpha < \frac{1}{\lambda_{\text{max}}}.
\]

Katz centrality computes the relative influence of a node within a network by measuring the number of the immediate neighbors (first degree nodes) and also all other nodes in the network that connect to the node under consideration through these immediate neighbors.

Extra weight can be provided to immediate neighbors through the parameter $\beta$. Connections made with distant neighbors are, however, penalized by an attenuation factor $\alpha$ which should be strictly less than the inverse largest eigenvalue of the adjacency matrix in order for the Katz centrality to be computed correctly. More information is provided in\(^1\).

**Parameters**

- $G$ (graph) – A NetworkX graph
- $\alpha$ (float) – Attenuation factor
- $\beta$ (scalar or dictionary, optional (default=1.0)) – Weight attributed to the immediate neighborhood. If not a scalar the dictionary must have an value for every node.
- normalized (bool) – If True normalize the resulting values.
- weight (None or string, optional) – If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

**Returns** nodes – Dictionary of nodes with Katz centrality as the value.

**Return type** dictionary

**Raises** NetworkXError – If the parameter $\beta$ is not a scalar but lacks a value for at least one node

**Examples**

```python
>>> import math
>>> G = nx.path_graph(4)
>>> phi = (1 + math.sqrt(5)) / 2.0  # largest eigenvalue of adj matrix
>>> centrality = nx.katz_centrality_numpy(G, 1/phi)
>>> for n, c in sorted(centrality.items()):
...    print("%d %.2f" % (n, c))
0 0.37
1 0.60
2 0.60
3 0.37
```

**See also:**

katz_centrality(), eigenvector_centrality_numpy(), eigenvector_centrality(), pagerank(), hits()

**Notes**

Katz centrality was introduced by\(^2\).

This algorithm uses a direct linear solver to solve the above equation. The parameter $\alpha$ should be strictly less than the inverse largest eigenvalue of the adjacency matrix for there to be a solution. You can use $\max (\text{nx.adjacency_spectrum}(G))$ to get $\lambda_{\text{max}}$ the largest eigenvalue of the adjacency matrix.

When $\alpha = 1/\lambda_{\text{max}}$ and $\beta = 0$, Katz centrality is the same as eigenvector centrality.

For directed graphs this finds “left” eigenvectors which corresponds to the in-edges in the graph. For out-edges Katz centrality first reverse the graph with $G$.reverse().

---


References

3.7.3 Closeness

closeness_centrality(G[, u, distance, ...])  Compute closeness centrality for nodes.
incremental_closeness_centrality(G, edge[,...])  Incremental closeness centrality for nodes.

networkx.algorithms.centrality.closeness_centrality

Closeness centrality of a node \( u \) is the reciprocal of the average shortest path distance to \( u \) over all \( n-1 \) reachable nodes.

\[
C(u) = \frac{n - 1}{\sum_{v=1}^{n-1} d(v, u)},
\]

where \( d(v, u) \) is the shortest-path distance between \( v \) and \( u \), and \( n \) is the number of nodes that can reach \( u \). Notice that the closeness distance function computes the incoming distance to \( u \) for directed graphs. To use outward distance, act on \( G.reverse() \).

Notice that higher values of closeness indicate higher centrality.

Wasserman and Faust propose an improved formula for graphs with more than one connected component. The result is “a ratio of the fraction of actors in the group who are reachable, to the average distance” from the reachable actors\(^2\). You might think this scale factor is inverted but it is not. As is, nodes from small components receive a smaller closeness value. Letting \( N \) denote the number of nodes in the graph,

\[
C_{WF}(u) = \frac{n - 1}{N - 1} \frac{n - 1}{\sum_{v=1}^{n-1} d(v, u)},
\]

Parameters

- \( G \) (graph) – A NetworkX graph
- \( u \) (node, optional) – Return only the value for node \( u \)
- \( distance \) (edge attribute key, optional (default=None)) – Use the specified edge attribute as the edge distance in shortest path calculations
- \( wf_improved \) (bool, optional (default=True)) – If True, scale by the fraction of nodes reachable. This gives the Wasserman and Faust improved formula. For single component graphs it is the same as the original formula.

Returns nodes – Dictionary of nodes with closeness centrality as the value.

Return type  dictionary

See also:

betweenness_centrality(), load_centrality(), eigenvector_centrality(), degree_centrality(), incremental_closeness_centrality()

---

Notes

The closeness centrality is normalized to \((n-1) / (\mid G \mid -1)\) where \(n\) is the number of nodes in the connected part of graph containing the node. If the graph is not completely connected, this algorithm computes the closeness centrality for each connected part separately scaled by that part's size.

If the ‘distance’ keyword is set to an edge attribute key then the shortest-path length will be computed using Dijkstra’s algorithm with that edge attribute as the edge weight.

The closeness centrality uses \textit{inward} distance to a node, not outward. If you want to use outward distances apply the function to \(G\).\text{reverse}()

In NetworkX 2.2 and earlier a bug caused Dijkstra’s algorithm to use the outward distance rather than the inward distance. If you use a ‘distance’ keyword and a DiGraph, your results will change between v2.2 and v2.3.

References

\texttt{networkx.algorithms.centrality.incremental_closeness_centrality}

\texttt{incremental_closeness_centrality}(G, edge, prev_cc=None, insertion=True, wf_improved=True)

Incremental closeness centrality for nodes.

Compute closeness centrality for nodes using level-based work filtering as described in Incremental Algorithms for Closeness Centrality by Sariyuce et al.

Level-based work filtering detects unnecessary updates to the closeness centrality and filters them out.

— From “Incremental Algorithms for Closeness Centrality”:

Theorem 1: Let \(G = (V, E)\) be a graph and \(u\) and \(v\) be two vertices in \(V\) such that there is no edge \((u, v)\) in \(E\). Let \(G' = (V, E \cup uv)\) Then \(cc[s] = cc'[s]\) if and only if \(|dG(s, u) - dG(s, v)| \leq 1\).

Where \(dG(u, v)\) denotes the length of the shortest path between two vertices \(u, v\) in a graph \(G\), \(cc[s]\) is the closeness centrality for a vertex \(s\) in \(V\), and \(cc'[s]\) is the closeness centrality for a vertex \(s\) in \(V\), with the \((u, v)\) edge added. —

We use Theorem 1 to filter out updates when adding or removing an edge. When adding an edge \((u, v)\), we compute the shortest path lengths from all other nodes to \(u\) and to \(v\) before the node is added. When removing an edge, we compute the shortest path lengths after the edge is removed. Then we apply Theorem 1 to use previously computed closeness centrality for nodes where \(|dG(s, u) - dG(s, v)| \leq 1\). This works only for undirected, unweighted graphs; the distance argument is not supported.

Closeness centrality\(^1\) of a node \(u\) is the reciprocal of the sum of the shortest path distances from \(u\) to all \(n-1\) other nodes. Since the sum of distances depends on the number of nodes in the graph, closeness is normalized by the sum of minimum possible distances \(n-1\).

\[
C(u) = \frac{n - 1}{\sum_{v=1}^{n-1} d(v, u)}
\]

where \(d(v, u)\) is the shortest-path distance between \(v\) and \(u\), and \(n\) is the number of nodes in the graph.

Notice that higher values of closeness indicate higher centrality.

Parameters

- \texttt{G} \texttt{(}graph\texttt{)} – A NetworkX graph
- \texttt{edge} \texttt{(}tuple\texttt{)} – The modified edge \((u, v)\) in the graph.

• **prev_cc** (*dictionary*) – The previous closeness centrality for all nodes in the graph.

• **insertion** (*bool, optional*) – If True (default) the edge was inserted, otherwise it was deleted from the graph.

• **wf_improved** (*bool, optional (default=True)*) – If True, scale by the fraction of nodes reachable. This gives the Wasserman and Faust improved formula. For single component graphs it is the same as the original formula.

**Returns** nodes – Dictionary of nodes with closeness centrality as the value.

**Return type** dictionary

**See also:**

*betweenness_centrality()*, *load_centrality()*, *eigenvector_centrality()*, *degree_centrality()*, *closeness_centrality()*

**Notes**

The closeness centrality is normalized to \((n-1)/(|G|-1)\) where \(n\) is the number of nodes in the connected part of graph containing the node. If the graph is not completely connected, this algorithm computes the closeness centrality for each connected part separately.

**References**

3.7.4 Current Flow Closeness

```python
current_flow_closeness_centrality(G[, weight=None, dtype=<class 'float'>, solver='lu'])
```

The function `current_flow_closeness_centrality` computes current-flow closeness centrality for nodes in a network. This metric is also known as information centrality.

**Parameters**

- **G** (*graph*) – A NetworkX graph.

- **weight** (*None or string, optional (default=None)*) – If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

- **dtype** (*data type (default=float)*) – Default data type for internal matrices. Set to np.float32 for lower memory consumption.

- **solver** (*string (default='lu')*) – Type of linear solver to use for computing the flow matrix. Options are “full” (uses most memory), “lu” (recommended), and “cg” (uses least memory).

**Returns** nodes – Dictionary of nodes with current flow closeness centrality as the value.

**Return type** dictionary
closeness_centrality()

Notes

The algorithm is from Brandes\(^1\).
See also\(^2\) for the original definition of information centrality.

References

networkx.algorithms.centrality.information_centrality

information_centrality\((G, \text{weight=\text{None}, dtype=\text{float}, solver=\text{lu}})\)

Compute current-flow closeness centrality for nodes.

Current-flow closeness centrality is variant of closeness centrality based on effective resistance between nodes in a network. This metric is also known as information centrality.

Parameters

- **G** (graph) – A NetworkX graph.
- **weight** (None or string, optional (default=\text{None})) – If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.
- **dtype** (\text{data type (default=float)}) – Default data type for internal matrices. Set to np.float32 for lower memory consumption.
- **solver** (string (default=\text{lu})) – Type of linear solver to use for computing the flow matrix. Options are “full” (uses most memory), “lu” (recommended), and “cg” (uses least memory).

Returns nodes – Dictionary of nodes with current flow closeness centrality as the value.

Return type dictionary

See also:
closeness_centrality()

Notes

The algorithm is from Brandes\(^1\).
See also\(^2\) for the original definition of information centrality.


3.7.5 (Shortest Path) Betweenness

```python
betweenness_centrality(G[, k, normalized, ...])
```
Compute the shortest-path betweenness centrality for nodes.

```python
dge_betweenness_centrality(G[, k, ...])
```
Compute betweenness centrality for edges.

```python
betweenness_centrality_subset(G, sources, ...)
```
Compute betweenness centrality for a subset of nodes.

```python
dge_betweenness_centrality_subset(G, ...[, ...])
```
Compute betweenness centrality for edges for a subset of nodes.

betweenness_centrality

Compute the shortest-path betweenness centrality for nodes.

Betweenness centrality of a node $v$ is the sum of the fraction of all-pairs shortest paths that pass through $v$

$$
c_B(v) = \sum_{s,t \in V} \frac{\sigma(s, t|v)}{\sigma(s, t)}
$$

where $V$ is the set of nodes, $\sigma(s, t)$ is the number of shortest $(s, t)$-paths, and $\sigma(s, t|v)$ is the number of those paths passing through some node $v$ other than $s, t$. If $s = t$, $\sigma(s, t) = 1$, and if $v \in s, t$, $\sigma(s, t|v) = 0^2$.

Parameters

- $G$ (graph) – A NetworkX graph.
- $k$ (int, optional (default=None)) – If $k$ is not None use $k$ node samples to estimate betweenness. The value of $k \leq n$ where $n$ is the number of nodes in the graph. Higher values give better approximation.
- normalized (bool, optional) – If True the betweenness values are normalized by $2/((n-1)(n-2))$ for graphs, and $1/((n-1)(n-2))$ for directed graphs where $n$ is the number of nodes in $G$.
- weight (None or string, optional (default=None)) – If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.
- endpoints (bool, optional) – If True include the endpoints in the shortest path counts.
- seed (integer, random_state, or None (default)) – Indicate of random number generation state. See Randomness. Note that this is only used if $k$ is not None.

Returns nodes – Dictionary of nodes with betweenness centrality as the value.

Return type dictionary

See also:

- edge_betweenness_centrality(), load_centrality()
Notes

The algorithm is from Ulrik Brandes¹. See⁴ for the original first published version and² for details on algorithms for variations and related metrics.

For approximate betweenness calculations set k=#samples to use k nodes (“pivots”) to estimate the betweenness values. For an estimate of the number of pivots needed see³.

For weighted graphs the edge weights must be greater than zero. Zero edge weights can produce an infinite number of equal length paths between pairs of nodes.

The total number of paths between source and target is counted differently for directed and undirected graphs. Directed paths are easy to count. Undirected paths are tricky: should a path from “u” to “v” count as 1 undirected path or as 2 directed paths?

For betweenness_centrality we report the number of undirected paths when G is undirected.

For betweenness_centrality_subset the reporting is different. If the source and target subsets are the same, then we want to count undirected paths. But if the source and target subsets differ – for example, if sources is {0} and targets is {1}, then we are only counting the paths in one direction. They are undirected paths but we are counting them in a directed way. To count them as undirected paths, each should count as half a path.

References

networkx.algorithms.centrality.edge_betweenness_centrality

edge_betweenness_centrality(G, k=None, normalized=True, weight=None, seed=None)

Compute betweenness centrality for edges.

Betweenness centrality of an edge \(e\) is the sum of the fraction of all-pairs shortest paths that pass through \(e\)

\[c_B(e) = \sum_{s,t\in V} \frac{\sigma(s,t|e)}{\sigma(s,t)}\]

where \(V\) is the set of nodes, \(\sigma(s,t)\) is the number of shortest \((s,t)\)-paths, and \(\sigma(s,t|e)\) is the number of those paths passing through edge \(e\)².

Parameters

- **G (graph)** – A NetworkX graph.
- **k (int, optional (default=None))** – If k is not None use k node samples to estimate betweenness. The value of k <= n where n is the number of nodes in the graph. Higher values give better approximation.
- **normalized (bool, optional)** – If True the betweenness values are normalized by \(2/(n(n-1))\) for graphs, and \(1/(n(n-1))\) for directed graphs where n is the number of nodes in G.
- **weight (None or string, optional (default=None))** – If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.
- **seed (integer, random_state, or None (default))** – Indicator of random number generation state. See Randomness. Note that this is only used if k is not None.


3.7. Centrality
Returns edges – Dictionary of edges with betweenness centrality as the value.

Return type dictionary

See also:

betweenness_centrality(), edge_load()

Notes

The algorithm is from Ulrik Brandes\(^1\).

For weighted graphs the edge weights must be greater than zero. Zero edge weights can produce an infinite number of equal length paths between pairs of nodes.

References

networkx.algorithms.centrality.betweenness_centrality_subset

betweenness_centrality_subset (G, sources, targets, normalized=False, weight=None)

Compute betweenness centrality for a subset of nodes.

\[ c_B(v) = \sum_{s \in S, t \in T} \frac{\sigma(s, t|v)}{\sigma(s, t)} \]

where \( S \) is the set of sources, \( T \) is the set of targets, \( \sigma(s, t) \) is the number of shortest \((s, t)\)-paths, and \( \sigma(s, t|v) \) is the number of those paths passing through some node \( v \) other than \( s, t \). If \( s = t \), \( \sigma(s, t) = 1 \), and if \( v \in s, t \), \( \sigma(s, t|v) = 0 \)\(^2\).

Parameters

- \( G \) (graph) – A NetworkX graph.
- sources (list of nodes) – Nodes to use as sources for shortest paths in betweenness
- targets (list of nodes) – Nodes to use as targets for shortest paths in betweenness
- normalized (bool, optional) – If True the betweenness values are normalized by \( 2/((n − 1)(n − 2)) \) for graphs, and \( 1/((n − 1)(n − 2)) \) for directed graphs where \( n \) is the number of nodes in \( G \).
- weight (None or string, optional (default=None)) – If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

Returns nodes – Dictionary of nodes with betweenness centrality as the value.

Return type dictionary

See also:

edge_betweenness_centrality(), load_centrality()

---


Notes

The basic algorithm is from\(^1\).

For weighted graphs the edge weights must be greater than zero. Zero edge weights can produce an infinite number of equal length paths between pairs of nodes.

The normalization might seem a little strange but it is designed to make \texttt{betweenness_centrality(G)} be the same as \texttt{betweenness_centrality_subset(G,sources=G.nodes(),targets=G.nodes())}.

The total number of paths between source and target is counted differently for directed and undirected graphs. Directed paths are easy to count. Undirected paths are tricky: should a path from “u” to “v” count as 1 undirected path or as 2 directed paths?

For \texttt{betweenness_centrality} we report the number of undirected paths when \(G\) is undirected.

For \texttt{betweenness_centrality_subset} the reporting is different. If the source and target subsets are the same, then we want to count undirected paths. But if the source and target subsets differ – for example, if sources is \{0\} and targets is \{1\}, then we are only counting the paths in one direction. They are undirected paths but we are counting them in a directed way. To count them as undirected paths, each should count as half a path.

References

\texttt{networkx.algorithms.centrality.edge_betweenness_centrality_subset}

\texttt{edge_betweenness_centrality_subset (G, sources, targets, normalized=False, weight=None)}

Compute betweenness centrality for edges for a subset of nodes.

\[
c_B(v) = \sum_{s \in S, t \in T} \frac{\sigma(s, t | e)}{\sigma(s, t)}
\]

where \(S\) is the set of sources, \(T\) is the set of targets, \(\sigma(s, t)\) is the number of shortest \((s, t)\)-paths, and \(\sigma(s, t | e)\) is the number of those paths passing through edge \(e\)\(^2\).

Parameters

- \(G\) (graph) – A \texttt{networkx} graph.
- \texttt{sources} (list of nodes) – Nodes to use as sources for shortest paths in betweenness
- \texttt{targets} (list of nodes) – Nodes to use as targets for shortest paths in betweenness
- \texttt{normalized} (bool, optional) – If True the betweenness values are normalized by \(2/ (n (n-1))\) for graphs, and \(1/ (n (n-1))\) for directed graphs where \(n\) is the number of nodes in \(G\).
- \texttt{weight} (None or string, optional (default=None)) – If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

Returns edges – Dictionary of edges with Betweenness centrality as the value.

Return type dictionary

See also:

\texttt{betweenness_centrality()}, \texttt{edge_load()}

---


3.7. Centrality 201
Notes

The basic algorithm is from\(^1\).

For weighted graphs the edge weights must be greater than zero. Zero edge weights can produce an infinite number of equal length paths between pairs of nodes.

The normalization might seem a little strange but it is the same as in edge_betweenness_centrality() and is designed to make edge_betweenness_centrality(G) be the same as edge_betweenness_centrality_subset(G, sources=G.nodes(), targets=G.nodes()).

References

3.7.6 Current Flow Betweenness

networkx.algorithms.centrality.current_flow_betweenness_centrality

current_flow_betweenness_centrality(G, normalized=True, weight=None, dtype=\'<class \'float\'>, solver='full')

Compute current-flow betweenness centrality for nodes.

Current-flow betweenness centrality uses an electrical current model for information spreading in contrast to betweenness centrality which uses shortest paths.

Current-flow betweenness centrality is also known as random-walk betweenness centrality\(^2\).

Parameters

- **G** (graph) – A NetworkX graph
- **normalized** (bool, optional (default=True)) – If True the betweenness values are normalized by \(2/[(n-1)(n-2)]\) where \(n\) is the number of nodes in \(G\).
- **weight** (string or None, optional (default=None)) – Key for edge data used as the edge weight. If None, then use 1 as each edge weight.
- **dtype** (data type (float)) – Default data type for internal matrices. Set to np.float32 for lower memory consumption.
- **solver** (string (default='lu')) – Type of linear solver to use for computing the flow matrix. Options are “full” (uses most memory), “lu” (recommended), and “cg” (uses least memory).

Returns nodes – Dictionary of nodes with betweenness centrality as the value.

Return type dictionary

---


See also:

approximate_current_flow_betweenness_centrality(), betweenness_centrality(),
edge_betweenness_centrality(), edge_current_flow_betweenness_centrality()

Notes

Current-flow betweenness can be computed in \(O(I(n-1) + mn \log n)\) time\(^1\), where \(I(n-1)\) is the time needed to compute the inverse Laplacian. For a full matrix this is \(O(n^3)\) but using sparse methods you can achieve \(O(m \sqrt{k})\) where \(k\) is the Laplacian matrix condition number.

The space required is \(O(nw)\) where \(w\) is the width of the sparse Laplacian matrix. Worse case is \(w = n\) for \(O(n^2)\).

If the edges have a ‘weight’ attribute they will be used as weights in this algorithm. Unspecified weights are set to 1.

References

networkx.algorithms.centrality.edge_current_flow_betweenness_centrality

dge_current_flow_betweenness_centrality(G, normalized=True, weight=None,
dtype=<class 'float'>, solver='full')

Compute current-flow betweenness centrality for edges.

Current-flow betweenness centrality uses an electrical current model for information spreading in contrast to betweenness centrality which uses shortest paths.

Current-flow betweenness centrality is also known as random-walk betweenness centrality\(^2\).

Parameters

- **G** (*graph*) – A NetworkX graph
- **normalized** (*bool*, *optional* (default=True)) – If True the betweenness values are normalized by \(2/[(n-1)(n-2)]\) where \(n\) is the number of nodes in \(G\).
- **weight** (*string* or *None*, *optional* (default=None)) – Key for edge data used as the edge weight. If None, then use 1 as each edge weight.
- **dtype** (*data type* (default=float)) – Default data type for internal matrices. Set to np.float32 for lower memory consumption.
- **solver** (*string* (default='lu')) – Type of linear solver to use for computing the flow matrix. Options are “full” (uses most memory), “lu” (recommended), and “cg” (uses least memory).

Returns **nodes** – Dictionary of edge tuples with betweenness centrality as the value.

Return type **dictionary**

Raises **NetworkXError** – The algorithm does not support DiGraphs. If the input graph is an instance of DiGraph class, NetworkXError is raised.

See also:

betweenness_centrality(), edge_betweenness_centrality(),
current_flow_betweenness_centrality()


Notes

Current-flow betweenness can be computed in $O(I(n-1) + mn \log n)$ time\(^1\), where $I(n-1)$ is the time needed to compute the inverse Laplacian. For a full matrix this is $O(n^3)$ but using sparse methods you can achieve $O(nm\sqrt{k})$ where $k$ is the Laplacian matrix condition number.

The space required is $O(nw)$ where $w$ is the width of the sparse Laplacian matrix. Worse case is $w = n$ for $O(n^2)$.

If the edges have a ‘weight’ attribute they will be used as weights in this algorithm. Unspecified weights are set to 1.

References

networkx.algorithms.centrality.approximate_current_flow_betweenness_centrality

approximate_current_flow_betweenness_centrality(G, normalized=True, weight=None, dtype=\langle\text{class 'float'}\rangle, solver='full', epsilon=0.5, kmax=10000, seed=None)

Compute the approximate current-flow betweenness centrality for nodes.

Approximates the current-flow betweenness centrality within absolute error of epsilon with high probability\(^1\).

Parameters

- **G** (graph) – A NetworkX graph
- **normalized** (bool, optional (default=True)) – If True the betweenness values are normalized by $2/[(n-1)(n-2)]$ where n is the number of nodes in G.
- **weight** (string or None, optional (default=None)) – Key for edge data used as the edge weight. If None, then use 1 as each edge weight.
- **dtype** (data type (float)) – Default data type for internal matrices. Set to np.float32 for lower memory consumption.
- **solver** (string (default=’lu’)) – Type of linear solver to use for computing the flow matrix. Options are “full” (uses most memory), “lu” (recommended), and “cg” (uses least memory).
- **epsilon** (float) – Absolute error tolerance.
- **kmax** (int) – Maximum number of sample node pairs to use for approximation.
- **seed** (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns

- **nodes** – Dictionary of nodes with betweenness centrality as the value.

Return type

dictionary

See also:

current_flow_betweenness_centrality()


Notes

The running time is \( O((1/\epsilon^2)m\sqrt{k}\log n) \) and the space required is \( O(m) \) for \( n \) nodes and \( m \) edges.

If the edges have a 'weight' attribute they will be used as weights in this algorithm. Unspecified weights are set to 1.

References

networkx.algorithms.centrality.current_flow_betweenness_centrality_subset

current_flow_betweenness_centrality_subset(G, sources, targets, normalized=True, weight=None, dtype=<class 'float'>, solver='lu')

Compute current-flow betweenness centrality for subsets of nodes.

Current-flow betweenness centrality uses an electrical current model for information spreading in contrast to betweenness centrality which uses shortest paths.

Current-flow betweenness centrality is also known as random-walk betweenness centrality\(^2\).

Parameters

- \( G \) (graph) – A NetworkX graph
- sources (list of nodes) – Nodes to use as sources for current
- targets (list of nodes) – Nodes to use as sinks for current
- normalized (bool, optional (default=True)) – If True the betweenness values are normalized by \( b=b/(n-1)(n-2) \) where \( n \) is the number of nodes in \( G \).
- weight (string or None, optional (default=None)) – Key for edge data used as the edge weight. If None, then use 1 as each edge weight.
- dtype (data type (float)) – Default data type for internal matrices. Set to np.float32 for lower memory consumption.
- solver (string (default='lu')) – Type of linear solver to use for computing the flow matrix. Options are “full” (uses most memory), “lu” (recommended), and “cg” (uses least memory).

Returns nodes – Dictionary of nodes with betweenness centrality as the value.

Return type dictionary

See also:

approximate_current_flow_betweenness_centrality(), betweenness_centrality(), edge_betweenness_centrality(), edge_current_flow_betweenness_centrality()

Notes

Current-flow betweenness can be computed in \( O(I(n - 1) + mn \log n) \) time\(^1\), where \( I(n - 1) \) is the time needed to compute the inverse Laplacian. For a full matrix this is \( O(n^3) \) but using sparse methods you can achieve \( O(mn\sqrt{k}) \) where \( k \) is the Laplacian matrix condition number.


The space required is $O(nw)$ where $w$ is the width of the sparse Laplacian matrix. Worse case is $w = n$ for $O(n^2)$.

If the edges have a ‘weight’ attribute they will be used as weights in this algorithm. Unspecified weights are set to 1.

References

networkx.algorithms.centrality.edge_current_flow_betweenness_centrality_subset

edge_current_flow_betweenness_centrality_subset (G, sources, targets, normalized=True, weight=None, dtype=<class 'float'>, solver='lu')

Compute current-flow betweenness centrality for edges using subsets of nodes.

Current-flow betweenness centrality uses an electrical current model for information spreading in contrast to betweenness centrality which uses shortest paths.

Current-flow betweenness centrality is also known as random-walk betweenness centrality$^2$.

Parameters

- **G** (graph) – A NetworkX graph
- **sources** (list of nodes) – Nodes to use as sources for current
- **targets** (list of nodes) – Nodes to use as sinks for current
- **normalized** (bool, optional (default=True)) – If True the betweenness values are normalized by $b=b/(n-1)(n-2)$ where n is the number of nodes in G.
- **weight** (string or None, optional (default=None)) – Key for edge data used as the edge weight. If None, then use 1 as each edge weight.
- **dtype** (data type (float)) – Default data type for internal matrices. Set to np.float32 for lower memory consumption.
- **solver** (string (default='lu')) – Type of linear solver to use for computing the flow matrix. Options are “full” (uses most memory), “lu” (recommended), and “cg” (uses least memory).

Returns **nodes** – Dictionary of edge tuples with betweenness centrality as the value.

Return type **dict**

See also:

betweenness_centrality(), edge_betweenness_centrality(), current_flow_betweenness_centrality()

Notes

Current-flow betweenness can be computed in $O(I(n-1) + mn \log n)$ time$^1$, where $I(n-1)$ is the time needed to compute the inverse Laplacian. For a full matrix this is $O(n^3)$ but using sparse methods you can achieve $O(nm\sqrt{k})$ where $k$ is the Laplacian matrix condition number.

The space required is $O(nw)$ where $w$ is the width of the sparse Laplacian matrix. Worse case is $w = n$ for $O(n^2)$.

---


If the edges have a ‘weight’ attribute they will be used as weights in this algorithm. Unspecified weights are set to 1.

References

3.7.7 Communicability Betweenness

```python
communicability_betweenness_centrality(G, normalized=True)
```

Returns subgraph communicability for all pairs of nodes in G.

networkx.algorithms.centrality.communicability_betweenness_centrality

**communicability_betweenness_centrality** *(G, normalized=True)*

Returns subgraph communicability for all pairs of nodes in G.

Communicability betweenness measure makes use of the number of walks connecting every pair of nodes as the basis of a betweenness centrality measure.

**Parameters**

* G *(graph)*

**Returns**

* nodes – Dictionary of nodes with communicability betweenness as the value.

* Return type – dictionary

**Raises**

* NetworkXError – If the graph is not undirected and simple.

**Notes**

Let $G=(V, E)$ be a simple undirected graph with $n$ nodes and $m$ edges, and $A$ denote the adjacency matrix of $G$.

Let $G(r)=(V, E(r))$ be the graph resulting from removing all edges connected to node $r$ but not the node itself.

The adjacency matrix for $G(r)$ is $A+E(r)$, where $E(r)$ has nonzeros only in row and column $r$.

The subgraph betweenness of a node $r$ is

$$\omega_r = \frac{1}{C} \sum_p \sum_q \frac{G_{prq}}{G_{pq}}, p \neq q, q \neq r,$$

where $G_{prq}=(e^{A})_{pq} - (e^{A+E(r)})_{pq}$ is the number of walks involving node $r$, $G_{pq}=(e^{A})_{pq}$ is the number of closed walks starting at node $p$ and ending at node $q$, and $C=(n-1)^2-(n-1)$ is a normalization factor equal to the number of terms in the sum.

The resulting $\omega_r$ takes values between zero and one. The lower bound cannot be attained for a connected graph, and the upper bound is attained in the star graph.

**References**

**Examples**

---

>>> G = nx.Graph([(0,1),(1,2),(1,5),(5,4),(2,4),(2,3),(4,3),(3,6)])
>>> cbc = nx.communicability_betweenness_centrality(G)

### 3.7.8 Group Centrality

<table>
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<tr>
<th>Function</th>
<th>Description</th>
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</thead>
<tbody>
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<td><code>group_betweenness_centrality(G, C[, ...])</code></td>
<td>Compute the group betweenness centrality for a group of nodes.</td>
</tr>
<tr>
<td><code>group_closeness_centrality(G, S[, weight])</code></td>
<td>Compute the group closeness centrality for a group of nodes.</td>
</tr>
<tr>
<td><code>group_degree_centrality(G, S)</code></td>
<td>Compute the group degree centrality for a group of nodes.</td>
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<tr>
<td><code>group_in_degree_centrality(G, S)</code></td>
<td>Compute the group in-degree centrality for a group of nodes.</td>
</tr>
<tr>
<td><code>group_out_degree_centrality(G, S)</code></td>
<td>Compute the group out-degree centrality for a group of nodes.</td>
</tr>
</tbody>
</table>

**networkx.algorithms.centrality.group_betweenness_centrality**

The function `group_betweenness_centrality` computes the group betweenness centrality of a group of nodes. The group betweenness centrality of a group of nodes $C$ is the sum of the fraction of all-pairs shortest paths that pass through any vertex in $C$.

$$c_B(C) = \sum_{s,t \in V - C; s < t} \frac{\sigma(s, t|C)}{\sigma(s, t)}$$

where $V$ is the set of nodes, $\sigma(s, t)$ is the number of shortest $(s, t)$-paths, and $\sigma(s, t|C)$ is the number of those paths passing through some node in group $C$. Note that $(s, t)$ are not members of the group ($V - C$ is the set of nodes in $V$ that are not in $C$).

**Parameters**

- **G** (*graph*) – A NetworkX graph.
- **C** (*list or set*) – C is a group of nodes which belong to G, for which group betweenness centrality is to be calculated.
- **normalized** (*bool, optional*) – If True, group betweenness is normalized by $2/((|V| - |C|)(|V| - |C| - 1))$ for graphs and $1/((|V| - |C|)(|V| - |C| - 1))$ for directed graphs where $|V|$ is the number of nodes in G and $|C|$ is the number of nodes in C.
- **weight** (*None or string, optional (default=None*)*) – If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

**Returns**

- **betweenness** – Group betweenness centrality of the group C.

**Return type**

float

**See also:**

`betweenness_centrality()`
Notes

The measure is described in\(^1\). The algorithm is an extension of the one proposed by Ulrik Brandes for betweenness centrality of nodes. Group betweenness is also mentioned in his paper\(^2\) along with the algorithm. The importance of the measure is discussed in\(^3\).

The number of nodes in the group must be a maximum of \(n - 2\) where \(n\) is the total number of nodes in the graph.

For weighted graphs the edge weights must be greater than zero. Zero edge weights can produce an infinite number of equal length paths between pairs of nodes.

References

networkx.algorithms.centrality.group_closeness_centrality

group_closeness_centrality\((G, S, weight=None)\)

Compute the group closeness centrality for a group of nodes.

Group closeness centrality of a group of nodes \(S\) is a measure of how close the group is to the other nodes in the graph.

\[
c_{\text{close}}(S) = \frac{|V - S|}{\sum_{v \in V - S} d_{S,v}}
\]

\[
d_{S,v} = \min_{u \in S} (d_{u,v})
\]

where \(V\) is the set of nodes, \(d_{S,v}\) is the distance of the group \(S\) from \(v\) defined as above. \((V - S)\) is the set of nodes in \(V\) that are not in \(S\).

Parameters

- \(G\) (graph) – A NetworkX graph.
- \(S\) (list or set) – \(S\) is a group of nodes which belong to \(G\), for which group closeness centrality is to be calculated.
- \(weight\) (None or string, optional (default=None)) – If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

Raises \NodeNotFound – If node(s) in \(S\) are not present in \(G\).

Returns \closeness – Group closeness centrality of the group \(S\).

Return type float

See also:
closeness_centrality()


Notes

The measure was introduced in\(^1\). The formula implemented here is described in\(^2\).
Higher values of closeness indicate greater centrality.

It is assumed that \(1/0\) is 0 (required in the case of directed graphs, or when a shortest path length is 0).
The number of nodes in the group must be a maximum of \(n - 1\) where \(n\) is the total number of nodes in the graph.

For directed graphs, the incoming distance is utilized here. To use the outward distance, act on \(G\).\text{reverse}().
For weighted graphs the edge weights must be greater than zero. Zero edge weights can produce an infinite number of equal length paths between pairs of nodes.

References

```
networkx.algorithms.centrality.group_degree_centrality
```

\textbf{group_degree_centrality}(G, S)
Compute the group degree centrality for a group of nodes.

Group degree centrality of a group of nodes \(S\) is the fraction of non-group members connected to group members.

**Parameters**

- \(G\) (graph) – A NetworkX graph.
- \(S\) (list or set) – \(S\) is a group of nodes which belong to \(G\), for which group degree centrality is to be calculated.

**Raises** \textit{NetworkXError} – If node(s) in \(S\) are not in \(G\).

**Returns** \textit{centrality} – Group degree centrality of the group \(S\).

**Return type** float

See also:

\textit{degree_centrality()}, \textit{group_in_degree_centrality()}, \textit{group_out_degree_centrality()}

Notes

The measure was introduced in\(^1\).

The number of nodes in the group must be a maximum of \(n - 1\) where \(n\) is the total number of nodes in the graph.

References

networkx.algorithms.centrality.group_in_degree_centrality

group_in_degree_centrality \((G, S)\)
Compute the group in-degree centrality for a group of nodes.

Group in-degree centrality of a group of nodes \(S\) is the fraction of non-group members connected to group members by incoming edges.

Parameters
- \(G\) (graph) – A NetworkX graph.
- \(S\) (list or set) – \(S\) is a group of nodes which belong to \(G\), for which group in-degree centrality is to be calculated.

Returns centrality – Group in-degree centrality of the group \(S\).

Return type float

Raises
- NetworkXNotImplemented – If \(G\) is undirected.
- NodeNotFound – If node(s) in \(S\) are not in \(G\).

See also:
degree_centrality(), group_degree_centrality(), group_out_degree_centrality()

Notes
The number of nodes in the group must be a maximum of \(n - 1\) where \(n\) is the total number of nodes in the graph.
\(G\).neighbors\(\{i\}\) gives nodes with an outward edge from \(i\), in a DiGraph, so for group in-degree centrality, the reverse graph is used.

networkx.algorithms.centrality.group_out_degree_centrality

group_out_degree_centrality \((G, S)\)
Compute the group out-degree centrality for a group of nodes.

Group out-degree centrality of a group of nodes \(S\) is the fraction of non-group members connected to group members by outgoing edges.

Parameters
- \(G\) (graph) – A NetworkX graph.
- \(S\) (list or set) – \(S\) is a group of nodes which belong to \(G\), for which group in-degree centrality is to be calculated.

Returns centrality – Group out-degree centrality of the group \(S\).

Return type float

Raises
- NetworkXNotImplemented – If \(G\) is undirected.
- NodeNotFound – If node(s) in \(S\) are not in \(G\).
See also: 

degree_centrality(), group_degree_centrality(), group_in_degree_centrality()

Notes

The number of nodes in the group must be a maximum of n - 1 where n is the total number of nodes in the graph.

G.neighbors(i) gives nodes with an outward edge from i, in a DiGraph, so for group out-degree centrality, the graph itself is used.

3.7.9 Load

<table>
<thead>
<tr>
<th>load_centrality(G[, v, cutoff, normalized, ...])</th>
<th>Compute load centrality for nodes.</th>
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</thead>
<tbody>
<tr>
<td>edge_load_centrality(G[, cutoff])</td>
<td>Compute edge load.</td>
</tr>
</tbody>
</table>

networkx.algorithms.centrality.load_centrality

load_centrality (G, v=None, cutoff=None, normalized=True, weight=None)

Compute load centrality for nodes.

The load centrality of a node is the fraction of all shortest paths that pass through that node.

Parameters

• G (graph) – A networkx graph.

• normalized (bool, optional (default=True)) – If True the betweenness values are normalized by b=b/(n-1)(n-2) where n is the number of nodes in G.

• weight (None or string, optional (default=None)) – If None, edge weights are ignored. Otherwise holds the name of the edge attribute used as weight.

• cutoff (bool, optional (default=None)) – If specified, only consider paths of length <= cut-off.

Returns nodes – Dictionary of nodes with centrality as the value.

Return type dictionary

See also: 

betweenness_centrality()

Notes

Load centrality is slightly different than betweenness. It was originally introduced by\(^2\). For this load algorithm see\(^1\).


References

networkx.algorithms.centrality.edge_load_centrality

edge_load_centrality \((G, \text{cutoff}=\text{False})\)

Compute edge load.

WARNING: This concept of edge load has not been analysed or discussed outside of NetworkX that we know of. It is based loosely on load_centrality in the sense that it counts the number of shortest paths which cross each edge. This function is for demonstration and testing purposes.

Parameters

- \(G\) (graph) – A networkx graph
- \(\text{cutoff}\) (bool, optional (default=\text{False})) – If specified, only consider paths of length \(\leq\) cutoff.

Returns

- A dict keyed by edge 2-tuple to the number of shortest paths which use that edge. Where more than one path is shortest the count is divided equally among paths.

3.7.10 Subgraph

<table>
<thead>
<tr>
<th>subgraph_centrality(G)</th>
<th>Returns subgraph centrality for each node in G.</th>
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<tbody>
<tr>
<td>subgraph_centrality_exp(G)</td>
<td>Returns the subgraph centrality for each node of G.</td>
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<tr>
<td>estrada_index(G)</td>
<td>Returns the Estrada index of a the graph G.</td>
</tr>
</tbody>
</table>

networkx.algorithms.centrality.subgraph_centrality

subgraph_centrality \((G)\)

Returns subgraph centrality for each node in G.

Subgraph centrality of a node \(n\) is the sum of weighted closed walks of all lengths starting and ending at node \(n\). The weights decrease with path length. Each closed walk is associated with a connected subgraph \(^1\).

Parameters \(G\) (graph)

Returns \(\text{nodes}\) – Dictionary of nodes with subgraph centrality as the value.

Return type \(\text{dictionary}\)

Raises \(\text{NetworkXError}\) – If the graph is not undirected and simple.

See also:

subgraph_centrality_exp() Alternative algorithm of the subgraph centrality for each node of G.

Notes

This version of the algorithm computes eigenvalues and eigenvectors of the adjacency matrix.

Subgraph centrality of a node $u$ in $G$ can be found using a spectral decomposition of the adjacency matrix $^1$, 

$$SC(u) = \sum_{j=1}^{N} (v_j^u)^2 e^{\lambda_j},$$

where $v_j$ is an eigenvector of the adjacency matrix $A$ of $G$ corresponding to the eigenvalue $\lambda_j$.

### Examples

(Example from $^1$) >>> G = nx.Graph([(1,2),(1,5),(1,8),(2,3),(2,8),(3,4),(3,6),(4,5),(4,7),(5,6),(6,7),(7,8)]) >>> sc = nx.subgraph_centrality(G) >>> print(['%s %0.2f'%(node,sc[node]) for node in sorted(sc)]) ['1 3.90', '2 3.90', '3 3.64', '4 3.71', '5 3.64', '6 3.71', '7 3.64', '8 3.90']

### References

networkx.algorithms.centrality.subgraph_centrality_exp

#### subgraph_centrality_exp(G)

Returns the subgraph centrality for each node of $G$.

Subgraph centrality of a node $n$ is the sum of weighted closed walks of all lengths starting and ending at node $n$. The weights decrease with path length. Each closed walk is associated with a connected subgraph ($^1$).

**Parameters**

- $G$ (graph)

**Returns**

- nodes – Dictionary of nodes with subgraph centrality as the value.

**Return type**

- dictionary

**Raises**

NetworkXError – If the graph is not undirected and simple.

See also:

- subgraph_centrality() Alternative algorithm of the subgraph centrality for each node of $G$.

### Notes

This version of the algorithm exponentiates the adjacency matrix.

The subgraph centrality of a node $u$ in $G$ can be found using the matrix exponential of the adjacency matrix of $G$ $^1$,

$$SC(u) = (e^A)_{uu}.$$
networkx.algorithms.centrality.estrada_index

**estrada_index**(*G*)

Returns the Estrada index of a the graph *G*.

The Estrada Index is a topological index of folding or 3D “compactness” (1).

**Parameters**

- *G* (*graph*)

**Returns**

- estrada index

**Return type** float

**Raises** NetworkXError – If the graph is not undirected and simple.

**Notes**

Let *G*=(*V*, *E*) be a simple undirected graph with *n* nodes and let

\[
\lambda_1 \leq \lambda_2 \leq \cdots \lambda_n \n\]

be a non-increasing ordering of the eigenvalues of its adjacency matrix *A*. The Estrada index is (1,2)

\[
EE(G) = \sum_{j=1}^{n} e^{\lambda_j}
\]

**References**


3.7.11 Harmonic Centrality

**harmonic_centrality**(*G*, nbunch=None, distance=None)

Compute harmonic centrality for nodes.

networkx.algorithms.centrality.harmonic_centrality

**harmonic_centrality**(*G*, *nbunch=None*, *distance=None*)

Calculate harmonic centrality for nodes.

Harmonic centrality\(^1\) of a node *u* is the sum of the reciprocal of the shortest path distances from all other nodes to *u*

\[
C(u) = \sum_{v \neq u} \frac{1}{d(v, u)}
\]

where \(d(v, u)\) is the shortest-path distance between *v* and *u*.

Notice that higher values indicate higher centrality.

---

Parameters

• \( G \) (graph) – A NetworkX graph
• \( nbunch \) (container) – Container of nodes. If provided harmonic centrality will be computed only over the nodes in \( nbunch \).
• \( distance \) (edge attribute key, optional (default=None)) – Use the specified edge attribute as the edge distance in shortest path calculations. If None, then each edge will have distance equal to 1.

Returns nodes – Dictionary of nodes with harmonic centrality as the value.

Return type dictionary

See also:

betweenness_centrality(), load_centrality(), eigenvector_centrality(), degree_centrality(), closeness_centrality()

Notes

If the ‘distance’ keyword is set to an edge attribute key then the shortest-path length will be computed using Dijkstra’s algorithm with that edge attribute as the edge weight.

References

3.7.12 Dispersion

dispersion(G[, u, v, normalized, alpha, b, c]) Calculate dispersion between \( u \) and \( v \) in \( G \).

networkx.algorithms.centrality.dispersion

dispersion (G, u=None, v=None, normalized=True, alpha=1.0, b=0.0, c=0.0)

Calculate dispersion between \( u \) and \( v \) in \( G \).

A link between two actors (\( u \) and \( v \)) has a high dispersion when their mutual ties (\( s \) and \( t \)) are not well connected with each other.

Parameters

• \( G \) (graph) – A NetworkX graph.
• \( u \) (node, optional) – The source for the dispersion score (e.g. ego node of the network).
• \( v \) (node, optional) – The target of the dispersion score if specified.
• \( normalized \) (bool) – If True (default) normalize by the embededness of the nodes (\( u \) and \( v \)).

Returns nodes – If \( u \) (\( v \)) is specified, returns a dictionary of nodes with dispersion score for all “target” (“source”) nodes. If neither \( u \) nor \( v \) is specified, returns a dictionary of dictionaries for all nodes \‘u\’ in the graph with a dispersion score for each node \‘v\’.

Return type dictionary
Notes

This implementation follows Lars Backstrom and Jon Kleinberg\(^1\). Typical usage would be to run dispersion on the ego network \(G_u\) if \(u\) were specified. Running \texttt{dispersion()} with neither \(u\) nor \(v\) specified can take some time to complete.

References

3.7.13 Reaching

\begin{verbatim}
local_reaching_centrality(G, v[, paths, weight, normalized])
global_reaching_centrality(G[, weight], normalized)
\end{verbatim}

\texttt{networkx.algorithms.centrality.local_reaching_centrality}

**local_reaching_centrality** \((G, v, paths=None, weight=None, normalized=True)\)

Returns the local reaching centrality of a node in a directed graph.

The local reaching centrality of a node in a directed graph is the proportion of other nodes reachable from that node\(^1\).

**Parameters**

- \(G\) (\texttt{DiGraph}) – A NetworkX DiGraph.
- \(v\) (\texttt{node}) – A node in the directed graph \(G\).
- \texttt{paths} (\texttt{dictionary} (\texttt{default=None})) – If this is not \texttt{None} it must be a dictionary representation of single-source shortest paths, as computed by, for example, \texttt{networkx.shortest_path()} with source node \(v\). Use this keyword argument if you intend to invoke this function many times but don’t want the paths to be recomputed each time.
- \texttt{weight} (\texttt{None} or \texttt{string}, optional (\texttt{default=None})) – Attribute to use for edge weights. If \texttt{None}, each edge weight is assumed to be one. A higher weight implies a stronger connection between nodes and a shorter path length.
- \texttt{normalized} (\texttt{bool}, optional (\texttt{default=True})) – Whether to normalize the edge weights by the total sum of edge weights.

**Returns** \(h\) – The local reaching centrality of the node \(v\) in the graph \(G\).

**Return type** \texttt{float}

**Examples**

```python
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_edges_from([(1, 2), (1, 3)])
>>> nx.local_reaching_centrality(G, 3)
```


NetworkX Reference, Release 2.5rc1.dev20191214022633

0.0
>>> G.add_edge(3, 2)
>>> nx.local_reaching_centrality(G, 3)
0.5

See also:

global_reaching_centrality()

References

networkx.algorithms.centrality.global_reaching_centrality

global_reaching_centrality(G, weight=None, normalized=True)

Returns the global reaching centrality of a directed graph.

The global reaching centrality of a weighted directed graph is the average over all nodes of the difference between the local reaching centrality of the node and the greatest local reaching centrality of any node in the graph\(^1\). For more information on the local reaching centrality, see local_reaching_centrality(). Informally, the local reaching centrality is the proportion of the graph that is reachable from the neighbors of the node.

Parameters

- **G** (DiGraph) – A networkx DiGraph.
- **weight** (None or string, optional (default=None)) – Attribute to use for edge weights. If None, each edge weight is assumed to be one. A higher weight implies a stronger connection between nodes and a shorter path length.
- **normalized** (bool, optional (default=True)) – Whether to normalize the edge weights by the total sum of edge weights.

Returns **h** – The global reaching centrality of the graph.

Return type **float**

Examples

```python
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_edge(1, 2)
>>> G.add_edge(1, 3)
>>> nx.global_reaching_centrality(G)
1.0
>>> G.add_edge(3, 2)
>>> nx.global_reaching_centrality(G)
0.75
```

See also:

local_reaching_centrality()

---

References

3.7.14 Percolation

```
percolation_centrality(G[, attribute, ...])  Compute the percolation centrality for nodes.
```

`networkx.algorithms.centrality.percolation_centrality`

```
percolation_centrality(G, attribute='percolation', states=None, weight=None)
```

Compute the percolation centrality for nodes.

Percolation centrality of a node \( v \), at a given time, is defined as the proportion of ‘percolated paths’ that go through that node.

This measure quantifies relative impact of nodes based on their topological connectivity, as well as their percolation states.

Percolation states of nodes are used to depict network percolation scenarios (such as during infection transmission in a social network of individuals, spreading of computer viruses on computer networks, or transmission of disease over a network of towns) over time. In this measure usually the percolation state is expressed as a decimal between 0.0 and 1.0.

When all nodes are in the same percolated state this measure is equivalent to betweenness centrality.

Parameters

- `G (graph)` – A NetworkX graph.
- `attribute (None or string, optional (default='percolation'))` – Name of the node attribute to use for percolation state, used if `states` is `None`.
- `states (None or dict, optional (default=None))` – Specify percolation states for the nodes, nodes as keys states as values.
- `weight (None or string, optional (default=None))` – If `None`, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

Returns `nodes` – Dictionary of nodes with percolation centrality as the value.

Return type `dictionary`

See also:

`betweenness_centrality()`

Notes

The algorithm is from Mahendra Piraveenan, Mikhail Prokopenko, and Liaquat Hossain\(^1\) Pair dependencies are calculated and accumulated using\(^2\)

For weighted graphs the edge weights must be greater than zero. Zero edge weights can produce an infinite number of equal length paths between pairs of nodes.

\(^1\) Mahendra Piraveenan, Mikhail Prokopenko, Liaquat Hossain Percolation Centrality: Quantifying Graph-Theoretic Impact of Nodes during Percolation in Networks http://journals.plos.org/plosone/article?id=10.1371/journal.pone.0053095

References

3.7.15 Second Order Centrality

```python
networkx.algorithms.centrality.second_order_centrality

second_order_centrality(G)  
Compute the second order centrality for nodes of G.

Parameters

G (graph) – A NetworkX connected and undirected graph.

Returns

nodes – Dictionary keyed by node with second order centrality as the value.

Return type

dictionary

Examples

```python
>>> G = nx.star_graph(10)
```print(sorted(soc.items(), key=lambda x:x[1])[0][0])
# pick first id
0

Raises

NetworkXException – If the graph G is empty, non connected or has negative weights.

See also:

betweenness_centrality()

Notes

Lower values of second order centrality indicate higher centrality.

The algorithm is from Kermarrec, Le Merrer, Sericola and Trédan.

This code implements the analytical version of the algorithm, i.e., there is no simulation of a random walk process involved. The random walk is here unbiased (corresponding to eq 6 of the paper), thus the centrality values are the standard deviations for random walk return times on the transformed input graph G (equal in-degree at each nodes by adding self-loops).

Complexity of this implementation, made to run locally on a single machine, is $O(n^3)$, with $n$ the size of G, which makes it viable only for small graphs.

References

3.7.16 VoteRank

```python
voterank(G[, number_of_nodes, max_iter])
```

Compute a list of seeds for the nodes in the graph using VoteRank

**networkx.algorithms.centrality.voterank**

**voterank** \((G, \text{number\_of\_nodes}=\text{None}, \text{max\_iter}=10000)\)

Compute a list of seeds for the nodes in the graph using VoteRank

VoteRank\(^1\) computes a ranking of the nodes in the graph \(G\) based on a voting scheme. With VoteRank, all nodes vote for each neighbour and the node with the highest score is elected iteratively. The voting ability of neighbors of elected nodes will be decreased in subsequent turn.

**Parameters**

- \(G\) (graph) – A NetworkX graph.
- \(\text{number\_of\_nodes}\) (integer, optional) – Number of ranked nodes to extract (default all nodes).
- \(\text{max\_iter}\) (integer, optional) – Maximum number of iterations to rank nodes.

**Returns**

- \(\text{voterank}\) – Ordered list of computed seeds.

**Return type**

- list

**Raises**

- NetworkXNotImplemented – If \(G\) is digraph.

**References**

3.8 Chains

Functions for finding chains in a graph.

```python
chain_decomposition(G[, root])
```

Returns the chain decomposition of a graph.

3.8.1 networkx.algorithms.chains.chain_decomposition

**chain_decomposition** \((G, \text{root}=\text{None})\)

Returns the chain decomposition of a graph.

The chain decomposition of a graph with respect a depth-first search tree is a set of cycles or paths derived from the set of fundamental cycles of the tree in the following manner. Consider each fundamental cycle with respect to the given tree, represented as a list of edges beginning with the non-tree edge oriented away from the root of the tree. For each fundamental cycle, if it overlaps with any previous fundamental cycle, just take the initial non-overlapping segment, which is a path instead of a cycle. Each cycle or path is called a chain. For more information, see\(^1\).

**Parameters**

- \(G\) (undirected graph)
- \(\text{root}\) (node (optional)) – A node in the graph \(G\). If specified, only the chain decomposition for the connected component containing this node will be returned. This node indicates the


Yields chain (list) – A list of edges representing a chain. There is no guarantee on the orientation of the edges in each chain (for example, if a chain includes the edge joining nodes 1 and 2, the chain may include either (1, 2) or (2, 1)).

Raises NodeNotFound – If root is not in the graph G.

Notes
The worst-case running time of this implementation is linear in the number of nodes and number of edges¹.

References

3.9 Chordal

Algorithms for chordal graphs.

A graph is chordal if every cycle of length at least 4 has a chord (an edge joining two nodes not adjacent in the cycle).

https://en.wikipedia.org/wiki/Chordal_graph

<table>
<thead>
<tr>
<th>is_chordal(G)</th>
<th>Checks whether G is a chordal graph.</th>
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<tr>
<td>chordal_graph_cliques(G)</td>
<td>Returns the set of maximal cliques of a chordal graph.</td>
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<td>Returns the treewidth of the chordal graph G.</td>
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<tr>
<td>find_induced_nodes(G, s, t[, treewidth_bound])</td>
<td>Returns the set of induced nodes in the path from s to t.</td>
</tr>
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</table>

3.9.1 networkx.algorithms.chordal.is_chordal

is_chordal(G)
Checks whether G is a chordal graph.

A graph is chordal if every cycle of length at least 4 has a chord (an edge joining two nodes not adjacent in the cycle).

Parameters G (graph) – A NetworkX graph.

Returns chordal – True if G is a chordal graph and False otherwise.

Return type bool

Raises NetworkXError – The algorithm does not support DiGraph, MultiGraph and MultiDiGraph. If the input graph is an instance of one of these classes, a NetworkXError is raised.

Examples

```python
>>> import networkx as nx
>>> e=[(1,2), (1,3), (2,3), (2,4), (3,4), (3,5), (3,6), (4,5), (4,6), (5,6)]
>>> G=nx.Graph(e)
>>> nx.is_chordal(G)
True
```
Notes

The routine tries to go through every node following maximum cardinality search. It returns False when it finds that the separator for any node is not a clique. Based on the algorithms in\(^1\).

References

3.9.2 networkx.algorithms.chordal.chordal_graph_cliques

chordal_graph_cliques\((G)\)
Returns the set of maximal cliques of a chordal graph.

The algorithm breaks the graph in connected components and performs a maximum cardinality search in each component to get the cliques.

Parameters  
\(G\) (graph) – A NetworkX graph

Returns cliques

Return type  
A set containing the maximal cliques in \(G\).

Raises  
NetworkXError – The algorithm does not support DiGraph, MultiGraph and MultiDiGraph. If the input graph is an instance of one of these classes, a NetworkXError is raised. The algorithm can only be applied to chordal graphs. If the input graph is found to be non-chordal, a NetworkXError is raised.

Examples

```python
>>> import networkx as nx
>>> e= [(1,2), (1,3), (2,3), (2,4), (3,4), (3,5), (3,6), (4,5), (4,6), (5,6), (7,8)]
>>> G = nx.Graph(e)
>>> G.add_node(9)
>>> setlist = nx.chordal_graph_cliques(G)
```

3.9.3 networkx.algorithms.chordal.chordal_graph_treewidth

chordal_graph_treewidth\((G)\)
Returns the treewidth of the chordal graph \(G\).

Parameters  
\(G\) (graph) – A NetworkX graph

Returns treewidth – The size of the largest clique in the graph minus one.

Return type  
int

Raises  
NetworkXError – The algorithm does not support DiGraph, MultiGraph and MultiDiGraph. If the input graph is an instance of one of these classes, a NetworkXError is raised. The algorithm can only be applied to chordal graphs. If the input graph is found to be non-chordal, a NetworkXError is raised.

---

Examples

```python
>>> import networkx as nx
>>> e = [(1, 2), (1, 3), (2, 3), (2, 4), (3, 4), (3, 5), (3, 6), (4, 5), (4, 6), (5, 6), (7, 8)]
>>> G = nx.Graph(e)
>>> G.add_node(9)
>>> nx.chordal_graph_treewidth(G)
3
```

References

3.9.4 `networkx.algorithms.chordal.find_induced_nodes`

`find_induced_nodes` \((G, s, t, \text{treewidth\_bound}=922372036854775807)\)

Returns the set of induced nodes in the path from \(s\) to \(t\).

Parameters

- \(G\) (graph) – A chordal NetworkX graph
- \(s\) (node) – Source node to look for induced nodes
- \(t\) (node) – Destination node to look for induced nodes
- \(\text{treewidth\_bound}\) (float) – Maximum treewidth acceptable for the graph \(H\). The search for induced nodes will end as soon as the \(\text{treewidth\_bound}\) is exceeded.

Returns **Induced nodes** – The set of induced nodes in the path from \(s\) to \(t\) in \(G\)

Return type Set of nodes

Raises `NetworkXError` – The algorithm does not support DiGraph, MultiGraph and MultiDiGraph. If the input graph is an instance of one of these classes, a `NetworkXError` is raised. The algorithm can only be applied to chordal graphs. If the input graph is found to be non-chordal, a `NetworkXError` is raised.

Examples

```python
>>> import networkx as nx
>>> G = nx.Graph()
>>> G = nx.generators.classic.path_graph(10)
>>> Induced_nodes = nx.find_induced_nodes(G, 1, 9, 2)
>>> sorted(Induced_nodes)
[1, 2, 3, 4, 5, 6, 7, 8, 9]
```

Notes

\(G\) must be a chordal graph and \((s,t)\) an edge that is not in \(G\).

If a \(\text{treewidth\_bound}\) is provided, the search for induced nodes will end as soon as the \(\text{treewidth\_bound}\) is exceeded.

The algorithm is inspired by Algorithm 4 in\(^1\). A formal definition of induced node can also be found on that reference.

---

3.10 Clique

Functions for finding and manipulating cliques.

Finding the largest clique in a graph is NP-complete problem, so most of these algorithms have an exponential running time; for more information, see the Wikipedia article on the clique problem\(^1\).

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>\texttt{enumerate_all_cliques(G)}</td>
<td>Returns all cliques in an undirected graph.</td>
</tr>
<tr>
<td>\texttt{find_cliques(G)}</td>
<td>Returns all maximal cliques in an undirected graph.</td>
</tr>
<tr>
<td>\texttt{make_max_clique_graph(G[, create_using])}</td>
<td>Returns the maximal clique graph of the given graph.</td>
</tr>
<tr>
<td>\texttt{make_clique_bipartite(G[, fpos, . . . ])}</td>
<td>Returns the bipartite clique graph corresponding to (G).</td>
</tr>
<tr>
<td>\texttt{graph_clique_number(G[, cliques])}</td>
<td>Returns the clique number of the graph.</td>
</tr>
<tr>
<td>\texttt{graph_number_of_cliques(G[, cliques])}</td>
<td>Returns the number of maximal cliques in the graph.</td>
</tr>
<tr>
<td>\texttt{node_clique_number(G[, nodes, cliques])}</td>
<td>Returns the size of the largest maximal clique containing each given node.</td>
</tr>
<tr>
<td>\texttt{number_of_cliques(G[, nodes, cliques])}</td>
<td>Returns the number of maximal cliques for each node.</td>
</tr>
<tr>
<td>\texttt{cliques_containing_node(G[, nodes, cliques])}</td>
<td>Returns a list of cliques containing the given node.</td>
</tr>
</tbody>
</table>

### 3.10.1 networkx.algorithms.clique.enumerate_all_cliques

\textbf{enumerate_all_cliques} \((G)\)

Returns all cliques in an undirected graph.

This function returns an iterator over cliques, each of which is a list of nodes. The iteration is ordered by cardinality of the cliques: first all cliques of size one, then all cliques of size two, etc.

**Parameters** \(G\) \((\text{NetworkX graph})\) – An undirected graph.

**Returns** An iterator over cliques, each of which is a list of nodes in \(G\). The cliques are ordered according to size.

**Return type** iterator

**Notes**

To obtain a list of all cliques, use \texttt{list(enumerate_all_cliques(G))}. However, be aware that in the worst-case, the length of this list can be exponential in the number of nodes in the graph (for example, when the graph is the complete graph). This function avoids storing all cliques in memory by only keeping current candidate node lists in memory during its search.

The implementation is adapted from the algorithm by Zhang, et al. (2005)\(^1\) to output all cliques discovered.

This algorithm ignores self-loops and parallel edges, since cliques are not conventionally defined with such edges.

---

\(^1\) clique problem:: \url{https://en.wikipedia.org/wiki/Clique_problem}

References

3.10.2 networkx.algorithms.clique.find_cliques

find_cliques(G)

Returns all maximal cliques in an undirected graph.

For each node v, a maximal clique for v is a largest complete subgraph containing v. The largest maximal clique is sometimes called the maximum clique.

This function returns an iterator over cliques, each of which is a list of nodes. It is an iterative implementation, so should not suffer from recursion depth issues.

Parameters G (NetworkX graph) – An undirected graph.

Returns An iterator over maximal cliques, each of which is a list of nodes in G. The order of cliques is arbitrary.

Return type iterator

See also:

find_cliques_recursive() A recursive version of the same algorithm.

Notes

To obtain a list of all maximal cliques, use list(find_cliques(G)). However, be aware that in the worst-case, the length of this list can be exponential in the number of nodes in the graph. This function avoids storing all cliques in memory by only keeping current candidate node lists in memory during its search.

This implementation is based on the algorithm published by Bron and Kerbosch (1973)¹, as adapted by Tomita, Tanaka and Takahashi (2006)² and discussed in Cazals and Karande (2008)³. It essentially unrolls the recursion used in the references to avoid issues of recursion stack depth (for a recursive implementation, see find_cliques_recursive()).

This algorithm ignores self-loops and parallel edges, since cliques are not conventionally defined with such edges.

References

3.10.3 networkx.algorithms.clique.make_max_clique_graph

make_max_clique_graph(G, create_using=None)

Returns the maximal clique graph of the given graph.

The nodes of the maximal clique graph of G are the cliques of G and an edge joins two cliques if the cliques are not disjoint.

Parameters

• G (NetworkX graph)

• **create_using** (*NetworkX graph constructor, optional (default=nx.Graph)*) – Graph type to create. If graph instance, then cleared before populated.

**Returns** A graph whose nodes are the cliques of G and whose edges join two cliques if they are not disjoint.

**Return type** *NetworkX graph*

**Notes**

This function behaves like the following code:

```python
import networkx as nx
G = nx.make_clique_bipartite(G)
cliques = [v for v in G.nodes() if G.nodes[v]['bipartite'] == 0]
G = nx.bipartite.project(G, cliques)
G = nx.relabel_nodes(G, {-v: v - 1 for v in G})
```

It should be faster, though, since it skips all the intermediate steps.

### 3.10.4 networkx.algorithms.clique.make_clique_bipartite

**make_clique_bipartite** (*G, fpos=None, create_using=None, name=None*)

Returns the bipartite clique graph corresponding to G.

In the returned bipartite graph, the “bottom” nodes are the nodes of G and the “top” nodes represent the maximal cliques of G. There is an edge from node v to clique C in the returned graph if and only if v is an element of C.

**Parameters**

- **G** (*NetworkX graph*) – An undirected graph.
- **fpos** (*bool*) – If True or not None, the returned graph will have an additional attribute, pos, a dictionary mapping node to position in the Euclidean plane.
- **create_using** (*NetworkX graph constructor, optional (default=nx.Graph]*) – Graph type to create. If graph instance, then cleared before populated.

**Returns**

A bipartite graph whose “bottom” set is the nodes of the graph G, whose “top” set is the cliques of G, and whose edges join nodes of G to the cliques that contain them.

The nodes of the graph G have the node attribute ‘bipartite’ set to 1 and the nodes representing cliques have the node attribute ‘bipartite’ set to 0, as is the convention for bipartite graphs in NetworkX.

**Return type** *NetworkX graph*

### 3.10.5 networkx.algorithms.clique.graph_clique_number

**graph_clique_number** (*G, cliques=None*)

Returns the clique number of the graph.

The clique number of a graph is the size of the largest clique in the graph.

**Parameters**

- **G** (*NetworkX graph*) – An undirected graph.
• **cliques (list)** – A list of cliques, each of which is itself a list of nodes. If not specified, the list of all cliques will be computed, as by `find_cliques()`.

**Returns** The size of the largest clique in \( G \).

**Return type** int

**Notes**

You should provide `cliques` if you have already computed the list of maximal cliques, in order to avoid an exponential time search for maximal cliques.

### 3.10.6 networkx.algorithms.clique.graph_number_of_cliques

**graph_number_of_cliques (G, cliques=None)**

Returns the number of maximal cliques in the graph.

**Parameters**

- **G (NetworkX graph)** – An undirected graph.
- **cliques (list)** – A list of cliques, each of which is itself a list of nodes. If not specified, the list of all cliques will be computed, as by `find_cliques()`.

**Returns** The number of maximal cliques in \( G \).

**Return type** int

**Notes**

You should provide `cliques` if you have already computed the list of maximal cliques, in order to avoid an exponential time search for maximal cliques.

### 3.10.7 networkx.algorithms.clique.node_clique_number

**node_clique_number (G, nodes=None, cliques=None)**

Returns the size of the largest maximal clique containing each given node.

Returns a single or list depending on input nodes. Optional list of cliques can be input if already computed.

### 3.10.8 networkx.algorithms.clique.number_of_cliques

**number_of_cliques (G, nodes=None, cliques=None)**

Returns the number of maximal cliques for each node.

Returns a single or list depending on input nodes. Optional list of cliques can be input if already computed.

### 3.10.9 networkx.algorithms.clique.cliques_containing_node

**cliques_containing_node (G, nodes=None, cliques=None)**

Returns a list of cliques containing the given node.

Returns a single list or list of lists depending on input nodes. Optional list of cliques can be input if already computed.
3.11 Clustering

Algorithms to characterize the number of triangles in a graph.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
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<td>triangles(G[, nodes])</td>
<td>Compute the number of triangles.</td>
</tr>
<tr>
<td>transitivity(G)</td>
<td>Compute graph transitivity, the fraction of all possible triangles present in G.</td>
</tr>
<tr>
<td>clustering(G[, nodes, weight])</td>
<td>Compute the clustering coefficient for nodes.</td>
</tr>
<tr>
<td>average_clustering(G[, nodes, weight, ...])</td>
<td>Compute the average clustering coefficient for the graph G.</td>
</tr>
<tr>
<td>square_clustering(G[, nodes])</td>
<td>Compute the squares clustering coefficient for nodes.</td>
</tr>
<tr>
<td>generalized_degree(G[, nodes])</td>
<td>Compute the generalized degree for nodes.</td>
</tr>
</tbody>
</table>

3.11.1 networkx.algorithms.cluster.triangles

triangles(G, nodes=None)

Compute the number of triangles.

Finds the number of triangles that include a node as one vertex.

Parameters:
- G (graph) – A networkx graph
- nodes (container of nodes, optional (default= all nodes in G)) – Compute triangles for nodes in this container.

Returns:
out – Number of triangles keyed by node label.

Return type: dictionary

Examples

```python
>>> G=nx.complete_graph(5)
>>> print(nx.triangles(G,0))
6
>>> print(nx.triangles(G))
{0: 6, 1: 6, 2: 6, 3: 6, 4: 6}
>>> print(list(nx.triangles(G,(0,1)).values()))
[6, 6]
```

Notes

When computing triangles for the entire graph each triangle is counted three times, once at each node. Self loops are ignored.

3.11.2 networkx.algorithms.cluster.transitivity

transitivity(G)

Compute graph transitivity, the fraction of all possible triangles present in G.

Possible triangles are identified by the number of “triads” (two edges with a shared vertex).
The transitivity is
\[ T = \frac{\text{\#triangles}}{\text{\#triads}}. \]

**Parameters** $G$ (graph)

**Returns** out – Transitivity

**Return type** float

**Examples**

```python
>>> G = nx.complete_graph(5)
>>> print(nx.transitivity(G))
1.0
```

### 3.11.3 networkx.algorithms.cluster.clustering

**clustering** ($G$, nodes=None, weight=None)

Compute the clustering coefficient for nodes.

For unweighted graphs, the clustering of a node $u$ is the fraction of possible triangles through that node that exist,

\[ c_u = \frac{2T(u)}{\text{deg}(u)(\text{deg}(u) - 1)} , \]

where $T(u)$ is the number of triangles through node $u$ and $\text{deg}(u)$ is the degree of $u$.

For weighted graphs, there are several ways to define clustering\(^1\). The one used here is defined as the geometric average of the subgraph edge weights\(^2\),

\[ c_u = \frac{1}{\text{deg}(u)(\text{deg}(u) - 1)} \sum_{uv} (\hat{w}_{uv}\hat{w}_{uw}\hat{w}_{vw})^{1/3} , \]

The edge weights $\hat{w}_{uv}$ are normalized by the maximum weight in the network $\hat{w}_{uv} = w_{uv} / \text{max}(w)$.

The value of $c_u$ is assigned to 0 if $\text{deg}(u) < 2$.

For directed graphs, the clustering is similarly defined as the fraction of all possible directed triangles or geometric average of the subgraph edge weights for unweighted and weighted directed graph respectively\(^3\).

\[ c_u = \frac{1}{\text{deg}^{\text{tot}}(u)(\text{deg}^{\text{tot}}(u) - 1) - 2\text{deg}^{\text{rs}}(u)T(u)} , \]

where $T(u)$ is the number of directed triangles through node $u$, $\text{deg}^{\text{tot}}(u)$ is the sum of in degree and out degree of $u$ and $\text{deg}^{\text{rs}}(u)$ is the reciprocal degree of $u$.

**Parameters**

- $G$ (graph)

---


• **nodes** *(container of nodes, optional (default=all nodes in G))* – Compute clustering for nodes in this container.

• **weight** *(string or None, optional (default=None))* – The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.

**Returns** `out` – Clustering coefficient at specified nodes

**Return type** `float`, or dictionary

### Examples

```python
g=nx.complete_graph(5)
print(nx.clustering(G,0))
1.0
print(nx.clustering(G))
{0: 1.0, 1: 1.0, 2: 1.0, 3: 1.0, 4: 1.0}
```

**Notes**

Self loops are ignored.

**References**

3.11.4 *networkx.algorithms.cluster.average_clustering*

`average_clustering(G, nodes=None, weight=None, count_zeros=True)`

Compute the average clustering coefficient for the graph `G`.

The clustering coefficient for the graph is the average,

\[
C = \frac{1}{n} \sum_{v \in G} c_v,
\]

where `n` is the number of nodes in `G`.

**Parameters**

• **G** *(graph)*

• **nodes** *(container of nodes, optional (default=all nodes in G))* – Compute average clustering for nodes in this container.

• **weight** *(string or None, optional (default=None))* – The edge attribute that holds the numerical value used as a weight. If None, then each edge has weight 1.

• **count_zeros** *(bool)* – If False include only the nodes with nonzero clustering in the average.

**Returns** `avg` – Average clustering

**Return type** `float`

### Examples

```python
g=nx.complete_graph(5)
print(nx.average_clustering(G))
1.0
```
Notes

This is a space saving routine; it might be faster to use the clustering function to get a list and then take the average.
Self loops are ignored.

References

3.11.5 networkx.algorithms.cluster.square_clustering

square_clustering \((G, \text{nodes}=\text{None})\)

Compute the squares clustering coefficient for nodes.

For each node return the fraction of possible squares that exist at the node\(^1\)

\[
C_4(v) = \frac{\sum_{u=1}^{k_v} \sum_{w=u+1}^{k_v} q_v(u, w)}{\sum_{u=1}^{k_v} \sum_{w=u+1}^{k_v} [a_v(u, w) + q_v(u, w)]},
\]

where \(q_v(u, w)\) are the number of common neighbors of \(u\) and \(w\) other than \(v\) (ie squares), and \(a_v(u, w) = (k_u - (1 + q_v(u, w) + \theta_{uw}))(k_w - (1 + q_v(u, w) + \theta_{uw}))\), where \(\theta_{uw} = 1\) if \(u\) and \(w\) are connected and 0 otherwise.

Parameters

- \(G\) (graph)
- \text{nodes} (container of nodes, optional (default=all nodes in \(G\)) – Compute clustering for nodes in this container.

Returns \(c4\) – A dictionary keyed by node with the square clustering coefficient value.

Return type dictionary

Examples

```python
>>> G=nx.complete_graph(5)
>>> print(nx.square_clustering(G,0))
1.0
>>> print(nx.square_clustering(G))
{0: 1.0, 1: 1.0, 2: 1.0, 3: 1.0, 4: 1.0}
```

Notes

While \(C_3(v)\) (triangle clustering) gives the probability that two neighbors of node \(v\) are connected with each other, \(C_4(v)\) is the probability that two neighbors of node \(v\) share a common neighbor different from \(v\). This algorithm can be applied to both bipartite and unipartite networks.

generalized_degree \( G, \text{nodes} = \text{None} \)

Compute the generalized degree for nodes.

For each node, the generalized degree shows how many edges of given triangle multiplicity the node is connected to. The triangle multiplicity of an edge is the number of triangles an edge participates in. The generalized degree of node \( i \) can be written as a vector \( k_i = (k_i^{(0)}, \ldots, k_i^{(N-2)}) \) where \( k_i^{(j)} \) is the number of edges attached to node \( i \) that participate in \( j \) triangles.

Parameters

- \( G \) (graph)
- \( \text{nodes} \) (container of nodes, optional (default=all nodes in \( G \)) – Compute the generalized degree for nodes in this container.

Returns \( \text{out} \) – Generalized degree of specified nodes. The Counter is keyed by edge triangle multiplicity.

Return type Counter, or dictionary of Counters

Examples

```python
g = nx.complete_graph(5)
print(nx.generalized_degree(g, 0))
Counter({3: 4})
print(nx.generalized_degree(g))
{0: Counter({3: 4}), 1: Counter({3: 4}), 2: Counter({3: 4}), 3: Counter({3: 4}), ...
4: Counter({3: 4})}
```

To recover the number of triangles attached to a node:

```python
k1 = nx.generalized_degree(g, 0)
sum([k*v for k,v in k1.items()])/2 == nx.triangles(g, 0)
True
```

Notes

In a network of \( N \) nodes, the highest triangle multiplicity an edge can have is \( N-2 \).

The return value does not include a zero entry if no edges of a particular triangle multiplicity are present.

The number of triangles node \( i \) is attached to can be recovered from the generalized degree \( k_i = (k_i^{(0)}, \ldots, k_i^{(N-2)}) \) by \( (k_i^{(1)} + 2k_i^{(2)} + \ldots + (N-2)k_i^{(N-2)})/2 \).

References

3.12 Coloring
NetworkX Reference, Release 2.5rc1.dev20191214022633

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>greedy_color(G[, strategy, interchange])</td>
<td>Color a graph using various strategies of greedy graph coloring.</td>
</tr>
<tr>
<td>equitable_color(G, num_colors)</td>
<td>Provides equitable (r + 1)-coloring for nodes of G in O(r * n^2) time if deg(G) &lt;= r.</td>
</tr>
</tbody>
</table>

3.12.1 networkx.algorithms.coloring.greedy_color

`greedy_color (G, strategy=’largest_first’, interchange=False)`

Color a graph using various strategies of greedy graph coloring.

Attempts to color a graph using as few colors as possible, where no neighbours of a node can have same color as the node itself. The given strategy determines the order in which nodes are colored.

The strategies are described in\(^1\), and smallest-last is based on\(^2\).

**Parameters**

- `G` (*NetworkX graph*)
- `strategy` (*string or function(G, colors]*) – A function (or a string representing a function) that provides the coloring strategy, by returning nodes in the ordering they should be colored. `G` is the graph, and `colors` is a dictionary of the currently assigned colors, keyed by nodes. The function must return an iterable over all the nodes in `G`.

If the strategy function is an iterator generator (that is, a function with `yield` statements), keep in mind that the `colors` dictionary will be updated after each `yield`, since this function chooses colors greedily.

If `strategy` is a string, it must be one of the following, each of which represents one of the built-in strategy functions.

- `'largest_first'`
- `'random_sequential'`
- `'smallest_last'`
- `'independent_set'`
- `'connected_sequential_bfs'`
- `'connected_sequential_dfs'`
- `'connected_sequential'` (alias for the previous strategy)
- `'saturation_largest_first'`
- `'DSATUR'` (alias for the previous strategy)

- `interchange` (*bool*) – Will use the color interchange algorithm described by\(^3\) if set to `True`.

Note that `saturation_largest_first` and `independent_set` do not work with interchange. Furthermore, if you use interchange with your own strategy function, you cannot rely on the values in the `colors` argument.

**Returns**

- A dictionary with keys representing nodes and values representing

---


corresponding coloring.

Examples

```python
>>> G = nx.cycle_graph(4)
>>> d = nx.coloring.greedy_color(G, strategy='largest_first')
>>> d
{0: 0, 1: 1, 2: 0, 3: 1}, {0: 1, 1: 0, 2: 1, 3: 0}
True
```

Raises `NetworkXPointlessConcept` – If `strategy` is `saturation_largest_first` or `independent_set` and `interchange` is `True`.

References

3.12.2 `networkx.algorithms.coloring.equitable_color`

equitable_color(G, num_colors)

Provides equitable (r + 1)-coloring for nodes of G in O(r * n^2) time if deg(G) <= r. The algorithm is described in Kierstead, H. A., Kostochka, A. V., Mydlarz, M., & Szemerédi, E. (2010). A fast algorithm for equitable coloring. Combinatorica, 30(2), 217-224.

Attempts to color a graph using r colors, where no neighbors of a node can have same color as the node itself and the number of nodes with each color differ by at most 1.

Parameters

- **G (networkX graph)** – The nodes of this graph will be colored.
- **num_colors (number of colors to use)** – This number must be at least one more than the maximum degree of nodes in the graph.

Returns

- A dictionary with keys representing nodes and values representing corresponding coloring.

Examples

```python
>>> G = nx.cycle_graph(4)
>>> d = nx.coloring.equitable_color(G, num_colors=3)
>>> nx.algorithms.coloring.equitable_coloring.is_equitable(G, d)
True
```

Raises `NetworkXAlgorithmError` – If the maximum degree of the graph G is greater than `num_colors`.

References

Some node ordering strategies are provided for use with `greedy_color()`.

---

NetworkX Reference, Release 2.5rc1.dev20191214022633

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>strategy_connected_sequential(G, colors, ...)</code></td>
<td>Returns an iterable over nodes in G in the order given by a breadth-first or depth-first traversal.</td>
</tr>
<tr>
<td><code>strategy_connected_sequential_dfs(G, colors)</code></td>
<td>Returns an iterable over nodes in G in the order given by a depth-first traversal.</td>
</tr>
<tr>
<td><code>strategy_connected_sequential_bfs(G, colors)</code></td>
<td>Returns an iterable over nodes in G in the order given by a breadth-first traversal.</td>
</tr>
<tr>
<td><code>strategy_independent_set(G, colors)</code></td>
<td>Uses a greedy independent set removal strategy to determine the colors.</td>
</tr>
<tr>
<td><code>strategy_largest_first(G, colors)</code></td>
<td>Returns a list of the nodes of G in decreasing order by degree.</td>
</tr>
<tr>
<td><code>strategy_random_sequential(G, colors[, seed])</code></td>
<td>Returns a random permutation of the nodes of G as a list.</td>
</tr>
<tr>
<td><code>strategy_saturation_largest_first(G, colors)</code></td>
<td>Iterates over all the nodes of G in &quot;saturation order&quot; (also known as “DSATUR”).</td>
</tr>
<tr>
<td><code>strategy_smallest_last(G, colors)</code></td>
<td>Returns a deque of the nodes of G, “smallest” last.</td>
</tr>
</tbody>
</table>

### 3.12.3 networkx.algorithms.coloring.strategy_connected_sequential

**strategy_connected_sequential** *(G, colors, traversal='bfs')*

Returns an iterable over nodes in G in the order given by a breadth-first or depth-first traversal.

- **traversal** must be one of the strings 'dfs' or 'bfs', representing depth-first traversal or breadth-first traversal, respectively.
- The generated sequence has the property that for each node except the first, at least one neighbor appeared earlier in the sequence.
- G is a NetworkX graph. colors is ignored.

### 3.12.4 networkx.algorithms.coloring.strategy_connected_sequential_dfs

**strategy_connected_sequential_dfs** *(G, colors)*

Returns an iterable over nodes in G in the order given by a depth-first traversal.

- The generated sequence has the property that for each node except the first, at least one neighbor appeared earlier in the sequence.
- G is a NetworkX graph. colors is ignored.

### 3.12.5 networkx.algorithms.coloring.strategy_connected_sequential_bfs

**strategy_connected_sequential_bfs** *(G, colors)*

Returns an iterable over nodes in G in the order given by a breadth-first traversal.

- The generated sequence has the property that for each node except the first, at least one neighbor appeared earlier in the sequence.
- G is a NetworkX graph. colors is ignored.

### 3.12.6 networkx.algorithms.coloring.strategy_independent_set

**strategy_independent_set** *(G, colors)*

Uses a greedy independent set removal strategy to determine the colors.
This function updates colors in-place and return None, unlike the other strategy functions in this module.

This algorithm repeatedly finds and removes a maximal independent set, assigning each node in the set an unused color.

G is a NetworkX graph.

This strategy is related to strategy_smallest_last(): in that strategy, an independent set of size one is chosen at each step instead of a maximal independent set.

3.12.7 networkx.algorithms.coloring.strategy_largest_first

strategy_largest_first (G, colors)
Returns a list of the nodes of G in decreasing order by degree.
G is a NetworkX graph. colors is ignored.

3.12.8 networkx.algorithms.coloring.strategy_random_sequential

strategy_random_sequential (G, colors, seed=None)
Returns a random permutation of the nodes of G as a list.
G is a NetworkX graph. colors is ignored.
seed [integer, random_state, or None (default)] Indicator of random number generation state. See Randomness.

3.12.9 networkx.algorithms.coloring.strategy_saturation_largest_first

strategy_saturation_largest_first (G, colors)
Iterates over all the nodes of G in “saturation order” (also known as “DSATUR”).
G is a NetworkX graph. colors is a dictionary mapping nodes of G to colors, for those nodes that have already been colored.

3.12.10 networkx.algorithms.coloring.strategy_smallest_last

strategy_smallest_last (G, colors)
Returns a deque of the nodes of G, “smallest” last.

Specifically, the degrees of each node are tracked in a bucket queue. From this, the node of minimum degree is repeatedly popped from the graph, updating its neighbors’ degrees.

G is a NetworkX graph. colors is ignored.

This implementation of the strategy runs in $O(n + m)$ time (ignoring polylogarithmic factors), where $n$ is the number of nodes and $m$ is the number of edges.

This strategy is related to strategy_independent_set(): if we interpret each node removed as an independent set of size one, then this strategy chooses an independent set of size one instead of a maximal independent set.
3.13 Communicability

Communicability.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>communicability(G)</code></td>
<td>Returns communicability between all pairs of nodes in G.</td>
</tr>
<tr>
<td><code>communicability_exp(G)</code></td>
<td>Returns communicability between all pairs of nodes in G.</td>
</tr>
</tbody>
</table>

3.13.1 networkx.algorithms.communicability_alg.communicability

`communicability(G)`

Returns communicability between all pairs of nodes in G.

The communicability between pairs of nodes in G is the sum of closed walks of different lengths starting at node u and ending at node v.

Parameters  

- **G** (graph)

Returns  

-omm – Dictionary of dictionaries keyed by nodes with communicability as the value.

Return type  

dictionary of dictionaries

Raises  

- NetworkXError – If the graph is not undirected and simple.

See also:

- `communicability_exp()`  
  Communicability between all pairs of nodes in G using spectral decomposition.
- `communicability_betweenness_centrality()`  
  Communicability betweenness centrality for each node in G.

Notes

This algorithm uses a spectral decomposition of the adjacency matrix. Let G=(V,E) be a simple undirected graph. Using the connection between the powers of the adjacency matrix and the number of walks in the graph, the communicability between nodes u and v based on the graph spectrum is:

\[
C(u, v) = \sum_{j=1}^{n} \phi_j(u)\phi_j(v)e^{\lambda_j},
\]

where \( \phi_j(u) \) is the urm\( j \) element of the jrm\( j \) orthonormal eigenvector of the adjacency matrix associated with the eigenvalue \( \lambda_j \).

References

Examples

```python
>>> G = nx.Graph([(0,1),(1,2),(1,5),(5,4),(2,4),(2,3),(4,3),(3,6)])
>>> c = nx.communicability(G)
```
3.13.2 networkx.algorithms.communicability_alg.communicability_exp

communicability_exp(G)
Returns communicability between all pairs of nodes in G.

Communicability between pair of node (u,v) of node in G is the sum of closed walks of different lengths starting at node u and ending at node v.

Parameters: G (graph)

Returns: comm – Dictionary of dictionaries keyed by nodes with communicability as the value.

Return type: dictionary of dictionaries

Raises: NetworkXError – If the graph is not undirected and simple.

See also:

communicability()  Communicability between pairs of nodes in G.

communicability_betweenness_centrality()  Communicability betweeness centrality for each node in G.

Notes

This algorithm uses matrix exponentiation of the adjacency matrix.

Let G=(V,E) be a simple undirected graph. Using the connection between the powers of the adjacency matrix and the number of walks in the graph, the communicability between nodes u and v is\(^1\),

\[
C(u, v) = (e^A)_{uv},
\]

where A is the adjacency matrix of G.

References

Examples

```python
>>> G = nx.Graph([(0,1),(1,2),(1,5),(5,4),(2,4),(2,3),(4,3),(3,6)])
>>> c = nx.communicability_exp(G)
```

3.14 Communities

Functions for computing and measuring community structure.

The functions in this class are not imported into the top-level networkx namespace. You can access these functions by importing the networkx.algorithms.community module, then accessing the functions as attributes of community. For example:

```python
>>> import networkx as nx
>>> from networkx.algorithms import community
>>> G = nx.barbell_graph(5, 1)
>>> communities_generator = community.girvan_newman(G)
```

3.14.1 Bipartitions

Functions for computing the Kernighan–Lin bipartition algorithm.

`kernighan_lin_bisection(G[, partition, ...])` Partition a graph into two blocks using the Kernighan–Lin algorithm.

Parameters

- **G** (graph)
- **partition** (tuple) – Pair of iterables containing an initial partition. If not specified, a random balanced partition is used.
- **max_iter** (int) – Maximum number of times to attempt swaps to find an improvement before giving up.
- **weight** (key) – Edge data key to use as weight. If None, the weights are all set to one.
- **seed** (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness. Only used if partition is None

Returns **partition** – A pair of sets of nodes representing the bipartition.

Return type tuple

Raises `NetworkXError` – If partition is not a valid partition of the nodes of the graph.

3.14.2 K-Clique

`k_clique_communities(G[, k, cliques])` Find k-clique communities in graph using the percolation method.

Parameters

- **G** (graph)
- **k** (int) – Size of the cliques to find. If None, the function will find cliques of all sizes.
- **cliques** (iterable) – If specified, only find communities with these cliques.

Returns **communities** – A list of communities, where each community is a list of nodes.

Return type list

Raises `NetworkXError` – If the graph is not connected.

References
nodes) k-cliques.

**Parameters**

- `G (NetworkX graph)`
- `k (int)` – Size of smallest clique
- `cliques (list or generator)` – Precomputed cliques (use networkx.find_cliques(G))

**Returns**

**Return type** Yields sets of nodes, one for each k-clique community.

**Examples**

```python
>>> from networkx.algorithms.community import k_clique_communities
>>> G = nx.complete_graph(5)
>>> K5 = nx.convert_node_labels_to_integers(G, first_label=2)
>>> G.add_edges_from(K5.edges())
>>> c = list(k_clique_communities(G, 4))
>>> sorted(list(c[0]))
[0, 1, 2, 3, 4, 5, 6]
>>> list(k_clique_communities(G, 6))
[]
```

**References**

3.14.3 Modularity-based communities

Functions for detecting communities based on modularity.

`networkx.algorithms.community.modularity_max.greedy_modularity_communities`

**greedy_modularity_communities** *(G[, weight])*  
Find communities in graph using Clauset-Newman-Moore greedy modularity maximization. This method currently supports the Graph class and does not consider edge weights.

Greedy modularity maximization begins with each node in its own community and joins the pair of communities that most increases modularity until no such pair exists.

**Parameters**

`G (NetworkX graph)`

**Returns**

**Return type** Yields sets of nodes, one for each community.

**Examples**

```python
>>> from networkx.algorithms.community import greedy_modularity_communities
>>> G = nx.karate_club_graph()
```
>>> c = list(greedy_modularity_communities(G))
>>> sorted(c[0])
[8, 14, 15, 18, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33]

References

3.14.4 Tree partitioning

Lukes Algorithm for exact optimal weighted tree partitioning.

\[ \text{lukes\_partitioning}(G, \text{max\_size}[....]) \]

Optimal partitioning of a weighted tree using the Lukes algorithm.

networkx.algorithms.community.lukes.lukes_partitioning

\[ \text{lukes\_partitioning}(G, \text{max\_size}: \text{int}, \text{node\_weight}:=\text{None}, \text{edge\_weight}:=\text{None}) \rightarrow \text{list} \]

Optimal partitioning of a weighted tree using the Lukes algorithm.

This algorithm partitions a connected, acyclic graph featuring integer node weights and float edge weights. The resulting clusters are such that the total weight of the nodes in each cluster does not exceed max_size and that the weight of the edges that are cut by the partition is minimum. The algorithm is based on LUKE[1].

Parameters

- \( G \) (graph)
- \text{max\_size} (int) – Maximum weight a partition can have in terms of sum of node\_weight for all nodes in the partition
- \text{edge\_weight} (key) – Edge data key to use as weight. If None, the weights are all set to one.
- \text{node\_weight} (key) – Node data key to use as weight. If None, the weights are all set to one.

The data must be int.

Returns\ partition – A list of sets of nodes representing the clusters of the partition.

Return type\ list

Raises

- \text{NotATree} – If G is not a tree.
- \text{TypeError} – If any of the values of node\_weight is not int.

References

3.14.5 Label propagation

Label propagation community detection algorithms.

\[ \text{asyn\_lpa\_communities}(G[, \text{weight}, \text{seed}]) \]

Returns communities in \( G \) as detected by asynchronous label propagation.

\[ \text{label\_propagation\_communities}(G) \]

Generates community sets determined by label propagation.
asyn_lpa_communities(G, weight=None, seed=None)

Returns communities in $G$ as detected by asynchronous label propagation.

The asynchronous label propagation algorithm is described in\textsuperscript{1}. The algorithm is probabilistic and the found communities may vary on different executions.

The algorithm proceeds as follows. After initializing each node with a unique label, the algorithm repeatedly sets the label of a node to be the label that appears most frequently among that node’s neighbors. The algorithm halts when each node has the label that appears most frequently among its neighbors. The algorithm is asynchronous because each node is updated without waiting for updates on the remaining nodes.

This generalized version of the algorithm in\textsuperscript{1} accepts edge weights.

Parameters

- G (Graph)
- weight (string) – The edge attribute representing the weight of an edge. If None, each edge is assumed to have weight one. In this algorithm, the weight of an edge is used in determining the frequency with which a label appears among the neighbors of a node: a higher weight means the label appears more often.
- seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns communities – Iterable of communities given as sets of nodes.

Return type iterable

Notes

Edge weight attributes must be numerical.

References
asyn_fluidc(G, k[, max_iter, seed]) Returns communities in $G$ as detected by Fluid Communities algorithm.

networkx.algorithms.community.asyn_fluid.asyn_fluidc

asyn_fluidc (G, k, max_iter=100, seed=None) Returns communities in $G$ as detected by Fluid Communities algorithm.

The asynchronous fluid communities algorithm is described in^1. The algorithm is based on the simple idea of fluids interacting in an environment, expanding and pushing each other. It’s initialization is random, so found communities may vary on different executions.

The algorithm proceeds as follows. First each of the initial $k$ communities is initialized in a random vertex in the graph. Then the algorithm iterates over all vertices in a random order, updating the community of each vertex based on its own community and the communities of its neighbours. This process is performed several times until convergence. At all times, each community has a total density of 1, which is equally distributed among the vertices it contains. If a vertex changes of community, vertex densities of affected communities are adjusted immediately. When a complete iteration over all vertices is done, such that no vertex changes the community it belongs to, the algorithm has converged and returns.

This is the original version of the algorithm described in^1. Unfortunately, it does not support weighted graphs yet.

Parameters

- **G** (Graph)
- **k** (integer) – The number of communities to be found.
- **max_iter** (integer) – The number of maximum iterations allowed. By default 15.
- **seed** (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns **communities** – Iterable of communities given as sets of nodes.

Return type iterable

Notes

$k$ variable is not an optional argument.

References

3.14.7 Measuring partitions

Functions for measuring the quality of a partition (into communities).

<table>
<thead>
<tr>
<th>function</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>coverage(G, partition)</td>
<td>Returns the coverage of a partition.</td>
</tr>
<tr>
<td>performance(G, partition)</td>
<td>Returns the performance of a partition.</td>
</tr>
</tbody>
</table>

---

networkx.algorithms.community.quality.coverage

coverage \( (G, \text{partition}) \)

Returns the coverage of a partition.

The coverage of a partition is the ratio of the number of intra-community edges to the total number of edges in the graph.

Parameters

- \( G \) (NetworkX graph)
- \( \text{partition} \) (sequence) – Partition of the nodes of \( G \), represented as a sequence of sets of nodes. Each block of the partition represents a community.

Returns  The coverage of the partition, as defined above.

Return type  float

Raises  NetworkXError – If \( \text{partition} \) is not a valid partition of the nodes of \( G \).

Notes

If \( G \) is a multigraph, the multiplicity of edges is counted.

References

networkx.algorithms.community.quality.performance

performance \( (G, \text{partition}) \)

Returns the performance of a partition.

The performance of a partition is the ratio of the number of intra-community edges plus inter-community non-edges with the total number of potential edges.

Parameters

- \( G \) (NetworkX graph) – A simple graph (directed or undirected).
- \( \text{partition} \) (sequence) – Partition of the nodes of \( G \), represented as a sequence of sets of nodes. Each block of the partition represents a community.

Returns  The performance of the partition, as defined above.

Return type  float

Raises  NetworkXError – If \( \text{partition} \) is not a valid partition of the nodes of \( G \).

References

3.14.8 Partitions via centrality measures

Functions for computing communities based on centrality notions.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>girvan_newman</td>
<td>Finds communities in a graph using the Girvan–Newman method.</td>
</tr>
</tbody>
</table>

3.14. Communities
networkx.algorithms.community.centrality.girvan_newman

**girvan_newman** *(G, most_valuable_edge=None)*

Finds communities in a graph using the Girvan–Newman method.

**Parameters**

- **G** *(NetworkX graph)*
- **most_valuable_edge** *(function)* — Function that takes a graph as input and outputs an edge. The edge returned by this function will be recomputed and removed at each iteration of the algorithm.

If not specified, the edge with the highest *networkx.edge_betweenness_centrality()* will be used.

**Returns** Iterator over tuples of sets of nodes in *G*. Each set of node is a community, each tuple is a sequence of communities at a particular level of the algorithm.

**Return type** iterator

**Examples**

To get the first pair of communities:

```python
>>> G = nx.path_graph(10)
>>> comp = girvan_newman(G)
>>> tuple(sorted(c) for c in next(comp))
([0, 1, 2, 3, 4], [5, 6, 7, 8, 9])
```

To get only the first *k* tuples of communities, use *itertools.islice()*:

```python
>>> import itertools
>>> G = nx.path_graph(8)
>>> k = 2
>>> comp = girvan_newman(G)
>>> for communities in itertools.islice(comp, k):
...     print(tuple(sorted(c) for c in communities))
... ([0, 1, 2, 3], [4, 5, 6, 7])
([0, 1], [2, 3], [4, 5, 6, 7])
```

To stop getting tuples of communities once the number of communities is greater than *k*, use *itertools.takewhile()*:

```python
>>> import itertools
>>> G = nx.path_graph(8)
>>> k = 4
>>> comp = girvan_newman(G)
>>> limited = itertools.takewhile(lambda c: len(c) <= k, comp)
>>> for communities in limited:
...     print(tuple(sorted(c) for c in communities))
... ([0, 1, 2, 3], [4, 5, 6, 7])
([0, 1], [2, 3], [4, 5, 6, 7])
([0, 1], [2, 3], [4, 5], [6, 7])
```

To just choose an edge to remove based on the weight:
>>> from operator import itemgetter
>>> G = nx.path_graph(10)
>>> edges = G.edges()
>>> nx.set_edge_attributes(G, {(u, v): v for u, v in edges}, 'weight')
>>> def heaviest(G):
...     u, v, w = max(G.edges(data='weight'), key=itemgetter(2))
...     return (u, v)
... 
>>> comp = girvan_newman(G, most_valuable_edge=heaviest)
>>> tuple(sorted(c) for c in next(comp))
([0, 1, 2, 3, 4, 5, 6, 7, 8], [9])

To utilize edge weights when choosing an edge with, for example, the highest betweenness centrality:

>>> from networkx import edge_betweenness_centrality as betweenness
>>> def most_central_edge(G):
...     centrality = betweenness(G, weight='weight')
...     return max(centrality, key=centrality.get)
... 
>>> G = nx.path_graph(10)
>>> comp = girvan_newman(G, most_valuable_edge=most_central_edge)
>>> tuple(sorted(c) for c in next(comp))
([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])

To specify a different ranking algorithm for edges, use the most_valuable_edge keyword argument:

>>> from networkx import edge_betweenness_centrality
>>> from random import random
>>> def most_central_edge(G):
...     centrality = edge_betweenness_centrality(G)
...     max_cent = max(centrality.values())
...     # Scale the centrality values so they are between 0 and 1,
...     # and add some random noise.
...     centrality = {e: c / max_cent for e, c in centrality.items()}
...     # Add some random noise.
...     centrality = {e: c + random() for e, c in centrality.items()}
...     return max(centrality, key=centrality.get)
... 
>>> G = nx.path_graph(10)
>>> comp = girvan_newman(G, most_valuable_edge=most_central_edge)

Notes

The Girvan–Newman algorithm detects communities by progressively removing edges from the original graph. The algorithm removes the “most valuable” edge, traditionally the edge with the highest betweenness centrality, at each step. As the graph breaks down into pieces, the tightly knit community structure is exposed and the result can be depicted as a dendrogram.

3.14.9 Validating partitions

Helper functions for community-finding algorithms.
is_partition(G, communities) Returns True if communities is a partition of the nodes of G.

networkx.algorithms.community.community_utils.is_partition

is_partition(G, communities)
Returns True if communities is a partition of the nodes of G.

A partition of a universe set is a family of pairwise disjoint sets whose union is the entire universe set.

Parameters

• G (NetworkX graph.)

• communities (list or iterable of sets of nodes) – If not a list, the iterable is converted internally to a list. If it is an iterator it is exhausted.

3.15 Components

3.15.1 Connectivity

is_connected(G) Returns True if the graph is connected, False otherwise.

number_connected_components(G) Returns the number of connected components.

connected_components(G) Generate connected components.

node_connected_component(G, n) Returns the set of nodes in the component of graph containing node n.

networkx.algorithms.components.is_connected

is_connected(G)
Returns True if the graph is connected, False otherwise.

Parameters

G (NetworkX Graph) – An undirected graph.

Returns connected – True if the graph is connected, false otherwise.

Return type bool

Raises NetworkXNotImplemented – If G is directed.

Examples

>>> G = nx.path_graph(4)
>>> print(nx.is_connected(G))
True

See also:
is_strongly_connected(), is_weakly_connected(), is_semi_connected(), is_biconnected(), connected_components()
Notes

For undirected graphs only.

networkx.algorithms.components.number_connected_components

number_connected_components(G)

Returns the number of connected components.

Parameters
- G (NetworkX graph) – An undirected graph.

Returns
- n – Number of connected components

Return type
- integer

See also:
- connected_components()
- number_weakly_connected_components()
- number_strongly_connected_components()

Notes

For undirected graphs only.

networkx.algorithms.components.connected_components

connected_components(G)

Generate connected components.

Parameters
- G (NetworkX graph) – An undirected graph

Returns
- comp – A generator of sets of nodes, one for each component of G.

Return type
- generator of sets

Raises
- NetworkXNotImplemented – If G is directed.

Examples

Generate a sorted list of connected components, largest first.

```python
>>> G = nx.path_graph(4)
>>> nx.add_path(G, [10, 11, 12])
>>> [len(c) for c in sorted(nx.connected_components(G), key=len, reverse=True)]
[4, 3]
```

If you only want the largest connected component, it’s more efficient to use max instead of sort.

```python
>>> largest_cc = max(nx.connected_components(G), key=len)
```

To create the induced subgraph of each component use:

```python
>>> S = [G.subgraph(c).copy() for c in connected_components(G)]
```

See also:

- strongly_connected_components()
- weakly_connected_components()
Notes

For undirected graphs only.

networkx.algorithms.components.node_connected_component

def node_connected_component(G, n):
    """Returns the set of nodes in the component of graph containing node n."
    
    Parameters
    
    • G (NetworkX Graph) – An undirected graph.
    • n (node label) – A node in G

    Returns comp – A set of nodes in the component of G containing node n.

    Return type set

    Raises NetworkXNotImplemented – If G is directed.

See also:
connected_components()

Notes

For undirected graphs only.

3.15.2 Strong connectivity

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>is_strongly_connected(G)</td>
<td>Test directed graph for strong connectivity.</td>
</tr>
<tr>
<td>number_strongly_connected_components(G)</td>
<td>Returns number of strongly connected components in graph.</td>
</tr>
<tr>
<td>strongly_connected_components(G)</td>
<td>Generate nodes in strongly connected components of graph.</td>
</tr>
<tr>
<td>strongly_connected_components_recursive(G)</td>
<td>Generate nodes in strongly connected components of graph.</td>
</tr>
<tr>
<td>kosaraju_strongly_connected_components(G)</td>
<td>Generate nodes in strongly connected components of graph.</td>
</tr>
<tr>
<td>condensation(G, scc)</td>
<td>Returns the condensation of G.</td>
</tr>
</tbody>
</table>

networkx.algorithms.components.is_strongly_connected

def is_strongly_connected(G):
    """Test directed graph for strong connectivity."
    
    A directed graph is strongly connected if and only if every vertex in the graph is reachable from every other vertex.

    Parameters G (NetworkX Graph) – A directed graph.
    
    Returns connected – True if the graph is strongly connected, False otherwise.

    Return type bool

    Raises NetworkXNotImplemented – If G is undirected.
See also:

```
is_weakly_connected(), is_semiconnected(), is_connected(), is_biconnected(),
strongly_connected_components()
```

Notes

For directed graphs only.

```python
networkx.algorithms.components.number_strongly_connected_components
```

```
number_strongly_connected_components(G)
```

Returns number of strongly connected components in graph.

- **Parameters**
  - `G (NetworkX graph)` – A directed graph.
- **Returns**
  - `n` – Number of strongly connected components
- **Return type**
  - integer
- **Raises**
  - `NetworkXNotImplemented` – If G is undirected.

See also:

```
strongly_connected_components(),
number_connected_components(),
number_weakly_connected_components()
```

Notes

For directed graphs only.

```python
networkx.algorithms.components.strongly_connected_components
```

```
strongly_connected_components(G)
```

Generate nodes in strongly connected components of graph.

- **Parameters**
  - `G (NetworkX Graph)` – A directed graph.
- **Returns**
  - `comp` – A generator of sets of nodes, one for each strongly connected component of G.
- **Return type**
  - generator of sets
- **Raises**
  - `NetworkXNotImplemented` – If G is undirected.

Examples

Generate a sorted list of strongly connected components, largest first.

```python
>>> G = nx.cycle_graph(4, create_using=nx.DiGraph())
>>> nx.add_cycle(G, [10, 11, 12])
>>> [len(c) for c in sorted(nx.strongly_connected_components(G),
...                        key=len, reverse=True)]
[4, 3]
```

If you only want the largest component, it’s more efficient to use max instead of sort.

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```
>>> largest = max(nx.strongly_connected_components(G), key=len)
```

See also:

- `connected_components()`,
- `weakly_connected_components()`,
- `kosaraju_strongly_connected_components()`

### Notes


### References

networkx.algorithms.components.strongly_connected_components_recursive

**strongly_connected_components_recursive** *(G)*

Generate nodes in strongly connected components of graph.

Recursive version of algorithm.

- **Parameters**
  - `G` *(NetworkX Graph)* – A directed graph.
- **Returns**
  - `comp` – A generator of sets of nodes, one for each strongly connected component of `G`.
- **Return type**
  - generator of sets
- **Raises** `NetworkXNotImplemented` – If `G` is undirected.

### Examples

Generate a sorted list of strongly connected components, largest first.

```
>>> G = nx.cycle_graph(4, create_using=nx.DiGraph())
>>> nx.add_cycle(G, [10, 11, 12])
>>> [len(c) for c in sorted(nx.strongly_connected_components_recursive(G), ...
...    key=len, reverse=True)]
[4, 3]
```

If you only want the largest component, it's more efficient to use `max` instead of `sort`.

```
>>> largest = max(nx.strongly_connected_components_recursive(G), key=len)
```

To create the induced subgraph of the components use:

```
>>> S = [G.subgraph(c).copy() for c in nx.weakly_connected_components(G)]
```

See also:

- `connected_components()`

### Notes

Uses Tarjan’s algorithm[1] with Nuutila’s modifications[2].
References

networkx.algorithms.components.kosaraju_strongly_connected_components

kosaraju_strongly_connected_components(G, source=None)
Generate nodes in strongly connected components of graph.

Parameters
- **G** *(NetworkX Graph)* – A directed graph.

Returns
- **comp** – A generator of sets of nodes, one for each strongly connected component of G.

Return type
generator of sets

Raises
NetworkXNotImplemented – If G is undirected.

Examples

Generate a sorted list of strongly connected components, largest first.

```python
>>> G = nx.cycle_graph(4, create_using=nx.DiGraph())
>>> nx.add_cycle(G, [10, 11, 12])
>>> [len(c) for c in sorted(nx.kosaraju_strongly_connected_components(G), ... key=len, reverse=True)]
[4, 3]
```

If you only want the largest component, it’s more efficient to use max instead of sort.

```python
>>> largest = max(nx.kosaraju_strongly_connected_components(G), key=len)
```

See also:

strongly_connected_components()

Notes

Uses Kosaraju’s algorithm.

networkx.algorithms.components.condensation

condensation(G, scc=None)
Returns the condensation of G.

The condensation of G is the graph with each of the strongly connected components contracted into a single node.

Parameters

- **G** *(NetworkX DiGraph)* – A directed graph.
- **scc** *(list or generator (optional, default=None))* – Strongly connected components. If provided, the elements in scc must partition the nodes in G. If not provided, it will be calculated as scc=nx.strongly_connected_components(G).

Returns
- **C** – The condensation graph C of G. The node labels are integers corresponding to the index of the component in the list of strongly connected components of G. C has a graph attribute named ‘mapping’ with a dictionary mapping the original nodes to the nodes in C to which they belong. Each node in C also has a node attribute ‘members’ with the set of original nodes in G that form the SCC that the node in C represents.

3.15. Components
Return type NetworkX DiGraph

Raises NetworkXNotImplemented – If G is undirected.

Notes
After contracting all strongly connected components to a single node, the resulting graph is a directed acyclic graph.

3.15.3 Weak connectivity

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>is_weakly_connected(G)</td>
<td>Test directed graph for weak connectivity.</td>
</tr>
<tr>
<td>number_weakly_connected_components(G)</td>
<td>Returns the number of weakly connected components in G.</td>
</tr>
<tr>
<td>weakly_connected_components(G)</td>
<td>Generate weakly connected components of G.</td>
</tr>
</tbody>
</table>

networkx.algorithms.components.is_weakly_connected

is_weakly_connected(G)
Test directed graph for weak connectivity.

A directed graph is weakly connected if and only if the graph is connected when the direction of the edge between nodes is ignored.

Note that if a graph is strongly connected (i.e. the graph is connected even when we account for directionality), it is by definition weakly connected as well.

Parameters G (NetworkX Graph) – A directed graph.

Returns connected – True if the graph is weakly connected, False otherwise.

Return type bool

Raises NetworkXNotImplemented – If G is undirected.

See also:
is_strongly_connected(), is_semiconnected(), is_connected(), is_biconnected(), weakly_connected_components()

Notes
For directed graphs only.

networkx.algorithms.components.number_weakly_connected_components

number_weakly_connected_components(G)
Returns the number of weakly connected components in G.

Parameters G (NetworkX graph) – A directed graph.

Returns n – Number of weakly connected components

Return type integer

Raises NetworkXNotImplemented – If G is undirected.
See also:

weakly_connected_components(), number_connected_components(), number_strongly_connected_components()

Notes

For directed graphs only.

networkx.algorithms.components.weakly_connected_components

weakly_connected_components(G)
Generate weakly connected components of G.

Parameters G (NetworkX graph) – A directed graph

Returns comp – A generator of sets of nodes, one for each weakly connected component of G.

Return type generator of sets

Raises NetworkXNotImplemented – If G is undirected.

Examples

Generate a sorted list of weakly connected components, largest first.

```python
>>> G = nx.path_graph(4, create_using=nx.DiGraph())
>>> nx.add_path(G, [10, 11, 12])
>>> [len(c) for c in sorted(nx.weakly_connected_components(G), key=len, reverse=True)]
[4, 3]
```

If you only want the largest component, it’s more efficient to use max instead of sort:

```python
>>> largest_cc = max(nx.weakly_connected_components(G), key=len)
```

See also:

c connected_components(), strongly_connected_components()

Notes

For directed graphs only.

3.15.4 Attracting components

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>is_attracting_component(G)</td>
<td>Returns True if G consists of a single attracting component.</td>
</tr>
<tr>
<td>number_attracting_components(G)</td>
<td>Returns the number of attracting components in G.</td>
</tr>
<tr>
<td>attracting_components(G)</td>
<td>Generates the attracting components in G.</td>
</tr>
</tbody>
</table>

3.15. Components
networkx.algorithms.components.is_attracting_component

is_attracting_component(G)

Returns True if G consists of a single attracting component.

Parameters G (DiGraph, MultiDiGraph) – The graph to be analyzed.

Returns attracting – True if G has a single attracting component. Otherwise, False.

Return type bool

Raises NetworkXNotImplemented – If the input graph is undirected.

See also:

attracting_components(), number_attracting_components()

networkx.algorithms.components.number_attracting_components

number_attracting_components(G)

Returns the number of attracting components in G.

Parameters G (DiGraph, MultiDiGraph) – The graph to be analyzed.

Returns n – The number of attracting components in G.

Return type int

Raises NetworkXNotImplemented – If the input graph is undirected.

See also:

attracting_components(), is_attracting_component()

networkx.algorithms.components.attracting_components

attracting_components(G)

Generates the attracting components in G.

An attracting component in a directed graph G is a strongly connected component with the property that a random walker on the graph will never leave the component, once it enters the component.

The nodes in attracting components can also be thought of as recurrent nodes. If a random walker enters the attractor containing the node, then the node will be visited infinitely often.

To obtain induced subgraphs on each component use: (G.subgraph(c).copy() for c in attracting_components(G))

Parameters G (DiGraph, MultiDiGraph) – The graph to be analyzed.

Returns attractors – A generator of sets of nodes, one for each attracting component of G.

Return type generator of sets

Raises NetworkXNotImplemented – If the input graph is undirected.

See also:

number_attracting_components(), is_attracting_component()
### 3.15.5 Biconnected components

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>is_biconnected(G)</code></td>
<td>Returns True if the graph is biconnected, False otherwise.</td>
</tr>
<tr>
<td><code>biconnected_components(G)</code></td>
<td>Returns a generator of sets of nodes, one set for each biconnected component of the graph.</td>
</tr>
<tr>
<td><code>biconnected_component_edges(G)</code></td>
<td>Returns a generator of lists of edges, one list for each biconnected component of the input graph.</td>
</tr>
<tr>
<td><code>articulation_points(G)</code></td>
<td>Yield the articulation points, or cut vertices, of a graph.</td>
</tr>
</tbody>
</table>

#### networkx.algorithms.components.is_biconnected

**is_biconnected(G)**

Returns True if the graph is biconnected, False otherwise.

A graph is biconnected if, and only if, it cannot be disconnected by removing only one node (and all edges incident on that node). If removing a node increases the number of disconnected components in the graph, that node is called an articulation point, or cut vertex. A biconnected graph has no articulation points.

**Parameters**  
G *(NetworkX Graph)* – An undirected graph.

**Returns**  
biconnected – True if the graph is biconnected, False otherwise.

**Return type**  
bool

**Raises**  
`NetworkXNotImplemented` – If the input graph is not undirected.

#### Examples

```python
>>> G = nx.path_graph(4)
>>> print(nx.is_biconnected(G))
False
>>> G.add_edge(0, 3)
>>> print(nx.is_biconnected(G))
True
```

**See also:**

`biconnected_components()`, `articulation_points()`, `biconnected_component_edges()`, `is_strongly_connected()`, `is_weakly_connected()`, `is_connected()`, `is_semiconnected()`

#### Notes

The algorithm to find articulation points and biconnected components is implemented using a non-recursive depth-first-search (DFS) that keeps track of the highest level that back edges reach in the DFS tree. A node \( n \) is an articulation point if, and only if, there exists a subtree rooted at \( n \) such that there is no back edge from any successor of \( n \) that links to a predecessor of \( n \) in the DFS tree. By keeping track of all the edges traversed by the DFS we can obtain the biconnected components because all edges of a bicomponent will be traversed consecutively between articulation points.
References

networkx.algorithms.components.biconnected_components

biconnected_components(G)

Returns a generator of sets of nodes, one set for each biconnected component of the graph.

Biconnected components are maximal subgraphs such that the removal of a node (and all edges incident on that node) will not disconnect the subgraph. Note that nodes may be part of more than one biconnected component. Those nodes are articulation points, or cut vertices. The removal of articulation points will increase the number of connected components of the graph.

Notice that by convention a dyad is considered a biconnected component.

Parameters

G (NetworkX Graph) – An undirected graph.

Returns

nodes – Generator of sets of nodes, one set for each biconnected component.

Return type

generator

Raises

NetworkXNotImplemented – If the input graph is not undirected.

See also:

k_components() this function is a special case where k=2

bridge_components() similar to this function, but is defined using 2-edge-connectivity instead of 2-node-connectivity.

Examples

```python
>>> G = nx.lollipop_graph(5, 1)
>>> print(nx.is_biconnected(G))
False
>>> bicomponents = list(nx.biconnected_components(G))
2
>>> G.add_edge(0, 5)
>>> print(nx.is_biconnected(G))
True
>>> bicomponents = list(nx.biconnected_components(G))
>>> len(bicomponents)
1
```

You can generate a sorted list of biconnected components, largest first, using sort.

```python
>>> G.remove_edge(0, 5)
>>> [len(c) for c in sorted(nx.biconnected_components(G), key=len, reverse=True)]
[5, 2]
```

If you only want the largest connected component, it’s more efficient to use max instead of sort.

```python
>>> Gc = max(nx.biconnected_components(G), key=len)
```

To create the components as subgraphs use: (G.subgraph(c).copy() for c in

biconnected_components(G))

See also:

is_biconnected(), articulation_points(), biconnected_component_edges()
Notes

The algorithm to find articulation points and biconnected components is implemented using a non-recursive depth-first-search (DFS) that keeps track of the highest level that back edges reach in the DFS tree. A node $n$ is an articulation point if, and only if, there exists a subtree rooted at $n$ such that there is no back edge from any successor of $n$ that links to a predecessor of $n$ in the DFS tree. By keeping track of all the edges traversed by the DFS we can obtain the biconnected components because all edges of a bicomponent will be traversed consecutively between articulation points.

References

networkx.algorithms.components.biconnected_component_edges

biconnected_component_edges $(G)$

Returns a generator of lists of edges, one list for each biconnected component of the input graph.

Biconnected components are maximal subgraphs such that the removal of a node (and all edges incident on that node) will not disconnect the subgraph. Note that nodes may be part of more than one biconnected component. Those nodes are articulation points, or cut vertices. However, each edge belongs to one, and only one, biconnected component.

Notice that by convention a dyad is considered a biconnected component.

Parameters $G$ *(NetworkX Graph)* – An undirected graph.

Returns edges – Generator of lists of edges, one list for each bicomponent.

Return type generator of lists

Raises NetworkXNotImplemented – If the input graph is not undirected.

Examples

```python
>>> G = nx.barbell_graph(4, 2)
>>> print(nx.is_biconnected(G))
False
>>> bicomponents_edges = list(nx.biconnected_component_edges(G))
>>> len(bicomponents_edges)
5
>>> G.add_edge(2, 8)
>>> print(nx.is_biconnected(G))
True
>>> bicomponents_edges = list(nx.biconnected_component_edges(G))
>>> len(bicomponents_edges)
1
```

See also:

is_biconnected(), biconnected_components(), articulation_points()

Notes

The algorithm to find articulation points and biconnected components is implemented using a non-recursive depth-first-search (DFS) that keeps track of the highest level that back edges reach in the DFS tree. A node $n$ is an articulation point if, and only if, there exists a subtree rooted at $n$ such that there is no back edge from any successor of $n$ that links to a predecessor of $n$ in the DFS tree. By keeping track of all the edges traversed
by the DFS we can obtain the biconnected components because all edges of a bicomponent will be traversed consecutively between articulation points.

References

networkx.algorithms.components.articulation_points

articulation_points(G)
Yield the articulation points, or cut vertices, of a graph.

An articulation point or cut vertex is any node whose removal (along with all its incident edges) increases the number of connected components of a graph. An undirected connected graph without articulation points is biconnected. Articulation points belong to more than one biconnected component of a graph.

Notice that by convention a dyad is considered a biconnected component.

Parameters G (NetworkX Graph) – An undirected graph.

Yields node – An articulation point in the graph.

Raises NetworkXNotImplemented – If the input graph is not undirected.

Examples

```python
>>> G = nx.barbell_graph(4, 2)
>>> print(nx.is_biconnected(G))
False
>>> len(list(nx.articulation_points(G)))
4
>>> G.add_edge(2, 8)
>>> print(nx.is_biconnected(G))
True
>>> len(list(nx.articulation_points(G)))
0
```

See also:
is_biconnected(), biconnected_components(), biconnected_component_edges()

Notes

The algorithm to find articulation points and biconnected components is implemented using a non-recursive depth-first-search (DFS) that keeps track of the highest level that back edges reach in the DFS tree. A node \( n \) is an articulation point if, and only if, there exists a subtree rooted at \( n \) such that there is no back edge from any successor of \( n \) that links to a predecessor of \( n \) in the DFS tree. By keeping track of all the edges traversed by the DFS we can obtain the biconnected components because all edges of a bicomponent will be traversed consecutively between articulation points.

References

3.15.6 Semiconnectedness
is_semiconnected(G[, topo_order]) Returns True if the graph is semiconnected, False otherwise.

networkx.algorithms.components.is_semiconnected

is_semiconnected(G, topo_order=None)
Returns True if the graph is semiconnected, False otherwise.

A graph is semiconnected if, and only if, for any pair of nodes, either one is reachable from the other, or they are mutually reachable.

Parameters

• G (NetworkX graph) – A directed graph.
• topo_order (list or tuple, optional) – A topological order for G (if None, the function will compute one)

Returns semiconnected – True if the graph is semiconnected, False otherwise.

Return type bool

Raises

• NetworkXNotImplemented – If the input graph is undirected.
• NetworkXPointlessConcept – If the graph is empty.

Examples

```python
>>> G=nx.path_graph(4,create_using=nx.DiGraph())
>>> print(nx.is_semiconnected(G))
True
>>> G=nx.DiGraph([(1, 2), (3, 2)])
>>> print(nx.is_semiconnected(G))
False
```

See also:
is_strongly_connected(), is_weakly_connected(), is_connected(), is_biconnected()

3.16 Connectivity

Connectivity and cut algorithms

3.16.1 Edge-augmentation

Algorithms for finding k-edge-augmentations

A k-edge-augmentation is a set of edges, that once added to a graph, ensures that the graph is k-edge-connected; i.e. the graph cannot be disconnected unless k or more edges are removed. Typically, the goal is to find the augmentation with minimum weight. In general, it is not guaranteed that a k-edge-augmentation exists.

See also:
edge_kcomponents algorithms for finding k-edge-connected components

connectivity algorithms for determining edge connectivity.

\[ \text{k_edge_augmentation}(G, k[, avail, weight, . . .]) \] Finds set of edges to k-edge-connect G.

\[ \text{is_k_edge_connected}(G, k) \] Tests to see if a graph is k-edge-connected.

\[ \text{is_locally_k_edge_connected}(G, s, t, k) \] Tests to see if an edge in a graph is locally k-edge-connected.

**networkx.algorithms.connectivity.edge_augmentation.k_edge_augmentation**

\[ \text{k_edge_augmentation}(G, \text{k}, \text{avail}=None, \text{weight}=None, \text{partial}=False) \] Finds set of edges to k-edge-connect G.

Adding edges from the augmentation to G make it impossible to disconnect G unless k or more edges are removed. This function uses the most efficient function available (depending on the value of k and if the problem is weighted or unweighted) to search for a minimum weight subset of available edges that k-edge-connects G. In general, finding a k-edge-augmentation is NP-hard, so solutions are not guaranteed to be minimal. Furthermore, a k-edge-augmentation may not exist.

**Parameters**

- **G** (NetworkX graph) – An undirected graph.
- **k** (integer) – Desired edge connectivity
- **avail** (dict or a set of 2 or 3 tuples) – The available edges that can be used in the augmentation.

If unspecified, then all edges in the complement of G are available. Otherwise, each item is an available edge (with an optional weight).

In the unweighted case, each item is an edge \((u, v)\).

In the weighted case, each item is a 3-tuple \((u, v, d)\) or a dict with items \((u, v): d\). The third item, \(d\), can be a dictionary or a real number. If \(d\) is a dictionary \(d[weight] \) correspondings to the weight.

- **weight** (string) – key to use to find weights if avail is a set of 3-tuples where the third item in each tuple is a dictionary.
- **partial** (boolean) – If partial is True and no feasible k-edge-augmentation exists, then all a partial k-edge-augmentation is generated. Adding the edges in a partial augmentation to G, minimizes the number of k-edge-connected components and maximizes the edge connectivity between those components. For details, see \text{partial_k_edge_augmentation}().

**Yields** edge (tuple) – Edges that, once added to G, would cause G to become k-edge-connected. If partial is False, an error is raised if this is not possible. Otherwise, generated edges form a partial augmentation, which k-edge-connects any part of G where it is possible, and maximally connects the remaining parts.

**Raises**

- **NetworkXUnfeasible** – If partial is False and no k-edge-augmentation exists.
- **NetworkXNotImplemented** – If the input graph is directed or a multigraph.
- **ValueError** – If k is less than 1
Notes

When k=1 this returns an optimal solution.

When k=2 and avail is None, this returns an optimal solution. Otherwise when k=2, this returns a 2-approximation of the optimal solution.

For k>3, this problem is NP-hard and this uses a randomized algorithm that produces a feasible solution, but provides no guarantees on the solution weight.

Example

```python
>>> # Unweighted cases
>>> G = nx.path_graph((1, 2, 3, 4))
>>> G.add_node(5)
>>> sorted(nx.k_edge_augmentation(G, k=1))
[(1, 5)]
>>> sorted(nx.k_edge_augmentation(G, k=2))
[(1, 5), (5, 4)]
>>> sorted(nx.k_edge_augmentation(G, k=3))
[(1, 4), (1, 5), (2, 5), (3, 5), (4, 5)]
>>> complement = list(nx.k_edge_augmentation(G, k=5, partial=True))
>>> G.add_edges_from(complement)
>>> nx.edge_connectivity(G)
4
```

```python
>>> # Weighted cases
>>> G = nx.path_graph((1, 2, 3, 4))
>>> G.add_node(5)
>>> # avail can be a tuple with a dict
>>> avail = [(1, 5, {'weight': 11}), (2, 5, {'weight': 10})]
>>> sorted(nx.k_edge_augmentation(G, k=1, avail=avail, weight='weight'))
[(2, 5)]
>>> # or avail can be a 3-tuple with a real number
>>> avail = [(1, 5, 11), (2, 5, 10), (4, 3, 1), (4, 5, 51)]
>>> sorted(nx.k_edge_augmentation(G, k=2, avail=avail))
[(1, 5), (2, 5), (4, 5)]
>>> # or avail can be a dict
>>> avail = {(1, 5): 11, (2, 5): 10, (4, 3): 1, (4, 5): 51}
>>> sorted(nx.k_edge_augmentation(G, k=2, avail=avail))
[(1, 5), (2, 5), (4, 5)]
>>> # If augmentation is infeasible, then a partial solution can be found
>>> avail = {(1, 5): 11}
>>> sorted(nx.k_edge_augmentation(G, k=2, avail=avail, partial=True))
[(1, 5)]
```

`networkx.algorithms.connectivity.edge_augmentation.is_k_edge_connected`

`is_k_edge_connected(G, k)`

Tests to see if a graph is k-edge-connected.

Is it impossible to disconnect the graph by removing fewer than k edges? If so, then G is k-edge-connected.
Parameters

- **G** *(NetworkX graph)* – An undirected graph.
- **k** *(integer)* – edge connectivity to test for

**Returns**  True if G is k-edge-connected.

**Return type**  boolean

See also:

*is_locally_k_edge_connected()*

**Example**

```python
>>> G = nx.barbell_graph(10, 0)
>>> nx.is_k_edge_connected(G, k=1)
True
>>> nx.is_k_edge_connected(G, k=2)
False
```

`networkx.algorithms.connectivity.edge_augmentation.is_locally_k_edge_connected`

**is_locally_k_edge_connected** *(G, s, t, k)*

Tests to see if an edge in a graph is locally k-edge-connected.

Is it impossible to disconnect s and t by removing fewer than k edges? If so, then s and t are locally k-edge-connected in G.

**Parameters**

- **G** *(NetworkX graph)* – An undirected graph.
- **s** *(node)* – Source node
- **t** *(node)* – Target node
- **k** *(integer)* – local edge connectivity for nodes s and t

**Returns**  True if s and t are locally k-edge-connected in G.

**Return type**  boolean

See also:

*is_k_edge_connected()*

**Example**

```python
>>> from networkx.algorithms.connectivity import is_locally_k_edge_connected
>>> G = nx.barbell_graph(10, 0)
>>> is_locally_k_edge_connected(G, 5, 15, k=1)
True
>>> is_locally_k_edge_connected(G, 5, 15, k=2)
False
>>> is_locally_k_edge_connected(G, 1, 5, k=2)
True
```
3.16.2 K-edge-components

Algorithms for finding k-edge-connected components and subgraphs.

A k-edge-connected component (k-edge-cc) is a maximal set of nodes in G, such that all pairs of node have an edge-connectivity of at least k.

A k-edge-connected subgraph (k-edge-subgraph) is a maximal set of nodes in G, such that the subgraph of G defined by the nodes has an edge-connectivity at least k.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>k_edge_components(G, k)</td>
<td>Generates nodes in each maximal k-edge-connected component in G.</td>
</tr>
<tr>
<td>k_edge_subgraphs(G, k)</td>
<td>Generates nodes in each maximal k-edge-connected subgraph in G.</td>
</tr>
<tr>
<td>bridge_components(G)</td>
<td>Finds all bridge-connected components G.</td>
</tr>
<tr>
<td>EdgeComponentAuxGraph</td>
<td>A simple algorithm to find all k-edge-connected components in a graph.</td>
</tr>
</tbody>
</table>

**networkx.algorithms.connectivity.edge_kcomponents.k_edge_components**

**k_edge_components**(G, k)
Generates nodes in each maximal k-edge-connected component in G.

**Parameters**

- G (NetworkX graph)
- k (Integer) – Desired edge connectivity

**Returns** k_edge_components – will have k-edge-connectivity in the graph G.

**Return type** a generator of k-edge-ccs. Each set of returned nodes

**See also:**
local_edge_connectivity()
k_edge_subgraphs() similar to this function, but the subgraph defined by the nodes must also have k-edge-connectivity.
k_components() similar to this function, but uses node-connectivity instead of edge-connectivity

**Raises**

- NetworkXNotImplemented – If the input graph is a multigraph.
- ValueError: – If k is less than 1

**Notes**

Attempts to use the most efficient implementation available based on k. If k=1, this is simply simply connected components for directed graphs and connected components for undirected graphs. If k=2 on an efficient bridge connected component algorithm from _[1] is run based on the chain decomposition. Otherwise, the algorithm from _[2] is used.
Example

```python
>>> import itertools as it
>>> from networkx.utils import pairwise

>>> paths = [
...   (1, 2, 4, 3, 1, 4),
...   (5, 6, 7, 8, 5, 7, 8, 6),
... ]

>>> G = nx.Graph()

>>> G.add_nodes_from(it.chain(*paths))

>>> G.add_edges_from(it.chain(*[pairwise(path) for path in paths]))

# note this returns {1, 4} unlike k_edge_subgraphs

>>> sorted(map(sorted, nx.k_edge_components(G, k=3)))
[[1, 4], [2], [3], [5, 6, 7, 8]]
```

References

networkx.algorithms.connectivity.edge_kcomponents.k_edge_subgraphs

**k_edge_subgraphs** *(G, k)*
Generates nodes in each maximal k-edge-connected subgraph in G.

**Parameters**

- G *(NetworkX graph)*
- k *(Integer)* – Desired edge connectivity

**Returns**

- **k_edge_subgraphs** – Each k-edge-subgraph is a maximal set of nodes that defines a subgraph of G that is k-edge-connected.

**Return type**

a generator of k-edge-subgraphs

See also:

d = edge_connectivity()

**k_edge_components** *(G)* similar to this function, but nodes only need to have k-edge-connectivity within the graph G and the subgraphs might not be k-edge-connected.

**Raises**

- **NetworkXNotImplemented** – If the input graph is a multigraph.
- **ValueError**: – If k is less than 1

**Notes**

Attempts to use the most efficient implementation available based on k. If k=1, or k=2 and the graph is undirected, then this simply calls **k_edge_components**. Otherwise the algorithm from _[1] is used.

**Example**
>>> import itertools as it
>>> from networkx.utils import pairwise

paths = [
    ...
    (1, 2, 4, 3, 1, 4),
    ...
    (5, 6, 7, 8, 5, 7, 8, 6),
    ...
]
>>> G = nx.Graph()
>>> G.add_nodes_from(it.chain(*paths))
>>> G.add_edges_from(it.chain(*[
    pairwise(path)
    for path in paths]))

# note this does not return {1, 4} unlike k_edge_components

>>> sorted(map(sorted, nx.k_edge_subgraphs(G, k=3)))
[[1], [2], [3], [4], [5, 6, 7, 8]]

References

networkx.algorithms.connectivity.edge_kcomponents.bridge_components

bridge_components(G)

Finds all bridge-connected components G.

Parameters

G (NetworkX undirected graph)

Returns

bridge_components

Return type

a generator of 2-edge-connected components

See also:

k_edge_subgraphs() this function is a special case for an undirected graph where k=2.
biconnected_components() similar to this function, but is defined using 2-node-connectivity instead of 2-edge-connectivity.

Raises NetworkXNotImplemented – If the input graph is directed or a multigraph.

Notes

Bridge-connected components are also known as 2-edge-connected components.

Example

>>> # The barbell graph with parameter zero has a single bridge
>>> G = nx.barbell_graph(5, 0)
>>> from networkx.algorithms.connectivity.edge_kcomponents import bridge_components
>>> sorted(map(sorted, bridge_components(G)))

[[0, 1, 2, 3, 4], [5, 6, 7, 8, 9]]

networkx.algorithms.connectivity.edge_kcomponents.EdgeComponentAuxGraph

class EdgeComponentAuxGraph

A simple algorithm to find all k-edge-connected components in a graph.
Constructing the AuxiliaryGraph (which may take some time) allows for the k-edge-ccs to be found in linear time for arbitrary k.

Notes

This implementation is based on\(^1\). The idea is to construct an auxiliary graph from which the k-edge-ccs can be extracted in linear time. The auxiliary graph is constructed in \(O(|V| \cdot F)\) operations, where F is the complexity of max flow. Querying the components takes an additional \(O(|V|)\) operations. This algorithm can be slow for large graphs, but it handles an arbitrary k and works for both directed and undirected inputs.

The undirected case for k=1 is exactly connected components. The undirected case for k=2 is exactly bridge connected components. The directed case for k=1 is exactly strongly connected components.

References

Example

```python
>>> import itertools as it
>>> from networkx.utils import pairwise
>>> from networkx.algorithms.connectivity import EdgeComponentAuxGraph

>>> # Build an interesting graph with multiple levels of k-edge-ccs
>>> paths = [
... (1, 2, 3, 4, 1, 3, 4, 2), # a 3-edge-cc (a 4 clique)
... (5, 6, 7, 5), # a 2-edge-cc (a 3 clique)
... (1, 5), # combine first two ccs into a 1-edge-cc
... (0,), # add an additional disconnected 1-edge-cc
... ]
>>> G = nx.Graph()
>>> G.add_nodes_from(it.chain(*paths))
>>> G.add_edges_from(it.chain(*[pairwise(path) for path in paths]))

# Constructing the AuxGraph takes about O(n ** 4)
>>> aux_graph = EdgeComponentAuxGraph.construct(G)
# Once constructed, querying takes O(n)
>>> sorted(map(sorted, aux_graph.k_edge_components(k=1)))
[[0], [1, 2, 3, 4, 5, 6, 7]]
>>> sorted(map(sorted, aux_graph.k_edge_components(k=2)))
[[0], [1, 2, 3, 4], [5, 6, 7]]
>>> sorted(map(sorted, aux_graph.k_edge_components(k=3)))
[[0], [1, 2, 3, 4], [5, 6], [7]]
>>> sorted(map(sorted, aux_graph.k_edge_components(k=4)))
[[0], [1], [2], [3], [4], [5], [6], [7]]
```

Example

```python
>>> # The auxiliary graph is primarily used for k-edge-ccs but it
>>> # can also speed up the queries of k-edge-subgraphs by refining the
>>> # search space.
>>> import itertools as it
>>> from networkx.utils import pairwise
>>> from networkx.algorithms.connectivity import EdgeComponentAuxGraph
>>> paths = [
```


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... (1, 2, 4, 3, 1, 4),
... ]
>>> G = nx.Graph()
>>> G.add_nodes_from(it.chain(*paths))
>>> G.add_edges_from(it.chain(*[pairwise(path) for path in paths]))
>>> aux_graph = EdgeComponentAuxGraph.construct(G)
>>> sorted(map(sorted, aux_graph.k_edge_subgraphs(k=3)))
[[[1], [2], [3], [4]]
[1, 4], [2], [3]]

__init__()  
Initialize self. See help(type(self)) for accurate signature.

Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
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<tbody>
<tr>
<td><strong>init</strong>()</td>
<td>Initialize self.</td>
</tr>
<tr>
<td>construct(G)</td>
<td>Builds an auxiliary graph encoding edge-connectivity between nodes.</td>
</tr>
<tr>
<td>k_edge_components(k)</td>
<td>Queries the auxiliary graph for k-edge-connected components.</td>
</tr>
<tr>
<td>k_edge_subgraphs(k)</td>
<td>Queries the auxiliary graph for k-edge-connected subgraphs.</td>
</tr>
</tbody>
</table>

3.16.3 K-node-components

Moody and White algorithm for k-components

__k_components(G[, flow_func])__ Returns the k-component structure of a graph G.

networkx.algorithms.connectivity.kcomponents.k_components

k_components(G, flow_func=None)  
Returns the k-component structure of a graph G.

A k-component is a maximal subgraph of a graph G that has, at least, node connectivity k: we need to remove at least k nodes to break it into more components. k-components have an inherent hierarchical structure because they are nested in terms of connectivity: a connected graph can contain several 2-components, each of which can contain one or more 3-components, and so forth.

Parameters

- G (NetworkX graph)
- flow_func (function) – Function to perform the underlying flow computations. Default value edmonds_karp(). This function performs better in sparse graphs with right tailed degree distributions. shortest_augmenting_path() will perform better in denser graphs.

Returns k_components – Dictionary with all connectivity levels k in the input Graph as keys and a list of sets of nodes that form a k-component of level k as values.

Return type  dict

Raises NetworkXNotImplemented – If the input graph is directed.
Examples

```python
>>> # Petersen graph has 10 nodes and it is triconnected, thus all
>>> # nodes are in a single component on all three connectivity levels
>>> G = nx.petersen_graph()
>>> k_components = nx.k_components(G)
```

Notes

Moody and White\(^1\) (appendix A) provide an algorithm for identifying k-components in a graph, which is based on Kanevsky’s algorithm\(^2\) for finding all minimum-size node cut-sets of a graph (implemented in `all_node_cuts()` function):

1. Compute node connectivity, k, of the input graph G.
2. Identify all k-cutsets at the current level of connectivity using Kanevsky’s algorithm.
3. Generate new graph components based on the removal of these cutsets. Nodes in a cutset belong to both sides of the induced cut.
4. If the graph is neither complete nor trivial, return to 1; else end.

This implementation also uses some heuristics (see\(^3\) for details) to speed up the computation.

See also:

- `node_connectivity()`, `all_node_cuts()`
- `biconnected_components()` special case of this function when k=2
- `k_edge_components()` similar to this function, but uses edge-connectivity instead of node-connectivity

References

3.16.4 K-node-cutsets

Kanevsky all minimum node k cutsets algorithm.

```python
all_node_cuts(G[, k, flow_func])
```

Returns all minimum k cutsets of an undirected graph G.

This implementation is based on Kanevsky’s algorithm\(^1\) for finding all minimum-size node cut-sets of an undirected graph G; ie the set (or sets) of nodes of cardinality equal to the node connectivity of G. Thus if removed, would break G into two or more connected components.

---


Parameters

- **G** *(NetworkX graph)* – Undirected graph
- **k** *(Integer)* – Node connectivity of the input graph. If k is None, then it is computed. Default value: None.
- **flow_func** *(function)* – Function to perform the underlying flow computations. Default value edmonds_karp. This function performs better in sparse graphs with right tailed degree distributions. shortest_augmenting_path will perform better in denser graphs.

Returns **cuts** – Each node cutset has cardinality equal to the node connectivity of the input graph.

Return type a generator of node cutsets

Examples

```python
>>> # A two-dimensional grid graph has 4 cutsets of cardinality 2
>>> G = nx.grid_2d_graph(5, 5)
>>> cutsets = list(nx.all_node_cuts(G))
>>> len(cutsets)
4
>>> all(2 == len(cutset) for cutset in cutsets)
True
>>> nx.node_connectivity(G)
2
```

Notes

This implementation is based on the sequential algorithm for finding all minimum-size separating vertex sets in a graph\(^1\). The main idea is to compute minimum cuts using local maximum flow computations among a set of nodes of highest degree and all other non-adjacent nodes in the Graph. Once we find a minimum cut, we add an edge between the high degree node and the target node of the local maximum flow computation to make sure that we will not find that minimum cut again.

See also:

node_connectivity(), edmonds_karp(), shortest_augmenting_path()

References

3.16.5 Flow-based disjoint paths

Flow based node and edge disjoint paths.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>edge_disjoint_paths(G, s, t[, flow_func, ...])</td>
<td>Returns the edges disjoint paths between source and target.</td>
</tr>
<tr>
<td>node_disjoint_paths(G, s, t[, flow_func, ...])</td>
<td>Computes node disjoint paths between source and target.</td>
</tr>
</tbody>
</table>

networkx.algorithms.connectivity.disjoint_paths.edge_disjoint_paths

edge_disjoint_paths (G, s, t, flow_func=None, cutoff=None, auxiliary=None, residual=None)  Returns the edges disjoint paths between source and target.
Edge disjoint paths are paths that do not share any edge. The number of edge disjoint paths between source and target is equal to their edge connectivity.

**Parameters**
- **G** (*NetworkX graph*)
- **s** (*node*) – Source node for the flow.
- **t** (*node*) – Sink node for the flow.
- **flow_func** (*function*) – A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see `maximum_flow()` for details). If `flow_func` is None, the default maximum flow function (`edmonds_karp()`) is used. The choice of the default function may change from version to version and should not be relied on. Default value: None.
- **cutoff** (*int*) – Maximum number of paths to yield. Some of the maximum flow algorithms, such as `edmonds_karp()` (the default) and `shortest_augmenting_path()` support the cutoff parameter, and will terminate when the flow value reaches or exceeds the cutoff. Other algorithms will ignore this parameter. Default value: None.
- **auxiliary** (*NetworkX DiGraph*) – Auxiliary digraph to compute flow based edge connectivity. It has to have a graph attribute called mapping with a dictionary mapping node names in G and in the auxiliary digraph. If provided it will be reused instead of recreated. Default value: None.
- **residual** (*NetworkX DiGraph*) – Residual network to compute maximum flow. If provided it will be reused instead of recreated. Default value: None.

**Returns** paths – A generator of edge independent paths.

**Return type** generator

**Raises**
- **NetworkXNoPath** – If there is no path between source and target.
- **NetworkXError** – If source or target are not in the graph G.

**See also:**
`node_disjoint_paths()`, `edge_connectivity()`, `maximum_flow()`, `edmonds_karp()`, `preflow_push()`, `shortest_augmenting_path()`

**Examples**

We use in this example the platonic icosahedral graph, which has node edge connectivity 5, thus there are 5 edge disjoint paths between any pair of nodes.

```python
>>> G = nx.icosahedral_graph()
>>> len(list(nx.edge_disjoint_paths(G, 0, 6)))
5
```

If you need to compute edge disjoint paths on several pairs of nodes in the same graph, it is recommended that you reuse the data structures that NetworkX uses in the computation: the auxiliary digraph for edge connectivity, and the residual network for the underlying maximum flow computation.

Example of how to compute edge disjoint paths among all pairs of nodes of the platonic icosahedral graph reusing the data structures.
You can also use alternative flow algorithms for computing edge disjoint paths. For instance, in dense networks the algorithm `shortest_augmenting_path()` will usually perform better than the default `edmonds_karp()` which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

```python
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> len(list(nx.edge_disjoint_paths(G, 0, 6, flow_func=shortest_augmenting_path)))
5
```

**Notes**

This is a flow based implementation of edge disjoint paths. We compute the maximum flow between source and target on an auxiliary directed network. The saturated edges in the residual network after running the maximum flow algorithm correspond to edge disjoint paths between source and target in the original network. This function handles both directed and undirected graphs, and can use all flow algorithms from NetworkX flow package.

**node_disjoint_paths** *(G, s, t, flow_func=None, cutoff=None, auxiliary=None, residual=None)*

Computes node disjoint paths between source and target.

Node disjoint paths are paths that only share their first and last nodes. The number of node independent paths between two nodes is equal to their local node connectivity.

**Parameters**

- **G** *(NetworkX graph)*
- **s** *(node)* – Source node.
- **t** *(node)* – Target node.
- **flow_func** *(function)* – A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see `maximum_flow()` for details). If `flow_func` is None, the default maximum flow function.
edmonds_karp() is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.

- **cutoff** *(int)* – Maximum number of paths to yield. Some of the maximum flow algorithms, such as edmonds_karp() (the default) and shortest_augmenting_path() support the cutoff parameter, and will terminate when the flow value reaches or exceeds the cutoff. Other algorithms will ignore this parameter. Default value: None.

- **auxiliary** *(NetworkX DiGraph)* – Auxiliary digraph to compute flow based node connectivity. It has to have a graph attribute called mapping with a dictionary mapping node names in G and in the auxiliary digraph. If provided it will be reused instead of recreated. Default value: None.

- **residual** *(NetworkX DiGraph)* – Residual network to compute maximum flow. If provided it will be reused instead of recreated. Default value: None.

**Returns** paths – Generator of node disjoint paths.

**Return type** generator

**Raises**

- **NetworkXNoPath** – If there is no path between source and target.

- **NetworkXError** – If source or target are not in the graph G.

**Examples**

We use in this example the platonic icosahedral graph, which has node node connectivity 5, thus there are 5 node disjoint paths between any pair of non neighbor nodes.

```python
>>> G = nx.icosahedral_graph()
>>> len(list(nx.node_disjoint_paths(G, 0, 6)))
5
```

If you need to compute node disjoint paths between several pairs of nodes in the same graph, it is recommended that you reuse the data structures that NetworkX uses in the computation: the auxiliary digraph for node connectivity and node cuts, and the residual network for the underlying maximum flow computation.

Example of how to compute node disjoint paths reusing the data structures:

```python
>>> # You also have to explicitly import the function for
>>> # building the auxiliary digraph from the connectivity package
>>> from networkx.algorithms.connectivity import build_auxiliary_node_connectivity
>>> H = build_auxiliary_node_connectivity(G)
>>> # And the function for building the residual network from the
>>> # flow package
>>> from networkx.algorithms.flow import build_residual_network
>>> R = build_residual_network(H, 'capacity')
>>> # Reuse the auxiliary digraph and the residual network by passing them
>>> # as arguments
>>> len(list(nx.node_disjoint_paths(G, 0, 6, auxiliary=H, residual=R)))
5
```

You can also use alternative flow algorithms for computing node disjoint paths. For instance, in dense networks the algorithm shortest_augmenting_path() will usually perform better than the default edmonds_karp() which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.
```python
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> len(list(nx.node_disjoint_paths(G, 0, 6, flow_func=shortest_augmenting_path)))
5
```

**Notes**

This is a flow based implementation of node disjoint paths. We compute the maximum flow between source and target on an auxiliary directed network. The saturated edges in the residual network after running the maximum flow algorithm correspond to node disjoint paths between source and target in the original network. This function handles both directed and undirected graphs, and can use all flow algorithms from NetworkX flow package.

**See also:**

`edge_disjoint_paths()`, `node_connectivity()`, `maximum_flow()`, `edmonds_karp()`, `preflow_push()`, `shortest_augmenting_path()`

### 3.16.6 Flow-based Connectivity

Flow based connectivity algorithms

```python
average_node_connectivity(G[, flow_func])
```

Returns the average connectivity of a graph G.

```python
all_pairs_node_connectivity(G[, nbunch, ...])
```

Compute node connectivity between all pairs of nodes of G.

```python
edge_connectivity(G[, s, t, flow_func, cutoff])
```

Returns the edge connectivity of the graph or digraph G.

```python
local_edge_connectivity(G, s, t[, ...])
```

Returns local edge connectivity for nodes s and t in G.

```python
local_node_connectivity(G, s, t[, ...])
```

Computes local node connectivity for nodes s and t.

```python
node_connectivity(G[, s, t, flow_func])
```

Returns node connectivity for a graph or digraph G.

#### networkx.algorithms.connectivity.connectivity.average_node_connectivity

**average_node_connectivity** *(G, flow_func=None)*

Returns the average connectivity of a graph G.

The average connectivity \( \bar{\kappa}(G) \) of a graph G is the average of local node connectivity over all pairs of nodes of G.

\[
\bar{\kappa}(G) = \frac{\sum_{u,v} \kappa_G(u,v)}{\binom{n}{2}}
\]

**Parameters**

- **G** *(NetworkX graph)* – Undirected graph
- **flow_func** *(function)* – A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see `maximum_flow()` for details). If flow_func is None, the default maximum flow function (`edmonds_karp()`) is used. See `local_node_connectivity()` for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.


**Returns** K – Average node connectivity

**Return type** float

See also:

`local_node_connectivity()`, `node_connectivity()`, `edge_connectivity()`, `maximum_flow()`, `edmonds_karp()`, `preflow_push()`, `shortest_augmenting_path()`

References

networkx.algorithms.connectivity.connectivity.all_pairs_node_connectivity

**all_pairs_node_connectivity** \((G, nbunch=None, flow_func=None)\)

Compute node connectivity between all pairs of nodes of \(G\).

**Parameters**

- **G** *(NetworkX graph)* – Undirected graph
- **nbunch** *(container)* – Container of nodes. If provided node connectivity will be computed only over pairs of nodes in nbunch.
- **flow_func** *(function)* – A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see `maximum_flow()` for details). If flow_func is None, the default maximum flow function (`edmonds_karp()`) is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.

**Returns** all_pairs – A dictionary with node connectivity between all pairs of nodes in \(G\), or in nbunch if provided.

**Return type** dict

See also:

`local_node_connectivity()`, `edge_connectivity()`, `local_edge_connectivity()`, `maximum_flow()`, `edmonds_karp()`, `preflow_push()`, `shortest_augmenting_path()`

networkx.algorithms.connectivity.connectivity.edge_connectivity

**edge_connectivity** \((G, s=None, t=None, flow_func=None, cutoff=None)\)

Returns the edge connectivity of the graph or digraph \(G\).

The edge connectivity is equal to the minimum number of edges that must be removed to disconnect \(G\) or render it trivial. If source and target nodes are provided, this function returns the local edge connectivity: the minimum number of edges that must be removed to break all paths from source to target in \(G\).

**Parameters**

- **G** *(NetworkX graph)* – Undirected or directed graph
- **s** *(node)* – Source node. Optional. Default value: None.
- **t** *(node)* – Target node. Optional. Default value: None.
- **flow_func** *(function)* – A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see `maximum_flow()` for details). If flow_func is None, the default maximum flow function
(edmonds_karp()) is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.

- **cutoff** *(integer, float)* – If specified, the maximum flow algorithm will terminate when the flow value reaches or exceeds the cutoff. This is only for the algorithms that support the cutoff parameter: edmonds_karp() and shortest_augmenting_path(). Other algorithms will ignore this parameter. Default value: None.

**Returns** K – Edge connectivity for G, or local edge connectivity if source and target were provided

**Return type** integer

**Examples**

```python
>>> # Platonic icosahedral graph is 5-edge-connected
>>> G = nx.icosahedral_graph()
>>> nx.edge_connectivity(G)
5
```

You can use alternative flow algorithms for the underlying maximum flow computation. In dense networks the algorithm shortest_augmenting_path() will usually perform better than the default edmonds_karp(), which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

```python
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> nx.edge_connectivity(G, flow_func=shortest_augmenting_path)
5
```

If you specify a pair of nodes (source and target) as parameters, this function returns the value of local edge connectivity.

```python
>>> nx.edge_connectivity(G, 3, 7)
5
```

If you need to perform several local computations among different pairs of nodes on the same graph, it is recommended that you reuse the data structures used in the maximum flow computations. See local_edge_connectivity() for details.

**Notes**

This is a flow based implementation of global edge connectivity. For undirected graphs the algorithm works by finding a ‘small’ dominating set of nodes of \(G\) (see algorithm 7 in\(^1\)) and computing local maximum flow (see local_edge_connectivity()) between an arbitrary node in the dominating set and the rest of nodes in it. This is an implementation of algorithm 6 in\(^1\). For directed graphs, the algorithm does \(n\) calls to the maximum flow function. This is an implementation of algorithm 8 in\(^1\).

**See also:**

local_edge_connectivity(), local_node_connectivity(), node_connectivity(), maximum_flow(), edmonds_karp(), preflow_push(), shortest_augmenting_path(), k_edge_components(), k_edge_subgraphs()

\(^1\) Abdol-Hossein Esfahanian. Connectivity Algorithms. [http://www.cse.msu.edu/~cse835/Papers/Graph_connectivity_revised.pdf](http://www.cse.msu.edu/~cse835/Papers/Graph_connectivity_revised.pdf)
References

networkx.algorithms.connectivity.connectivity.local_edge_connectivity

local_edge_connectivity (G, s, t, flow_func=None, auxiliary=None, residual=None, cutoff=None)

Returns local edge connectivity for nodes s and t in G.

Local edge connectivity for two nodes s and t is the minimum number of edges that must be removed to disconnect them.

This is a flow based implementation of edge connectivity. We compute the maximum flow on an auxiliary digraph build from the original network (see below for details). This is equal to the local edge connectivity because the value of a maximum s-t-flow is equal to the capacity of a minimum s-t-cut (Ford and Fulkerson theorem)\(^1\).

Parameters

- G (NetworkX graph) – Undirected or directed graph
- s (node) – Source node
- t (node) – Target node
- flow_func (function) – A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see maximum_flow() for details). If flow_func is None, the default maximum flow function (edmonds_karp()) is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.
- auxiliary (NetworkX DiGraph) – Auxiliary digraph for computing flow based edge connectivity. If provided it will be reused instead of recreated. Default value: None.
- residual (NetworkX DiGraph) – Residual network to compute maximum flow. If provided it will be reused instead of recreated. Default value: None.
- cutoff (integer, float) – If specified, the maximum flow algorithm will terminate when the flow value reaches or exceeds the cutoff. This is only for the algorithms that support the cutoff parameter: edmonds_karp() and shortest_augmenting_path(). Other algorithms will ignore this parameter. Default value: None.

Returns K – local edge connectivity for nodes s and t.

Return type integer

Examples

This function is not imported in the base NetworkX namespace, so you have to explicitly import it from the connectivity package:

```python
>>> from networkx.algorithms.connectivity import local_edge_connectivity
```

We use in this example the platonic icosahedral graph, which has edge connectivity 5.

```python
>>> G = nx.icosahedral_graph()
>>> local_edge_connectivity(G, 0, 6)
5
```

If you need to compute local connectivity on several pairs of nodes in the same graph, it is recommended that you reuse the data structures that NetworkX uses in the computation: the auxiliary digraph for edge connectivity, and the residual network for the underlying maximum flow computation.

Example of how to compute local edge connectivity among all pairs of nodes of the platonic icosahedral graph reusing the data structures.

```python
>>> import itertools
>>> # You also have to explicitly import the function for
>>> # building the auxiliary digraph from the connectivity package
>>> from networkx.algorithms.connectivity import build_auxiliary_edge_connectivity
>>> H = build_auxiliary_edge_connectivity(G)
>>> # And the function for building the residual network from the
>>> # flow package
>>> from networkx.algorithms.flow import build_residual_network
>>> R = build_residual_network(H, 'capacity')
>>> for u, v in itertools.combinations(G, 2):
...    k = local_edge_connectivity(G, u, v, auxiliary=H, residual=R)
...    result[u][v] = k
>>> all(result[u][v] == 5 for u, v in itertools.combinations(G, 2))
True
```

You can also use alternative flow algorithms for computing edge connectivity. For instance, in dense networks the algorithm `shortest_augmenting_path()` will usually perform better than the default `edmonds_karp()` which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

```python
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> local_edge_connectivity(G, 0, 6, flow_func=shortest_augmenting_path)
5
```

**Notes**

This is a flow based implementation of edge connectivity. We compute the maximum flow using, by default, the `edmonds_karp()` algorithm on an auxiliary digraph build from the original input graph:

If the input graph is undirected, we replace each edge (u,'v') with two reciprocal arcs (u, v) and (v, u) and then we set the attribute 'capacity' for each arc to 1. If the input graph is directed we simply add the 'capacity' attribute. This is an implementation of algorithm 1 in

The maximum flow in the auxiliary network is equal to the local edge connectivity because the value of a maximum s-t-flow is equal to the capacity of a minimum s-t-cut (Ford and Fulkerson theorem).

See also:

`edge_connectivity()`, `local_node_connectivity()`, `node_connectivity()`, `maximum_flow()`, `edmonds_karp()`, `preflow_push()`, `shortest_augmenting_path()`
local_node_connectivity \( (G, s, t, \text{flow\_func}=\text{None}, \text{auxiliary}=\text{None}, \text{residual}=\text{None}, \text{cutoff}=\text{None}) \)

Computes local node connectivity for nodes s and t.

Local node connectivity for two non adjacent nodes s and t is the minimum number of nodes that must be removed (along with their incident edges) to disconnect them.

This is a flow based implementation of node connectivity. We compute the maximum flow on an auxiliary digraph build from the original input graph (see below for details).

Parameters

- **G** (*NetworkX graph*) – Undirected graph
- **s** (*node*) – Source node
- **t** (*node*) – Target node
- **flow\_func** (*function*) – A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see maximum\_flow() for details). If flow\_func is None, the default maximum flow function (edmonds\_karp()) is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.
- **auxiliary** (*NetworkX DiGraph*) – Auxiliary digraph to compute flow based node connectivity. It has to have a graph attribute called mapping with a dictionary mapping node names in G and in the auxiliary digraph. If provided it will be reused instead of recreated. Default value: None.
- **residual** (*NetworkX DiGraph*) – Residual network to compute maximum flow. If provided it will be reused instead of recreated. Default value: None.
- **cutoff** (*integer, float*) – If specified, the maximum flow algorithm will terminate when the flow value reaches or exceeds the cutoff. This is only for the algorithms that support the cutoff parameter: edmonds\_karp() and shortest\_augmenting\_path(). Other algorithms will ignore this parameter. Default value: None.

Returns **K** – local node connectivity for nodes s and t

Return type integer

Examples

This function is not imported in the base NetworkX namespace, so you have to explicitly import it from the connectivity package:

```python
>>> from networkx.algorithms.connectivity import local_node_connectivity
```

We use in this example the platonic icosahedral graph, which has node connectivity 5.

```python
>>> G = nx.icosahedral_graph()
>>> local_node_connectivity(G, 0, 6)
5
```
If you need to compute local connectivity on several pairs of nodes in the same graph, it is recommended that you reuse the data structures that NetworkX uses in the computation: the auxiliary digraph for node connectivity, and the residual network for the underlying maximum flow computation.

Example of how to compute local node connectivity among all pairs of nodes of the platonic icosahedral graph reusing the data structures.

```python
>>> import itertools

>>> # You also have to explicitly import the function for
>>> # building the auxiliary digraph from the connectivity package
>>> from networkx.algorithms.connectivity import (...
   build_auxiliary_node_connectivity)
...
>>> H = build_auxiliary_node_connectivity(G)
>>> # And the function for building the residual network from the
>>> # flow package
>>> from networkx.algorithms.flow import build_residual_network
>>> # Note that the auxiliary digraph has an edge attribute named capacity
>>> R = build_residual_network(H, 'capacity')
>>> result = dict.fromkeys(G, dict())
>>> # Reuse the auxiliary digraph and the residual network by passing them
>>> # as parameters
>>> for u, v in itertools.combinations(G, 2):
   ... k = local_node_connectivity(G, u, v, auxiliary=H, residual=R)
   ...
   ... result[u][v] = k
   ...
>>> all(result[u][v] == 5 for u, v in itertools.combinations(G, 2))
True
```

You can also use alternative flow algorithms for computing node connectivity. For instance, in dense networks the algorithm `shortest_augmenting_path()` will usually perform better than the default `edmonds_karp()` which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

```python
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> local_node_connectivity(G, 0, 6, flow_func=shortest_augmenting_path)
5
```

**Notes**

This is a flow based implementation of node connectivity. We compute the maximum flow using, by default, the `edmonds_karp()` algorithm (see: `maximum_flow()`) on an auxiliary digraph build from the original input graph:

For an undirected graph G having n nodes and m edges we derive a directed graph H with 2n nodes and 2m+n arcs by replacing each original node v with two nodes v_A, v_B linked by an (internal) arc in H. Then for each edge (u, v) in G we add two arcs (u_B, v_A) and (v_B, u_A) in H. Finally we set the attribute capacity = 1 for each arc in H\(^1\).

For a directed graph G having n nodes and m arcs we derive a directed graph H with 2n nodes and m+n arcs by replacing each original node v with two nodes v_A, v_B linked by an (internal) arc (v_A, v_B) in H. Then for each arc (u, v) in G we add one arc (u_B, v_A) in H. Finally we set the attribute capacity = 1 for each arc in H.

This is equal to the local node connectivity because the value of a maximum s-t-flow is equal to the capacity of a minimum s-t-cut.

See also:

local_edge_connectivity(), node_connectivity(), minimum_node_cut(), maximum_flow(), edmonds_karp(), preflow_push(), shortest_augmenting_path()

References

networkx.algorithms.connectivity.connectivity.node_connectivity

def node_connectivity(G, s=None, t=None, flow_func=None):

    Returns node connectivity for a graph or digraph G.

    Node connectivity is equal to the minimum number of nodes that must be removed to disconnect G or render it trivial. If source and target nodes are provided, this function returns the local node connectivity: the minimum number of nodes that must be removed to break all paths from source to target in G.

    Parameters

    • G (NetworkX graph) – Undirected graph
    • s (node) – Source node. Optional. Default value: None.
    • t (node) – Target node. Optional. Default value: None.
    • flow_func (function) – A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see maximum_flow() for details). If flow_func is None, the default maximum flow function (edmonds_karp()) is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.

    Returns K – Node connectivity of G, or local node connectivity if source and target are provided.

    Return type integer

Examples

>>> # Platonic icosahedral graph is 5-node-connected
>>> G = nx.icosahedral_graph()
>>> nx.node_connectivity(G)
5

You can use alternative flow algorithms for the underlying maximum flow computation. In dense networks the algorithm shortest_augmenting_path() will usually perform better than the default edmonds_karp(), which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> nx.node_connectivity(G, flow_func=shortest_augmenting_path)
5

If you specify a pair of nodes (source and target) as parameters, this function returns the value of local node connectivity.
If you need to perform several local computations among different pairs of nodes on the same graph, it is recommended that you reuse the data structures used in the maximum flow computations. See `local_node_connectivity()` for details.

**Notes**

This is a flow based implementation of node connectivity. The algorithm works by solving $O((n - \delta - 1 + \delta(\delta - 1)/2))$ maximum flow problems on an auxiliary digraph. Where $\delta$ is the minimum degree of G. For details about the auxiliary digraph and the computation of local node connectivity see `local_node_connectivity()`. This implementation is based on algorithm 11 in\(^1\).

See also:

`local_node_connectivity()`, `edge_connectivity()`, `maximum_flow()`, `edmonds_karp()`, `preflow_push()`, `shortest_augmenting_path()`

**References**

3.16.7 Flow-based Minimum Cuts

Flow based cut algorithms

<table>
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<tr>
<th>Function</th>
<th>Description</th>
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<tr>
<td><code>minimum_edge_cut(G, s, t, flow_func)</code></td>
<td>Returns a set of edges of minimum cardinality that disconnects G.</td>
</tr>
<tr>
<td><code>minimum_node_cut(G, s, t, flow_func)</code></td>
<td>Returns a set of nodes of minimum cardinality that disconnects G.</td>
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<tr>
<td><code>minimum_st_edge_cut(G, s, t[, flow_func, ...])</code></td>
<td>Returns the edges of the cut-set of a minimum (s, t)-cut.</td>
</tr>
<tr>
<td><code>minimum_st_node_cut(G, s, t[, flow_func, ...])</code></td>
<td>Returns a set of nodes of minimum cardinality that disconnect source from target in G.</td>
</tr>
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`networkx.algorithms.connectivity.cuts.minimum_edge_cut`

`minimum_edge_cut(G, s=None, t=None, flow_func=None)`

Returns a set of edges of minimum cardinality that disconnects G.

If source and target nodes are provided, this function returns the set of edges of minimum cardinality that, if removed, would break all paths among source and target in G. If not, it returns a set of edges of minimum cardinality that disconnects G.

**Parameters**

- G (NetworkX graph)
- s (node) – Source node. Optional. Default value: None.
- t (node) – Target node. Optional. Default value: None.
- flow_func (function) – A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions

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\(^1\) Abdol-Hossein Esfahanian. Connectivity Algorithms. [http://www.cse.msu.edu/~cse835/Papers/Graph_connectivity_revised.pdf](http://www.cse.msu.edu/~cse835/Papers/Graph_connectivity_revised.pdf)
maximum_flow() for details). If flow_func is None, the default maximum flow function (edmonds_karp()) is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.

Returns cutset – Set of edges that, if removed, would disconnect G. If source and target nodes are provided, the set contains the edges that if removed, would destroy all paths between source and target.

Return type set

Examples

```python
>>> # Platonic icosahedral graph has edge connectivity 5
>>> G = nx.icosahedral_graph()
>>> len(nx.minimum_edge_cut(G))
5
```

You can use alternative flow algorithms for the underlying maximum flow computation. In dense networks the algorithm shortest_augmenting_path() will usually perform better than the default edmonds_karp(), which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

```python
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> len(nx.minimum_edge_cut(G, flow_func=shortest_augmenting_path))
5
```

If you specify a pair of nodes (source and target) as parameters, this function returns the value of local edge connectivity.

```python
>>> nx.edge_connectivity(G, 3, 7)
5
```

If you need to perform several local computations among different pairs of nodes on the same graph, it is recommended that you reuse the data structures used in the maximum flow computations. See local_edge_connectivity() for details.

Notes

This is a flow based implementation of minimum edge cut. For undirected graphs the algorithm works by finding a ‘small’ dominating set of nodes of G (see algorithm 7 in\(^1\)) and computing the maximum flow between an arbitrary node in the dominating set and the rest of nodes in it. This is an implementation of algorithm 6 in\(^1\). For directed graphs, the algorithm does \(n\) calls to the max flow function. The function raises an error if the directed graph is not weakly connected and returns an empty set if it is weakly connected. It is an implementation of algorithm 8 in\(^1\).

See also:

minimum_st_edge_cut(), minimum_node_cut(), stoer_wagner(), node_connectivity(), edge_connectivity(), maximum_flow(), edmonds_karp(), preflow_push(), shortest_augmenting_path()

---

\(^1\) Abdol-Hossein Esfahanian. Connectivity Algorithms. [http://www.cse.msu.edu/~cse835/Papers/Graph_connectivity_revised.pdf](http://www.cse.msu.edu/~cse835/Papers/Graph_connectivity_revised.pdf)
References

networkx.algorithms.connectivity.cuts.minimum_node_cut

minimum_node_cut(G, s=None, t=None, flow_func=None)

Returns a set of nodes of minimum cardinality that disconnects G.

If source and target nodes are provided, this function returns the set of nodes of minimum cardinality that, if removed, would destroy all paths among source and target in G. If not, it returns a set of nodes of minimum cardinality that disconnects G.

Parameters

- G (NetworkX graph)
- s (node) – Source node. Optional. Default value: None.
- t (node) – Target node. Optional. Default value: None.
- flow_func (function) – A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see maximum_flow() for details). If flow_func is None, the default maximum flow function (edmonds_karp()) is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.

Returns cutset – Set of nodes that, if removed, would disconnect G. If source and target nodes are provided, the set contains the nodes that if removed, would destroy all paths between source and target.

Return type set

Examples

```python
>>> # Platonic icosahedral graph has node connectivity 5
>>> G = nx.icosahedral_graph()
>>> node_cut = nx.minimum_node_cut(G)
>>> len(node_cut)
5
```

You can use alternative flow algorithms for the underlying maximum flow computation. In dense networks the algorithm shortest_augmenting_path() will usually perform better than the default edmonds_karp(), which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

```python
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> node_cut == nx.minimum_node_cut(G, flow_func=shortest_augmenting_path)
True
```

If you specify a pair of nodes (source and target) as parameters, this function returns a local st node cut.

```python
>>> len(nx.minimum_node_cut(G, 3, 7))
5
```

If you need to perform several local st cuts among different pairs of nodes on the same graph, it is recommended that you reuse the data structures used in the maximum flow computations. See minimum_st_node_cut() for details.
Notes

This is a flow based implementation of minimum node cut. The algorithm is based in solving a number of maximum flow computations to determine the capacity of the minimum cut on an auxiliary directed network that corresponds to the minimum node cut of G. It handles both directed and undirected graphs. This implementation is based on algorithm 11 in 1.

See also:

minimum_st_node_cut(), minimum_cut(), minimum_edge_cut(), stoer_wagner(), node_connectivity(), edge_connectivity(), maximum_flow(), edmonds_karp(), preflow_push(), shortest_augmenting_path()

References

networkx.algorithms.connectivity.cuts.minimum_st_edge_cut

minimum_st_edge_cut (G, s, t, flow_func=None, auxiliary=None, residual=None)

Returns the edges of the cut-set of a minimum (s, t)-cut.

This function returns the set of edges of minimum cardinality that, if removed, would destroy all paths among source and target in G. Edge weights are not considered. See minimum_cut() for computing minimum cuts considering edge weights.

Parameters

- G (NetworkX graph)
- s (node) – Source node for the flow.
- t (node) – Sink node for the flow.
- auxiliary (NetworkX DiGraph) – Auxiliary digraph to compute flow based node connectivity. It has to have a graph attribute called mapping with a dictionary mapping node names in G and in the auxiliary digraph. If provided it will be reused instead of recreated. Default value: None.
- flow_func (function) – A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see maximum_flow() for details). If flow_func is None, the default maximum flow function (edmonds_karp()) is used. See node_connectivity() for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.
- residual (NetworkX DiGraph) – Residual network to compute maximum flow. If provided it will be reused instead of recreated. Default value: None.

Returns cutset – Set of edges that, if removed from the graph, will disconnect it.

Return type set

See also:

minimum_cut(), minimum_node_cut(), minimum_edge_cut(), stoer_wagner(), node_connectivity(), edge_connectivity(), maximum_flow(), edmonds_karp(), preflow_push(), shortest_augmenting_path()

Examples

This function is not imported in the base NetworkX namespace, so you have to explicitly import it from the connectivity package:

```python
>>> from networkx.algorithms.connectivity import minimum_st_edge_cut
```

We use in this example the platonic icosahedral graph, which has edge connectivity 5.

```python
>>> G = nx.icosahedral_graph()
>>> len(minimum_st_edge_cut(G, 0, 6))
5
```

If you need to compute local edge cuts on several pairs of nodes in the same graph, it is recommended that you reuse the data structures that NetworkX uses in the computation: the auxiliary digraph for edge connectivity, and the residual network for the underlying maximum flow computation.

Example of how to compute local edge cuts among all pairs of nodes of the platonic icosahedral graph reusing the data structures.

```python
>>> import itertools

>>> # You also have to explicitly import the function for
>>> # building the auxiliary digraph from the connectivity package
>>> from networkx.algorithms.connectivity import (...

...    build_auxiliary_edge_connectivity)
>>> H = build_auxiliary_edge_connectivity(G)
>>> # And the function for building the residual network from the
>>> # flow package
>>> from networkx.algorithms.flow import build_residual_network

>>> R = build_residual_network(H, 'capacity')
>>> result = dict.fromkeys(G, dict())

>>> # Reuse the auxiliary digraph and the residual network by passing them
>>> # as parameters
>>> for u, v in itertools.combinations(G, 2):
...    k = len(minimum_st_edge_cut(G, u, v, auxiliary=H, residual=R))
...    result[u][v] = k
>>> all(result[u][v] == 5 for u, v in itertools.combinations(G, 2))
True
```

You can also use alternative flow algorithms for computing edge cuts. For instance, in dense networks the algorithm `shortest_augmenting_path()` will usually perform better than the default `edmonds_karp()` which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

```python
>>> from networkx.algorithms.flow import shortest_augmenting_path

>>> len(minimum_st_edge_cut(G, 0, 6, flow_func=shortest_augmenting_path))
5
```

networkx.algorithms.connectivity.cuts.minimum_st_node_cut

```python
minimum_st_node_cut(G, s, t, flow_func=None, auxiliary=None, residual=None)
```

Returns a set of nodes of minimum cardinality that disconnect source from target in G.

This function returns the set of nodes of minimum cardinality that, if removed, would destroy all paths among source and target in G.
Parameters

- **G** (*NetworkX graph*)
- **s** (*node*) – Source node.
- **t** (*node*) – Target node.
- **flow_func** (*function*) – A function for computing the maximum flow among a pair of nodes. The function has to accept at least three parameters: a Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see `maximum_flow()` for details). If `flow_func` is None, the default maximum flow function (`edmonds_karp()`) is used. See below for details. The choice of the default function may change from version to version and should not be relied on. Default value: None.
- **auxiliary** (*NetworkX DiGraph*) – Auxiliary digraph to compute flow based node connectivity. It has to have a graph attribute called mapping with a dictionary mapping node names in G and in the auxiliary digraph. If provided it will be reused instead of recreated. Default value: None.
- **residual** (*NetworkX DiGraph*) – Residual network to compute maximum flow. If provided it will be reused instead of recreated. Default value: None.

**Returns**
- **cutset** – Set of nodes that, if removed, would destroy all paths between source and target in G.

**Return type** `set`

**Examples**

This function is not imported in the base NetworkX namespace, so you have to explicitly import it from the connectivity package:

```python
>>> from networkx.algorithms.connectivity import minimum_st_node_cut
```

We use in this example the platonic icosahedral graph, which has node connectivity 5.

```python
>>> G = nx.icosahedral_graph()
>>> len(minimum_st_node_cut(G, 0, 6))
5
```

If you need to compute local st cuts between several pairs of nodes in the same graph, it is recommended that you reuse the data structures that NetworkX uses in the computation: the auxiliary digraph for node connectivity and node cuts, and the residual network for the underlying maximum flow computation.

Example of how to compute local st node cuts reusing the data structures:

```python
>>> # You also have to explicitly import the function for building the auxiliary digraph from the connectivity package
>>> from networkx.algorithms.connectivity import (...
...    build_auxiliary_node_connectivity)
>>> H = build_auxiliary_node_connectivity(G)
>>> # And the function for building the residual network from the flow package
>>> from networkx.algorithms.flow import build_residual_network
>>> R = build_residual_network(H, 'capacity')
>>> # Reuse the auxiliary digraph and the residual network by passing them as parameters
```
You can also use alternative flow algorithms for computing minimum st node cuts. For instance, in dense networks the algorithm `shortest_augmenting_path()` will usually perform better than the default `edmonds_karp()` which is faster for sparse networks with highly skewed degree distributions. Alternative flow functions have to be explicitly imported from the flow package.

```python
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> len(minimum_st_node_cut(G, 0, 6, flow_func=shortest_augmenting_path))
5
```

**Notes**

This is a flow based implementation of minimum node cut. The algorithm is based in solving a number of maximum flow computations to determine the capacity of the minimum cut on an auxiliary directed network that corresponds to the minimum node cut of G. It handles both directed and undirected graphs. This implementation is based on algorithm 11 in\(^1\).

**See also:**

- `minimum_node_cut()`, `minimum_edge_cut()`, `stoer_wagner()`, `node_connectivity()`, `edge_connectivity()`, `maximum_flow()`, `edmonds_karp()`, `preflow_push()`, `shortest_augmenting_path()`

**References**

3.16.8 **Stoer-Wagner minimum cut**

Stoer-Wagner minimum cut algorithm.

```python
stoer_wagner(G[, weight, heap])
```

Returns the weighted minimum edge cut using the Stoer-Wagner algorithm.

**Parameters**

1. Abdol-Hossein Esfahanian. Connectivity Algorithms. [http://www.cse.msu.edu/~cse835/Papers/Graph_connectivity_revised.pdf](http://www.cse.msu.edu/~cse835/Papers/Graph_connectivity_revised.pdf)
• **G** (*NetworkX graph*) – Edges of the graph are expected to have an attribute named by the weight parameter below. If this attribute is not present, the edge is considered to have unit weight.

• **weight** (*string*) – Name of the weight attribute of the edges. If the attribute is not present, unit weight is assumed. Default value: ‘weight’.

• **heap** (*class*) – Type of heap to be used in the algorithm. It should be a subclass of `MinHeap` or implement a compatible interface.

If a stock heap implementation is to be used, `BinaryHeap` is recommended over `PairingHeap` for Python implementations without optimized attribute accesses (e.g., CPython) despite a slower asymptotic running time. For Python implementations with optimized attribute accesses (e.g., PyPy), `PairingHeap` provides better performance. Default value: `BinaryHeap`.

**Returns**

• **cut_value** (*integer or float*) – The sum of weights of edges in a minimum cut.

• **partition** (*pair of node lists*) – A partitioning of the nodes that defines a minimum cut.

**Raises**

• **NetworkXNotImplemented** – If the graph is directed or a multigraph.

• **NetworkXError** – If the graph has less than two nodes, is not connected or has a negative-weighted edge.

**Examples**

```python
>>> G = nx.Graph()
>>> G.add_edge('x', 'a', weight=3)
>>> G.add_edge('x', 'b', weight=1)
>>> G.add_edge('a', 'c', weight=3)
>>> G.add_edge('b', 'c', weight=5)
>>> G.add_edge('b', 'd', weight=4)
>>> G.add_edge('d', 'e', weight=2)
>>> G.add_edge('c', 'y', weight=2)
>>> G.add_edge('e', 'y', weight=3)
>>> cut_value, partition = nx.stoer_wagner(G)
>>> cut_value
4
```

### 3.16.9 Utilities for flow-based connectivity

Utilities for connectivity package

<table>
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<tr>
<th>Function</th>
<th>Description</th>
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<td><code>build_auxiliary_edge_connectivity(G)</code></td>
<td>Auxiliary digraph for computing flow based edge connectivity</td>
</tr>
<tr>
<td><code>build_auxiliary_node_connectivity(G)</code></td>
<td>Creates a directed graph D from an undirected graph G to compute flow based node connectivity.</td>
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</table>
networkx.algorithms.connectivity.utils.build_auxiliary_edge_connectivity

`build_auxiliary_edge_connectivity(G)`

Auxiliary digraph for computing flow based edge connectivity

If the input graph is undirected, we replace each edge \((u, v)\) with two reciprocal arcs \((u, v)\) and \((v, u)\) and then we set the attribute ‘capacity’ for each arc to 1. If the input graph is directed we simply add the ‘capacity’ attribute. Part of algorithm 1 in\(^1\).

References

networkx.algorithms.connectivity.utils.build_auxiliary_node_connectivity

`build_auxiliary_node_connectivity(G)`

Creates a directed graph \(D\) from an undirected graph \(G\) to compute flow based node connectivity.

For an undirected graph \(G\) having \(n\) nodes and \(m\) edges we derive a directed graph \(D\) with \(2n\) nodes and \(2m+n\) arcs by replacing each original node \(v\) with two nodes \(v_A, v_B\) linked by an (internal) arc in \(D\). Then for each edge \((u, v)\) in \(G\) we add two arcs \((u_B, v_A)\) and \((v_B, u_A)\) in \(D\). Finally we set the attribute capacity = 1 for each arc in \(D\).

For a directed graph having \(n\) nodes and \(m\) arcs we derive a directed graph \(D\) with \(2n\) nodes and \(m+n\) arcs by replacing each original node \(v\) with two nodes \(v_A, v_B\) linked by an (internal) arc \((v_A, v_B)\) in \(D\). Then for each arc \((u, v)\) in \(G\) we add one arc \((u_B, v_A)\) in \(D\). Finally we set the attribute capacity = 1 for each arc in \(D\).

A dictionary with a mapping between nodes in the original graph and the auxiliary digraph is stored as a graph attribute: \(H\.\text{graph}[\text{‘mapping’}]\).

References

3.17 Cores

Find the k-cores of a graph.

The k-core is found by recursively pruning nodes with degrees less than k.

See the following references for details:


For directed graphs a more general notion is that of D-cores which looks at \((k, l)\) restrictions on (in, out) degree. The \((k, k)\) D-core is the k-core.


Multi-scale structure and topological anomaly detection via a new network statistic: The onion decomposition L. Hébert-Dufresne, J. A. Grochow, and A. Allard Scientific Reports 6, 31708 (2016) http://doi.org/10.1038/srep31708

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\(^1\) Abdol-Hossein Esfahanian. Connectivity Algorithms. (this is a chapter, look for the reference of the book). http://www.cse.msu.edu/~cse835/Papers/Graph_connectivity_revised.pdf

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<table>
<thead>
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<tr>
<td><code>core_number(G)</code></td>
<td>Returns the core number for each vertex.</td>
</tr>
<tr>
<td><code>k_core(G[, k, core_number])</code></td>
<td>Returns the k-core of G.</td>
</tr>
<tr>
<td><code>k_shell(G[, k, core_number])</code></td>
<td>Returns the k-shell of G.</td>
</tr>
<tr>
<td><code>k_crust(G[, k, core_number])</code></td>
<td>Returns the k-crust of G.</td>
</tr>
<tr>
<td><code>k_corona(G, k[, core_number])</code></td>
<td>Returns the k-corona of G.</td>
</tr>
<tr>
<td><code>k_truss(G, k)</code></td>
<td>Returns the k-truss of G.</td>
</tr>
<tr>
<td><code>onion_layers(G)</code></td>
<td>Returns the layer of each vertex in an onion decomposition of the graph.</td>
</tr>
</tbody>
</table>

### 3.17.1 networkx.algorithms.core.core_number

**core_number** *(G)*  
Returns the core number for each vertex.

A k-core is a maximal subgraph that contains nodes of degree k or more.

The core number of a node is the largest value k of a k-core containing that node.

**Parameters**
- `G` *(NetworkX graph)* – A graph or directed graph

**Returns**
- `core_number` – A dictionary keyed by node to the core number.

**Return type**
- dictionary

**Raises** `NetworkXError` – The k-core is not implemented for graphs with self loops or parallel edges.

**Notes**

Not implemented for graphs with parallel edges or self loops.

For directed graphs the node degree is defined to be the in-degree + out-degree.

**References**

### 3.17.2 networkx.algorithms.core.k_core

**k_core** *(G, k==None, core_number=None)*  
Returns the k-core of G.

A k-core is a maximal subgraph that contains nodes of degree k or more.

**Parameters**
- `G` *(NetworkX graph)* – A graph or directed graph
- `k` *(int, optional)* – The order of the core. If not specified return the main core.
- `core_number` *(dictionary, optional)* – Precomputed core numbers for the graph G.

**Returns**
- `G` – The k-core subgraph

**Return type**
- NetworkX graph

**Raises** `NetworkXError` – The k-core is not defined for graphs with self loops or parallel edges.
Notes
The main core is the core with the largest degree.
Not implemented for graphs with parallel edges or self loops.
For directed graphs the node degree is defined to be the in-degree + out-degree.
Graph, node, and edge attributes are copied to the subgraph.

See also:
core_number()

References
3.17.3 networkx.algorithms.core.k_shell

k_shell \((G, k=None, \text{core}\_\text{number}=\text{None})\)
Returns the k-shell of G.
The k-shell is the subgraph induced by nodes with core number k. That is, nodes in the k-core that are not in the (k+1)-core.

Parameters
- G (NetworkX graph) – A graph or directed graph.
- k (int, optional) – The order of the shell. If not specified return the outer shell.
- core_number (dictionary, optional) – Precomputed core numbers for the graph G.

Returns G – The k-shell subgraph

Return type NetworkX graph

Raises NetworkXError – The k-shell is not implemented for graphs with self loops or parallel edges.

Notes
This is similar to k_corona but in that case only neighbors in the k-core are considered.
Not implemented for graphs with parallel edges or self loops.
For directed graphs the node degree is defined to be the in-degree + out-degree.
Graph, node, and edge attributes are copied to the subgraph.

See also:
core_number(), k_corona()

References
3.17.4 networkx.algorithms.core.k_crust

k_crust \((G, k=None, \text{core}\_\text{number}=\text{None})\)
Returns the k-crust of G.
The k-crust is the graph G with the k-core removed.
Parameters

- \( G \) (NetworkX graph) – A graph or directed graph.
- \( k \) (int, optional) – The order of the shell. If not specified return the main crust.
- \( \text{core_number} \) (dictionary, optional) – Precomputed core numbers for the graph \( G \).

Returns \( G \) – The k-crust subgraph

Return type NetworkX graph

Raises \( \text{NetworkXError} \) – The k-crust is not implemented for graphs with self loops or parallel edges.

Notes

This definition of k-crust is different than the definition in\(^1 \). The k-crust in\(^1 \) is equivalent to the k+1 crust of this algorithm.

Not implemented for graphs with parallel edges or self loops.

For directed graphs the node degree is defined to be the in-degree + out-degree.

Graph, node, and edge attributes are copied to the subgraph.

See also:

\( \text{core_number}() \)

References

3.17.5 networkx.algorithms.core.k_corona

\( \text{k_corona}(G, k, \text{core_number}=\text{None}) \)

Returns the k-corona of \( G \).

The k-corona is the subgraph of nodes in the k-core which have exactly k neighbours in the k-core.

Parameters

- \( G \) (NetworkX graph) – A graph or directed graph
- \( k \) (int) – The order of the corona.
- \( \text{core_number} \) (dictionary, optional) – Precomputed core numbers for the graph \( G \).

Returns \( G \) – The k-corona subgraph

Return type NetworkX graph

Raises \( \text{NetworkXError} \) – The k-corona is not defined for graphs with self loops or parallel edges.

Notes

Not implemented for graphs with parallel edges or self loops.

For directed graphs the node degree is defined to be the in-degree + out-degree.

Graph, node, and edge attributes are copied to the subgraph.

---

\(^1\) A model of Internet topology using k-shell decomposition Shai Carmi, Shlomo Havlin, Scott Kirkpatrick, Yuval Shavitt, and Eran Shir, PNAS July 3, 2007 vol. 104 no. 27 11150-11154 http://www.pnas.org/content/104/27/11150.full
See also:

`core_number()`

References

### 3.17.6 networkx.algorithms.core.k_truss

**k_truss** ($G, k$)

Returns the k-truss of $G$.

The k-truss is the maximal induced subgraph of $G$ which contains at least three vertices where every edge is incident to at least $k-2$ triangles.

**Parameters**

- $G$ (*NetworkX graph*) – An undirected graph
- $k$ (*int*) – The order of the truss

**Returns** $H$ – The k-truss subgraph

**Return type** NetworkX graph

**Raises** `NetworkXError` – The k-truss is not defined for graphs with self loops or parallel edges or directed graphs.

**Notes**

A k-clique is a (k-2)-truss and a k-truss is a (k+1)-core.

Not implemented for digraphs or graphs with parallel edges or self loops.

Graph, node, and edge attributes are copied to the subgraph.

K-trusses were originally defined in [2] which states that the k-truss is the maximal induced subgraph where each edge belongs to at least $k-2$ triangles. A more recent paper, [1], uses a slightly different definition requiring that each edge belong to at least $k$ triangles. This implementation uses the original definition of $k-2$ triangles.

**References**

### 3.17.7 networkx.algorithms.core.onion_layers

**onion_layers** ($G$)

Returns the layer of each vertex in an onion decomposition of the graph.

The onion decomposition refines the k-core decomposition by providing information on the internal organization of each k-shell. It is usually used alongside the core numbers.

**Parameters** $G$ (*NetworkX graph*) – A simple graph without self loops or parallel edges

**Returns** `od_layers` – A dictionary keyed by vertex to the onion layer. The layers are contiguous integers starting at 1.

**Return type** dictionary

**Raises** `NetworkXError` – The onion decomposition is not implemented for graphs with self loops or parallel edges or for directed graphs.
Notes

Not implemented for graphs with parallel edges or self loops.
Not implemented for directed graphs.

See also:

core_number()

References

3.18 Covering

Functions related to graph covers.

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<th>Function</th>
<th>Description</th>
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<td>min_edge_cover(G[, matching_algorithm])</td>
<td>Returns a set of edges which constitutes the minimum edge cover of the graph.</td>
</tr>
<tr>
<td>is_edge_cover(G, cover)</td>
<td>Decides whether a set of edges is a valid edge cover of the graph.</td>
</tr>
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</table>

### 3.18.1 networkx.algorithms.covering.min_edge_cover

**min_edge_cover** *(G, matching_algorithm=None)*

Returns a set of edges which constitutes the minimum edge cover of the graph.

A smallest edge cover can be found in polynomial time by finding a maximum matching and extending it greedily so that all nodes are covered.

**Parameters**

- **G** *(NetworkX graph)* – An undirected bipartite graph.
- **matching_algorithm** *(function)* – A function that returns a maximum cardinality matching in a given bipartite graph. The function must take one input, the graph G, and return a dictionary mapping each node to its mate. If not specified, hopcroft_karp_matching() will be used. Other possibilities include eppstein_matching(), or matching algorithms in the networkx.algorithms.matching module.

**Returns** **min_cover** – It contains all the edges of minimum edge cover in form of tuples. It contains both the edges *(u, v)* and *(v, u)* for given nodes *u* and *v* among the edges of minimum edge cover.

**Return type** **set**

**Notes**

An edge cover of a graph is a set of edges such that every node of the graph is incident to at least one edge of the set. The minimum edge cover is an edge covering of smallest cardinality.

Due to its implementation, the worst-case running time of this algorithm is bounded by the worst-case running time of the function matching_algorithm.

Minimum edge cover for bipartite graph can also be found using the function present in networkx.algorithms.bipartite.covering
3.18.2 networkx.algorithms.covering.is_edge_cover

is_edge_cover (G, cover)

Decides whether a set of edges is a valid edge cover of the graph.

Given a set of edges, whether it is an edge covering can be decided if we just check whether all nodes of the
graph has an edge from the set, incident on it.

Parameters

- G (NetworkX graph) – An undirected bipartite graph.
- cover (set) – Set of edges to be checked.

Returns

Whether the set of edges is a valid edge cover of the graph.

Return type bool

Notes

An edge cover of a graph is a set of edges such that every node of the graph is incident to at least one edge of
the set.

3.19 Cycles

3.19.1 Cycle finding algorithms

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<td>cycle_basis(G[, root])</td>
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<td>minimum_cycle_basis(G[, weight])</td>
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3.19.2 networkx.algorithms.cycles.cycle_basis

cycle_basis (G, root=None)

Returns a list of cycles which form a basis for cycles of G.

A basis for cycles of a network is a minimal collection of cycles such that any cycle in the network can be written
as a sum of cycles in the basis. Here summation of cycles is defined as “exclusive or” of the edges. Cycle bases
are useful, e.g. when deriving equations for electric circuits using Kirchhoff’s Laws.

Parameters

- G (NetworkX Graph)
- root (node, optional) – Specify starting node for basis.

Returns

- A list of cycle lists. Each cycle list is a list of nodes
- which forms a cycle (loop) in G.
Examples

```python
>>> G = nx.Graph()
>>> nx.add_cycle(G, [0, 1, 2, 3])
>>> nx.add_cycle(G, [0, 3, 4, 5])
>>> print(nx.cycle_basis(G, 0))
[[3, 4, 5, 0], [1, 2, 3, 0]]
```

Notes

This is adapted from algorithm CACM 491\(^1\).

References

See also:

`simple_cycles()`

### 3.19.3 networkx.algorithms.cycles.simple_cycles

**simple_cycles(G)**

Find simple cycles (elementary circuits) of a directed graph.

A simple cycle, or elementary circuit, is a closed path where no node appears twice. Two elementary circuits are distinct if they are not cyclic permutations of each other.

This is a nonrecursive, iterator/generator version of Johnson’s algorithm\(^1\). There may be better algorithms for some cases\(^2\).

**Parameters**

- **G (NetworkX DiGraph)** – A directed graph

**Returns**

- **cycle_generator** – A generator that produces elementary cycles of the graph. Each cycle is represented by a list of nodes along the cycle.

**Return type**

generator

Examples

```python
>>> edges = [(0, 0), (0, 1), (0, 2), (1, 2), (2, 0), (2, 1), (2, 2)]
>>> G = nx.DiGraph(edges)
>>> len(list(nx.simple_cycles(G))))
5
```

To filter the cycles so that they don’t include certain nodes or edges, copy your graph and eliminate those nodes or edges before calling

---

Notes

The implementation follows pp. 79-80 in\(^1\). The time complexity is \(O((n + e)(c + 1))\) for \(n\) nodes, \(e\) edges and \(c\) elementary circuits.

References

See also:

cycle_basis()

3.19.4 networkx.algorithms.cycles.find_cycle

find_cycle \((G, \text{source}=\text{None}, \text{orientation}=\text{None})\)

Returns a cycle found via depth-first traversal.

The cycle is a list of edges indicating the cyclic path. Orientation of directed edges is controlled by orientation.

Parameters

- \(G\) (graph) – A directed/undirected graph/multigraph.
- \(\text{source}\) (node, list of nodes) – The node from which the traversal begins. If None, then a source is chosen arbitrarily and repeatedly until all edges from each node in the graph are searched.
- \(\text{orientation}\) (none | ‘original’ | ‘reverse’ | ‘ignore’ (default: none)) – For directed graphs and directed multigraphs, edge traversals need not respect the original orientation of the edges. When set to ‘reverse’ every edge is traversed in the reverse direction. When set to ‘ignore’, every edge is treated as undirected. When set to ‘original’, every edge is treated as directed. In all three cases, the yielded edge tuples add a last entry to indicate the direction in which that edge was traversed. If orientation is None, the yielded edge has no direction indicated. The direction is respected, but not reported.

Returns edges – A list of directed edges indicating the path taken for the loop. If no cycle is found, then an exception is raised. For graphs, an edge is of the form \((u, v)\) where \(u\) and \(v\) are the tail and head of the edge as determined by the traversal. For multigraphs, an edge is of the form \((u, v, \text{key})\), where \(\text{key}\) is the key of the edge. When the graph is directed, then \(u\) and \(v\) are always in the order of the actual directed edge. If orientation is not None then the edge tuple is extended to include the direction of traversal (‘forward’ or ‘reverse’) on that edge.

Return type  directed edges

Raises NetworkXNoCycle – If no cycle was found.
Examples

In this example, we construct a DAG and find, in the first call, that there are no directed cycles, and so an exception is raised. In the second call, we ignore edge orientations and find that there is an undirected cycle. Note that the second call finds a directed cycle while effectively traversing an undirected graph, and so, we found an “undirected cycle”. This means that this DAG structure does not form a directed tree (which is also known as a polytree).

```python
>>> import networkx as nx
>>> G = nx.DiGraph([(0, 1), (0, 2), (1, 2)])
>>> try:
...    nx.find_cycle(G, orientation='original')
... except:
...    pass
...
>>> list(nx.find_cycle(G, orientation='ignore'))
[(0, 1, 'forward'), (1, 2, 'forward'), (0, 2, 'reverse')]
```

3.19.5 networkx.algorithms.cycles.minimum_cycle_basis

minimum_cycle_basis (G, weight=None)

Returns a minimum weight cycle basis for G

Minimum weight means a cycle basis for which the total weight (length for unweighted graphs) of all the cycles is minimum.

Parameters

- G (NetworkX Graph)
- weight (string) – name of the edge attribute to use for edge weights

Returns

- A list of cycle lists. Each cycle list is a list of nodes
- which forms a cycle (loop) in G. Note that the nodes are not necessarily returned in a order by which they appear in the cycle

Examples

```python
>>> G=nx.Graph()
>>> nx.add_cycle(G, [0,1,2,3])
>>> nx.add_cycle(G, [0,3,4,5])
>>> print([sorted(c) for c in nx.minimum_cycle_basis(G)])
[[0, 1, 2, 3], [0, 3, 4, 5]]
```

References


See also:

- simple_cycles()
- cycle_basis()
## 3.20 Cuts

Functions for finding and evaluating cuts in a graph.

<table>
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<th>Function</th>
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<td><code>boundary_expansion(G, S)</code></td>
<td>Returns the boundary expansion of the set ( S ).</td>
</tr>
<tr>
<td><code>conductance(G, S[, T, weight])</code></td>
<td>Returns the conductance of two sets of nodes.</td>
</tr>
<tr>
<td><code>cut_size(G, S[, T, weight])</code></td>
<td>Returns the size of the cut between two sets of nodes.</td>
</tr>
<tr>
<td><code>edge_expansion(G, S[, T, weight])</code></td>
<td>Returns the edge expansion between two node sets.</td>
</tr>
<tr>
<td><code>mixing_expansion(G, S[, T, weight])</code></td>
<td>Returns the mixing expansion between two node sets.</td>
</tr>
<tr>
<td><code>node_expansion(G, S)</code></td>
<td>Returns the node expansion of the set ( S ).</td>
</tr>
<tr>
<td><code>normalized_cut_size(G, S[, T, weight])</code></td>
<td>Returns the normalized size of the cut between two sets of nodes.</td>
</tr>
<tr>
<td><code>volume(G, S[, weight])</code></td>
<td>Returns the volume of a set of nodes.</td>
</tr>
</tbody>
</table>

### 3.20.1 networkx.algorithms.cuts.boundary_expansion

**boundary_expansion** \((G, S)\)

Returns the boundary expansion of the set \( S \).

The *boundary expansion* is the quotient of the size of the edge boundary and the cardinality of \( S \). [1]

**Parameters**

- \( G \) (NetworkX graph)
- \( S \) (sequence) – A sequence of nodes in \( G \).

**Returns** The boundary expansion of the set \( S \).

**Return type** number

**See also:**

`edge_expansion()`, `mixing_expansion()`, `node_expansion()`

### 3.20.2 networkx.algorithms.cuts.conductance

**conductance** \((G, S, T=None, weight=None)\)

Returns the conductance of two sets of nodes.

The *conductance* is the quotient of the cut size and the smaller of the volumes of the two sets. [1]

**Parameters**

- \( G \) (NetworkX graph)
- \( S \) (sequence) – A sequence of nodes in \( G \).
- \( T \) (sequence) – A sequence of nodes in \( G \).
- \( weight \) (object) – Edge attribute key to use as weight. If not specified, edges have weight one.

**Returns** The conductance between the two sets \( S \) and \( T \).

**Return type** number
See also:

\texttt{cut\_size()}, \texttt{edge\_expansion()}, \texttt{normalized\_cut\_size()}, \texttt{volume()}

References

3.20.3 networkx.algorithms.cuts.cut_size

\texttt{cut\_size}(G, S=T=None, weight=None)

Returns the size of the cut between two sets of nodes.

A \textit{cut} is a partition of the nodes of a graph into two sets. The \textit{cut size} is the sum of the weights of the edges “between” the two sets of nodes.

Parameters

\begin{itemize}
  \item \texttt{G} (\textit{NetworkX graph})
  \item \texttt{S} (\textit{sequence}) – A sequence of nodes in \texttt{G}.
  \item \texttt{T} (\textit{sequence}) – A sequence of nodes in \texttt{G}. If not specified, this is taken to be the set complement of \texttt{S}.
  \item \texttt{weight} (\textit{object}) – Edge attribute key to use as weight. If not specified, edges have weight one.
\end{itemize}

Returns Total weight of all edges from nodes in set \texttt{S} to nodes in set \texttt{T} (and, in the case of directed graphs, all edges from nodes in \texttt{T} to nodes in \texttt{S}).

Return type number

Examples

In the graph with two cliques joined by a single edges, the natural bipartition of the graph into two blocks, one for each clique, yields a cut of weight one:

```
>>> G = nx.barbell_graph(3, 0)
>>> S = {0, 1, 2}
>>> T = {3, 4, 5}
>>> nx.cut_size(G, S, T)
1
```

Each parallel edge in a multigraph is counted when determining the cut size:

```
>>> G = nx.MultiGraph(['ab', 'ab'])
>>> S = {'a'}
>>> T = {'b'}
>>> nx.cut_size(G, S, T)
2
```

Notes

In a multigraph, the cut size is the total weight of edges including multiplicity.
3.20.4 `networkx.algorithms.cuts.edge Expansion`

`edge Expansion(G, S, T=None, weight=None)`

Returns the edge expansion between two node sets.

The edge expansion is the quotient of the cut size and the smaller of the cardinalities of the two sets. [1]

**Parameters**

- `G` (*NetworkX graph*)
- `S` (*sequence*) – A sequence of nodes in `G`.
- `T` (*sequence*) – A sequence of nodes in `G`.
- `weight` (*object*) – Edge attribute key to use as weight. If not specified, edges have weight one.

**Returns** The edge expansion between the two sets `S` and `T`.

**Return type** number

See also: `boundary Expansion()`, `mixing Expansion()`, `node Expansion()`

**References**

3.20.5 `networkx.algorithms.cuts.mixing Expansion`

`mixing Expansion(G, S, T=None, weight=None)`

Returns the mixing expansion between two node sets.

The mixing expansion is the quotient of the cut size and twice the number of edges in the graph. [1]

**Parameters**

- `G` (*NetworkX graph*)
- `S` (*sequence*) – A sequence of nodes in `G`.
- `T` (*sequence*) – A sequence of nodes in `G`.
- `weight` (*object*) – Edge attribute key to use as weight. If not specified, edges have weight one.

**Returns** The mixing expansion between the two sets `S` and `T`.

**Return type** number

See also: `boundary Expansion()`, `edge Expansion()`, `node Expansion()`

**References**

3.20.6 `networkx.algorithms.cuts.node Expansion`

`node Expansion(G, S)`

Returns the node expansion of the set `S`.

The node expansion is the quotient of the size of the node boundary of `S` and the cardinality of `S`. [1]
Parameters

- \( G \) (NetworkX graph)
- \( S \) (sequence) – A sequence of nodes in \( G \).

Returns

The node expansion of the set \( S \).

Return type

number

See also:

boundary_expansion(), edgeExpansion(), mixing_expansion()

References

3.20.7 networkx.algorithms.cuts.normalized_cut_size

**normalized_cut_size** \((G, S, T=None, weight=None)\)

Returns the normalized size of the cut between two sets of nodes.

The normalized cut size is the cut size times the sum of the reciprocal sizes of the volumes of the two sets. [1]

Parameters

- \( G \) (NetworkX graph)
- \( S \) (sequence) – A sequence of nodes in \( G \).
- \( T \) (sequence) – A sequence of nodes in \( G \).
- \( weight \) (object) – Edge attribute key to use as weight. If not specified, edges have weight one.

Returns

The normalized cut size between the two sets \( S \) and \( T \).

Return type

number

Notes

In a multigraph, the cut size is the total weight of edges including multiplicity.

See also:

conductance(), cut_size(), edgeExpansion(), volume()

References

3.20.8 networkx.algorithms.cuts.volume

**volume** \((G, S, weight=None)\)

Returns the volume of a set of nodes.

The volume of a set \( S \) is the sum of the (out-)degrees of nodes in \( S \) (taking into account parallel edges in multigraphs). [1]

Parameters

- \( G \) (NetworkX graph)
- \( S \) (sequence) – A sequence of nodes in \( G \).
• **weight** *(object)* – Edge attribute key to use as weight. If not specified, edges have weight one.

**Returns** The volume of the set of nodes represented by \( S \) in the graph \( G \).

**Return type** number

**See also:**

- `conductance()`, `cut_size()`, `edge_expansion()`, `edge_boundary()`, `normalized_cut_size()`

### References

#### 3.21 Directed Acyclic Graphs

Algorithms for directed acyclic graphs (DAGs).

Note that most of these functions are only guaranteed to work for DAGs. In general, these functions do not check for acyclic-ness, so it is up to the user to check for that.

<table>
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<th>Description</th>
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<td><code>ancestors(G, source)</code></td>
<td>Returns all nodes having a path to <code>source</code> in <code>G</code></td>
</tr>
<tr>
<td><code>descendants(G, source)</code></td>
<td>Returns all nodes reachable from <code>source</code> in <code>G</code></td>
</tr>
<tr>
<td><code>topological_sort(G)</code></td>
<td>Returns a generator of nodes in topologically sorted order</td>
</tr>
<tr>
<td><code>all_topological_sorts(G)</code></td>
<td>Returns a generator of <em>all</em> topological sorts of the directed graph <code>G</code></td>
</tr>
<tr>
<td><code>lexicographical_topological_sort(G[, key])</code></td>
<td>Returns a generator of nodes in lexicographically topologically sorted order</td>
</tr>
<tr>
<td><code>is_directed_acyclic_graph(G)</code></td>
<td>Returns True if the graph <code>G</code> is a directed acyclic graph (DAG) or False if not</td>
</tr>
<tr>
<td><code>is_aperiodic(G)</code></td>
<td>Returns True if <code>G</code> is aperiodic</td>
</tr>
<tr>
<td><code>transitive_closure(G[, reflexive])</code></td>
<td>Returns transitive closure of a directed graph</td>
</tr>
<tr>
<td><code>transitive_reduction(G)</code></td>
<td>Returns transitive reduction of a directed graph</td>
</tr>
<tr>
<td><code>antichains(G[, topo_order])</code></td>
<td>Generates antichains from a directed acyclic graph (DAG)</td>
</tr>
<tr>
<td><code>dag_longest_path(G[, weight, ...])</code></td>
<td>Returns the longest path in a directed acyclic graph (DAG)</td>
</tr>
<tr>
<td><code>dag_longest_path_length(G[, weight, ...])</code></td>
<td>Returns the longest path length in a DAG</td>
</tr>
<tr>
<td><code>dag_to_branching(G)</code></td>
<td>Returns a branching representing all (overlapping) paths from root nodes to leaf nodes in the given directed acyclic graph</td>
</tr>
</tbody>
</table>

#### 3.21.1 networkx.algorithms.dag.ancestors

**ancestors** *(G, source)*

Returns all nodes having a path to `source` in `G`.

**Parameters**

- `G` *(NetworkX DiGraph)* – A directed acyclic graph (DAG)
- `source` *(node in G)*

**Returns** The ancestors of source in `G`
Return type  set()

3.21.2 networkx.algorithms.dag.descendants
descendants (G, source)
    Returns all nodes reachable from source in G.
Parameters
    • G (NetworkX DiGraph) – A directed acyclic graph (DAG)
    • source (node in G)
Returns  The descendants of source in G
Return type  set()

3.21.3 networkx.algorithms.dag.topological_sort
topological_sort (G)
    Returns a generator of nodes in topologically sorted order.
    A topological sort is a nonunique permutation of the nodes such that an edge from u to v implies that u appears
    before v in the topological sort order.
Parameters G (NetworkX digraph) – A directed acyclic graph (DAG)
Returns  An iterable of node names in topological sorted order.
Return type  iterable
Raises
    • NetworkXError – Topological sort is defined for directed graphs only. If the graph G is
      undirected, a NetworkXError is raised.
    • NetworkXUnfeasible – If G is not a directed acyclic graph (DAG) no topological sort
      exists and a NetworkXUnfeasible exception is raised. This can also be raised if G is
      changed while the returned iterator is being processed
    • RuntimeError – If G is changed while the returned iterator is being processed.

Examples
To get the reverse order of the topological sort:

```python
e> DG = nx.DiGraph([(1, 2), (2, 3)])
e> list(reversed(list(nx.topological_sort(DG)))).
[3, 2, 1]
```

If your DiGraph naturally has the edges representing tasks/inputs and nodes representing people/processes that
initiate tasks, then topological_sort is not quite what you need. You will have to change the tasks to nodes with
dependence reflected by edges. The result is a kind of topological sort of the edges. This can be done with
networkx.line_graph() as follows:

```python
e> list(nx.topological_sort(nx.line_graph(DG))
[[(1, 2), (2, 3)]
```
Notes

This algorithm is based on a description and proof in “Introduction to Algorithms: A Creative Approach”\(^1\).

See also:

- `is_directed_acyclic_graph()`, `lexicographical_topological_sort()`

References

3.21.4 networkx.algorithms.dag.all_topological_sorts

`all_topological_sorts(G)`

Returns a generator of all topological sorts of the directed graph G.

A topological sort is a nonunique permutation of the nodes such that an edge from u to v implies that u appears before v in the topological sort order.

**Parameters**

- `G (NetworkX DiGraph)` – A directed graph

**Returns**

All topological sorts of the digraph G

**Return type**

generator

**Raises**

- `NetworkXNotImplemented` – If G is not directed
- `NetworkXUnfeasible` – If G is not acyclic

Examples

To enumerate all topological sorts of directed graph:

```python
>>> DG = nx.DiGraph([(1, 2), (2, 3), (2, 4)])
>>> list(nx.all_topological_sorts(DG))
[[1, 2, 4, 3], [1, 2, 3, 4]]
```

Notes

Implements an iterative version of the algorithm given in [1].

References

3.21.5 networkx.algorithms.dag.lexicographical_topological_sort

`lexicographical_topological_sort(G, key=None)`

Returns a generator of nodes in lexicographically topologically sorted order.

A topological sort is a nonunique permutation of the nodes such that an edge from u to v implies that u appears before v in the topological sort order.

**Parameters**

- `G (NetworkX digraph)` – A directed acyclic graph (DAG)

---

• **key (function, optional)** – This function maps nodes to keys with which to resolve ambiguities in the sort order. Defaults to the identity function.

**Returns** An iterable of node names in lexicographical topological sort order.

**Return type** iterable

**Raises**

• **NetworkXError** – Topological sort is defined for directed graphs only. If the graph G is undirected, a NetworkXError is raised.

• **NetworkXUnfeasible** – If G is not a directed acyclic graph (DAG) no topological sort exists and a NetworkXUnfeasible exception is raised. This can also be raised if G is changed while the returned iterator is being processed

• **RuntimeError** – If G is changed while the returned iterator is being processed.

**Notes**

This algorithm is based on a description and proof in “Introduction to Algorithms: A Creative Approach”\(^1\).

See also:

topological_sort()

**References**

3.21.6 networkx.algorithms.dag.is_directed_acyclic_graph

is_directed_acyclic_graph (G)

Returns True if the graph G is a directed acyclic graph (DAG) or False if not.

**Parameters** G (NetworkX graph)

**Returns** True if G is a DAG, False otherwise

**Return type** bool

3.21.7 networkx.algorithms.dag.is_aperiodic

is_aperiodic (G)

Returns True if G is aperiodic.

A directed graph is aperiodic if there is no integer k > 1 that divides the length of every cycle in the graph.

**Parameters** G (NetworkX DiGraph) – A directed graph

**Returns** True if the graph is aperiodic False otherwise

**Return type** bool

**Raises** NetworkXError – If G is not directed

Notes

This uses the method outlined in \(^1\), which runs in \(O(m)\) time given \(m\) edges in \(G\). Note that a graph is not aperiodic if it is acyclic as every integer trivial divides length 0 cycles.

References

3.21.8 networkx.algorithms.dag.transitive_closure

transitive_closure \((G, \text{reflexive}=\text{False})\)

Returns transitive closure of a directed graph

The transitive closure of \(G = (V,E)\) is a graph \(G^+ = (V,E^+)\) such that for all \(v, w\) in \(V\) there is an edge \((v, w)\) in \(E^+\) if and only if there is a path from \(v\) to \(w\) in \(G\).

Handling of paths from \(v\) to \(v\) has some flexibility within this definition. A reflexive transitive closure creates a self-loop for the path from \(v\) to \(v\) of length 0. The usual transitive closure creates a self-loop only if a cycle exists (a path from \(v\) to \(v\) with length > 0). We also allow an option for no self-loops.

**Parameters**

- \(G\) (NetworkX DiGraph) – A directed graph

- \text{reflexive} (Bool or None, optional (default: False)) – Determines when cycles create self-loops in the Transitive Closure. If True, trivial cycles (length 0) create self-loops. The result is a reflexive transitive closure of \(G\). If False (the default) non-trivial cycles create self-loops. If None, self-loops are not created.

**Returns** The transitive closure of \(G\)

**Return type** NetworkX DiGraph

**Raises** NetworkXNotImplemented – If \(G\) is not directed

References

TODO this function applies to all directed graphs and is probably misplaced here in dag.py

3.21.9 networkx.algorithms.dag.transitive_reduction

transitive_reduction \((G)\)

Returns transitive reduction of a directed graph

The transitive reduction of \(G = (V,E)\) is a graph \(G^- = (V,E^-)\) such that for all \(v,w\) in \(V\) there is an edge \((v,w)\) in \(E^-\) if and only if \((v,w)\) is in \(E\) and there is no path from \(v\) to \(w\) in \(G\) with length greater than 1.

**Parameters** \(G\) (NetworkX DiGraph) – A directed acyclic graph (DAG)

**Returns** The transitive reduction of \(G\)

**Return type** NetworkX DiGraph

**Raises** NetworkXError – If \(G\) is not a directed acyclic graph (DAG) transitive reduction is not uniquely defined and a NetworkXError exception is raised.

---


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3.21.10 networkx.algorithms.dag.antichains

antichains \((G, \text{topo\_order}=\text{None})\)

Generates antichains from a directed acyclic graph (DAG).

An antichain is a subset of a partially ordered set such that any two elements in the subset are incomparable.

Parameters

- \(G\) (NetworkX DiGraph) – A directed acyclic graph (DAG)
- \(\text{topo\_order}\) (list or tuple, optional) – A topological order for \(G\) (if None, the function will compute one)

Returns

Return type generator object

Raises

- \(\text{NetworkXNotImplemented}\) – If \(G\) is not directed
- \(\text{NetworkXUnfeasible}\) – If \(G\) contains a cycle

Notes

This function was originally developed by Peter Jipsen and Franco Saliola for the SAGE project. It’s included in NetworkX with permission from the authors. Original SAGE code at:

https://github.com/sagemath/sage/blob/master/src/sage/combinat/posets/hasse_diagram.py

References

3.21.11 networkx.algorithms.dag.dag_longest_path

dag_longest_path \((G, \text{weight}='weight', \text{default\_weight}=1, \text{topo\_order}=\text{None})\)

Returns the longest path in a directed acyclic graph (DAG).

If \(G\) has edges with weight attribute the edge data are used as weight values.

Parameters

- \(G\) (NetworkX DiGraph) – A directed acyclic graph (DAG)
- \(\text{weight}\) (str, optional) – Edge data key to use for weight
- \(\text{default\_weight}\) (int, optional) – The weight of edges that do not have a weight attribute
- \(\text{topo\_order}\) (list or tuple, optional) – A topological order for \(G\) (if None, the function will compute one)

Returns Longest path

Return type list

Raises \(\text{NetworkXNotImplemented}\) – If \(G\) is not directed
See also:

\texttt{dag\_longest\_path\_length()}

### 3.21.12 \texttt{networkx.algorithms.dag.dag\_longest\_path\_length}

\texttt{dag\_longest\_path\_length}(G, weight='weight', default_weight=1)

Returns the longest path length in a DAG

**Parameters**

- \(G\) (\textit{NetworkX} \texttt{DiGraph}) – A directed acyclic graph (DAG)
- \texttt{weight} (\textit{string, optional}) – Edge data key to use for weight
- \texttt{default\_weight} (\textit{int, optional}) – The weight of edges that do not have a weight attribute

**Returns**

Longest path length

**Return type** \texttt{int}

**Raises** \texttt{NetworkXNotImplemented} – If \(G\) is not directed

See also:

\texttt{dag\_longest\_path()}

### 3.21.13 \texttt{networkx.algorithms.dag.dag\_to\_branching}

\texttt{dag\_to\_branching}(G)

Returns a branching representing all (overlapping) paths from root nodes to leaf nodes in the given directed acyclic graph.

As described in \textit{networkx.algorithms.tree.recognition}, a branching is a directed forest in which each node has at most one parent. In other words, a branching is a disjoint union of arborescences. For this function, each node of in-degree zero in \(G\) becomes a root of one of the arborescences, and there will be one leaf node for each distinct path from that root to a leaf node in \(G\).

Each node \(v\) in \(G\) with \(k\) parents becomes \(k\) distinct nodes in the returned branching, one for each parent, and the sub-DAG rooted at \(v\) is duplicated for each copy. The algorithm then recurses on the children of each copy of \(v\).

**Parameters** \(G\) (\textit{NetworkX} \textit{graph}) – A directed acyclic graph.

**Returns**

The branching in which there is a bijection between root-to-leaf paths in \(G\) (in which multiple paths may share the same leaf) and root-to-leaf paths in the branching (in which there is a unique path from a root to a leaf).

Each node has an attribute ‘source’ whose value is the original node to which this node corresponds. No other graph, node, or edge attributes are copied into this new graph.

**Return type** \texttt{DiGraph}

**Raises**

- \texttt{NetworkXNotImplemented} – If \(G\) is not directed, or if \(G\) is a multigraph.
- \texttt{HasACycle} – If \(G\) is not acyclic.
Examples

To examine which nodes in the returned branching were produced by which original node in the directed acyclic graph, we can collect the mapping from source node to new nodes into a dictionary. For example, consider the directed diamond graph:

```python
>>> from collections import defaultdict
>>> from operator import itemgetter

>>> G = nx.DiGraph(nx.utils.pairwise('abd'))
>>> G.add_edges_from(nx.utils.pairwise('acd'))
>>> B = nx.dag_to_branching(G)

>>> sources = defaultdict(set)
>>> for v, source in B.nodes(data='source'):
...    sources[source].add(v)

>>> len(sources['a'])
1
>>> len(sources['d'])
2
```

To copy node attributes from the original graph to the new graph, you can use a dictionary like the one constructed in the above example:

```python
>>> for source, nodes in sources.items():
...    for v in nodes:
...        B.nodes[v].update(G.nodes[source])
```

Notes

This function is not idempotent in the sense that the node labels in the returned branching may be uniquely generated each time the function is invoked. In fact, the node labels may not be integers; in order to relabel the nodes to be more readable, you can use the `networkx.convert_node_labels_to_integers()` function.

The current implementation of this function uses `networkx.prefix_tree()`, so it is subject to the limitations of that function.

### 3.22 Distance Measures

Graph diameter, radius, eccentricity and other properties.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>barycenter(G[, weight, attr, sp])</code></td>
<td>Calculate barycenter of a connected graph, optionally with edge weights.</td>
</tr>
<tr>
<td><code>center(G[, e, usebounds])</code></td>
<td>Returns the center of the graph G.</td>
</tr>
<tr>
<td><code>diameter(G[, e, usebounds])</code></td>
<td>Returns the diameter of the graph G.</td>
</tr>
<tr>
<td><code>eccentricity(G[, v, sp])</code></td>
<td>Returns the eccentricity of nodes in G.</td>
</tr>
<tr>
<td><code>extrema_bounding(G[, compute])</code></td>
<td>Compute requested extreme distance metric of undirected graph G.</td>
</tr>
<tr>
<td><code>periphery(G[, e, usebounds])</code></td>
<td>Returns the periphery of the graph G.</td>
</tr>
<tr>
<td><code>radius(G[, e, usebounds])</code></td>
<td>Returns the radius of the graph G.</td>
</tr>
</tbody>
</table>
Table 83 – continued from previous page

| resistance_distance(G, nodeA, nodeB[, ...]) | Returns the resistance distance between node A and node B on graph G. |

### 3.22.1 networkx.algorithms.distance_measures.barycenter

**barycenter** *(G, weight=None, attr=None, sp=None)*

Calculate barycenter of a connected graph, optionally with edge weights.

The barycenter a connected graph *G* is the subgraph induced by the set of its nodes *v* minimizing the objective function

\[
\sum_{u \in V(G)} d_G(u, v),
\]

where *d_G* is the (possibly weighted) path length. The barycenter is also called the median. See [?], p. 78.

**Parameters**
- *G* *(networkx.Graph)* – The connected graph *G*.
- *weight* *(str, optional)* – Passed through to shortest_path_length().
- *attr* *(str, optional)* – If given, write the value of the objective function to each node’s *attr* attribute. Otherwise do not store the value.
- *sp* *(dict of dicts, optional)* – All pairs shortest path lengths as a dictionary of dictionaries

**Returns** Nodes of *G* that induce the barycenter of *G*.

**Return type** list

**Raises**
- *NetworkXNoPath* – If *G* is disconnected. *G* may appear disconnected to barycenter() if *sp* is given but is missing shortest path lengths for any pairs.
- *ValueError* – If *sp* and *weight* are both given.

**See also:**
- center(), periphery()

### 3.22.2 networkx.algorithms.distance_measures.center

**center** *(G, e=None, usebounds=False)*

Returns the center of the graph *G*.

The center is the set of nodes with eccentricity equal to radius.

**Parameters**
- *G* *(NetworkX graph)* – A graph
- *e* *(eccentricity dictionary, optional)* – A precomputed dictionary of eccentricities.

**Returns** *c* – List of nodes in center

**Return type** list

**See also:**
- barycenter(), periphery()
3.22.3 networkx.algorithms.distance_measures.diameter

diameter \( (G, e=None, usebounds=False) \)
Returns the diameter of the graph \( G \).

The diameter is the maximum eccentricity.

Parameters

- \( G \) (NetworkX graph) – A graph
- \( e \) (eccentricity dictionary, optional) – A precomputed dictionary of eccentricities.

Returns \( d \) – Diameter of graph

Return type integer

See also:

eccentricity()

3.22.4 networkx.algorithms.distance_measures.eccentricity

eccentricity \( (G, v=None, sp=None) \)
Returns the eccentricity of nodes in \( G \).

The eccentricity of a node \( v \) is the maximum distance from \( v \) to all other nodes in \( G \).

Parameters

- \( G \) (NetworkX graph) – A graph
- \( v \) (node, optional) – Return value of specified node
- \( sp \) (dict of dicts, optional) – All pairs shortest path lengths as a dictionary of dictionaries

Returns \( ecc \) – A dictionary of eccentricity values keyed by node.

Return type dictionary

3.22.5 networkx.algorithms.distance_measures.extrema_bounding

extrema_bounding \( (G, compute='diameter') \)
Compute requested extreme distance metric of undirected graph \( G \)

Computation is based on smart lower and upper bounds, and in practice linear in the number of nodes, rather than quadratic (except for some border cases such as complete graphs or circle shaped graphs).

Parameters

- \( G \) (NetworkX graph) – An undirected graph
- \( compute \) (string denoting the requesting metric) – “diameter” for the maximal eccentricity value, “radius” for the minimal eccentricity value, “periphery” for the set of nodes with eccentricity equal to the diameter “center” for the set of nodes with eccentricity equal to the radius

Returns \( value \) – int for “diameter” and “radius” or list of nodes for “center” and “periphery”

Return type value of the requested metric

Raises NetworkXError – If the graph consists of multiple components
Notes

This algorithm was proposed in the following papers:


3.22.6 networkx.algorithms.distance_measures.periphery

periphery (G, e=None, usebounds=False)

Returns the periphery of the graph G.

The periphery is the set of nodes with eccentricity equal to the diameter.

Parameters

• G (NetworkX graph) – A graph
• e (eccentricity dictionary, optional) – A precomputed dictionary of eccentricities.

Returns p – List of nodes in periphery

Return type list

See also:

barycenter(), center()

3.22.7 networkx.algorithms.distance_measures.radius

radius (G, e=None, usebounds=False)

Returns the radius of the graph G.

The radius is the minimum eccentricity.

Parameters

• G (NetworkX graph) – A graph
• e (eccentricity dictionary, optional) – A precomputed dictionary of eccentricities.

Returns r – Radius of graph

Return type integer

3.22.8 networkx.algorithms.distance_measures.resistance_distance

resistance_distance (G, nodeA, nodeB, weight=None, invert_weight=True)

Returns the resistance distance between node A and node B on graph G.

The resistance distance between two nodes of a graph is akin to treating the graph as a grid of resistors with a resistance equal to the provided weight.
If weight is not provided, then a weight of 1 is used for all edges.

**Parameters**

- `G (NetworkX graph)` – A graph
- `nodeA (node)` – A node within graph G.
- `nodeB (node)` – A node within graph G, exclusive of Node A.
- `weight (string or None, optional (default=None))` – The edge data key used to compute the resistance distance. If None, then each edge has weight 1.
- `invert_weight (boolean (default=True))` – Proper calculation of resistance distance requires building the Laplacian matrix with the reciprocal of the weight. Not required if the weight is already inverted. Weight cannot be zero.

**Returns** `rd` – Value of effective resistance distance

**Return type** `float`

**Notes**


### 3.23 Distance-Regular Graphs

#### 3.23.1 Distance-regular graphs

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>is_distance_regular(G)</code></td>
<td>Returns True if the graph is distance regular, False otherwise.</td>
</tr>
<tr>
<td><code>is_strongly_regular(G)</code></td>
<td>Returns True if and only if the given graph is strongly regular.</td>
</tr>
<tr>
<td><code>intersection_array(G)</code></td>
<td>Returns the intersection array of a distance-regular graph.</td>
</tr>
<tr>
<td><code>global_parameters(b, c)</code></td>
<td>Returns global parameters for a given intersection array.</td>
</tr>
</tbody>
</table>

#### 3.23.2 networkx.algorithms.distance_regular.is_distance_regular

`is_distance_regular(G)`

Returns True if the graph is distance regular, False otherwise.

A connected graph G is distance-regular if for any nodes x, y and any integers i, j = 0, 1, …, d (where d is the graph diameter), the number of vertices at distance i from x and distance j from y depends only on i, j and the graph distance between x and y, independently of the choice of x and y.

**Parameters**

- `G (Networkx graph (undirected))`

**Returns** True if the graph is Distance Regular, False otherwise

**Return type** `bool`
Examples

```python
>>> G = nx.hypercube_graph(6)
>>> nx.is_distance_regular(G)
True
```

See also:

`intersection_array()`, `global_parameters()`

Notes

For undirected and simple graphs only

References

3.23.3 networkx.algorithms.distance_regular.is_strongly_regular

`is_strongly_regular(G)`

Returns True if and only if the given graph is strongly regular.

An undirected graph is strongly regular if

- it is regular,
- each pair of adjacent vertices has the same number of neighbors in common,
- each pair of nonadjacent vertices has the same number of neighbors in common.

Each strongly regular graph is a distance-regular graph. Conversely, if a distance-regular graph has diameter two, then it is a strongly regular graph. For more information on distance-regular graphs, see `is_distance_regular()`.

Parameters

- `G (NetworkX graph)` – An undirected graph.

Returns

Whether `G` is strongly regular.

Return type

`bool`

Examples

The cycle graph on five vertices is strongly regular. It is two-regular, each pair of adjacent vertices has no shared neighbors, and each pair of nonadjacent vertices has one shared neighbor:

```python
>>> import networkx as nx
>>> G = nx.cycle_graph(5)
>>> nx.is_strongly_regular(G)
True
```

3.23.4 networkx.algorithms.distance_regular.intersection_array

`intersection_array(G)`

Returns the intersection array of a distance-regular graph.
Given a distance-regular graph $G$ with integers $b_i$, $c_i$, $i = 0, \ldots, d$ such that for any 2 vertices $x, y$ in $G$ at a distance $i = d(x, y)$, there are exactly $c_i$ neighbors of $y$ at a distance of $i-1$ from $x$ and $b_i$ neighbors of $y$ at a distance of $i+1$ from $x$.

A distance regular graph’s intersection array is given by, $[b_0, b_1, \ldots, b_{d-1}; c_1, c_2, \ldots, c_d]$

**Parameters**

- $G$ (*Networkx graph (undirected)*)

**Returns**

- $b, c$

**Return type**

tuple of lists

**Examples**

```python
>>> G = nx.icosahedral_graph()
>>> nx.intersection_array(G)
([[5, 2, 1], [1, 2, 5]])
```

**References**

See also: `global_parameters()`

### 3.23.5 networkx.algorithms.distance_regular.global_parameters

**global_parameters** $(b, c)$

Returns global parameters for a given intersection array.

Given a distance-regular graph $G$ with integers $b_i$, $c_i$, $i = 0, \ldots, d$ such that for any 2 vertices $x, y$ in $G$ at a distance $i = d(x, y)$, there are exactly $c_i$ neighbors of $y$ at a distance of $i-1$ from $x$ and $b_i$ neighbors of $y$ at a distance of $i+1$ from $x$.

Thus, a distance regular graph has the global parameters, $[[c_0, a_0, b_0], [c_1, a_1, b_1], \ldots, [c_d, a_d, b_d]]$ for the intersection array $[b_0, b_1, \ldots, b_{d-1}; c_1, c_2, \ldots, c_d]$ where $a_i + b_i + c_i = k$, $k$= degree of every vertex.

**Parameters**

- $b$ (*list*)
- $c$ (*list*)

**Returns**

An iterable over three tuples.

**Return type**

iterable

**Examples**

```python
>>> G = nx.dodecahedral_graph()
>>> b, c = nx.intersection_array(G)
>>> list(nx.global_parameters(b, c))
[(0, 0, 3), (1, 0, 2), (1, 1, 1), (1, 1, 1), (2, 0, 1), (3, 0, 0)]
```
References

See also:

\textit{intersection\_array()}

## 3.24 Dominance

Dominance algorithms.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>\texttt{immediate_dominators(G, start)}</td>
<td>Returns the immediate dominators of all nodes of a directed graph.</td>
</tr>
<tr>
<td>\texttt{dominance_frontiers(G, start)}</td>
<td>Returns the dominance frontiers of all nodes of a directed graph.</td>
</tr>
</tbody>
</table>

### 3.24.1 networkx.algorithms.dominance.immediate\_dominators

\texttt{immediate\_dominators}(G, start)

Returns the immediate dominators of all nodes of a directed graph.

**Parameters**

- \texttt{G} \textit{(a DiGraph or MultiDiGraph)} – The graph where dominance is to be computed.
- \texttt{start} \textit{(node)} – The start node of dominance computation.

**Returns** \texttt{idom} – A dict containing the immediate dominators of each node reachable from \texttt{start}.

**Return type** dict keyed by nodes

**Raises**

- \texttt{NetworkXNotImplemented} – If \texttt{G} is undirected.
- \texttt{NetworkXError} – If \texttt{start} is not in \texttt{G}.

**Notes**

Except for \texttt{start}, the immediate dominators are the parents of their corresponding nodes in the dominator tree.

**Examples**

```python
>>> G = nx.DiGraph([(1, 2), (1, 3), (2, 5), (3, 4), (4, 5)])
>>> sorted(nx.immediate_dominators(G, 1).items())
[(1, 1), (2, 1), (3, 1), (4, 3), (5, 1)]
```

**References**

### 3.24.2 networkx.algorithms.dominance.dominance\_frontiers

\texttt{dominance\_frontiers}(G, start)

Returns the dominance frontiers of all nodes of a directed graph.
Parameters

- **G** *(a DiGraph or MultiDiGraph)* – The graph where dominance is to be computed.
- **start** *(node)* – The start node of dominance computation.

Returns **df** – A dict containing the dominance frontiers of each node reachable from *start* as lists.

Return type  *dict keyed by nodes*

Raises

- **NetworkXNotImplemented** – If *G* is undirected.
- **NetworkXError** – If *start* is not in *G*.

Examples

```python
>>> G = nx.DiGraph([(1, 2), (1, 3), (2, 5), (3, 4), (4, 5)])
>>> sorted((u, sorted(df)) for u, df in nx.dominance_frontiers(G, 1).items())
[(1, []), (2, [5]), (3, [5]), (4, [5]), (5, [])]
```

References

3.25 Dominating Sets

Functions for computing dominating sets in a graph.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>dominating_set(G [, start_with=None])</code></td>
<td>Finds a dominating set for the graph <em>G</em>.</td>
</tr>
<tr>
<td><code>is_dominating_set(G, nbunch)</code></td>
<td>Checks if <em>nbunch</em> is a dominating set for <em>G</em>.</td>
</tr>
</tbody>
</table>

3.25.1 networkx.algorithms.dominating.dominating_set

`dominating_set (G, start_with=None)`

Finds a dominating set for the graph *G*.

A *dominating set* for a graph with node set *V* is a subset *D* of *V* such that every node not in *D* is adjacent to at least one member of *D*.

Parameters

- **G** *(NetworkX graph)*
- **start_with** *(node (default=None))* – Node to use as a starting point for the algorithm.

Returns **D** – A dominating set for *G*.

Return type  *set*

Notes

This function is an implementation of algorithm 7 in\(^1\) which finds some dominating set, not necessarily the smallest one.

---


\(^2\) Abdol-Hossein Esfahanian. Connectivity Algorithms. [http://www.cse.msu.edu/~cse835/Papers/Graph_connectivity_revised.pdf](http://www.cse.msu.edu/~cse835/Papers/Graph_connectivity_revised.pdf)
See also:

is_dominating_set()

References

3.25.2 networkx.algorithms.dominating.is_dominating_set

is_dominating_set(G, nbunch)
Checks if nbunch is a dominating set for G.

A dominating set for a graph with node set V is a subset D of V such that every node not in D is adjacent to at least one member of D.1

Parameters

• G (NetworkX graph)
• nbunch (iterable) – An iterable of nodes in the graph G.

See also:

dominating_set()

References

3.26 Efficiency

Provides functions for computing the efficiency of nodes and graphs.

<table>
<thead>
<tr>
<th>function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>efficiency(G, u, v)</td>
<td>Returns the efficiency of a pair of nodes in a graph.</td>
</tr>
<tr>
<td>local_efficiency(G)</td>
<td>Returns the average local efficiency of the graph.</td>
</tr>
<tr>
<td>global_efficiency(G)</td>
<td>Returns the average global efficiency of the graph.</td>
</tr>
</tbody>
</table>

3.26.1 networkx.algorithms.efficiency_measures.efficiency

efficiency(G, u, v)
Returns the efficiency of a pair of nodes in a graph.

The efficiency of a pair of nodes is the multiplicative inverse of the shortest path distance between the nodes. Returns 0 if no path between nodes.

Parameters

• G (networkx.Graph) – An undirected graph for which to compute the average local efficiency.
• u, v (node) – Nodes in the graph G.

Returns  Multiplicative inverse of the shortest path distance between the nodes.

Return type  float

1 https://en.wikipedia.org/wiki/Dominating_set  
Notes

Edge weights are ignored when computing the shortest path distances.

See also:

local_efficiency(), global_efficiency()

References

3.26.2 networkx.algorithms.efficiency_measures.local_efficiency

local_efficiency(G)

Returns the average local efficiency of the graph.

The efficiency of a pair of nodes in a graph is the multiplicative inverse of the shortest path distance between the nodes. The local efficiency of a node in the graph is the average global efficiency of the subgraph induced by the neighbors of the node. The average local efficiency is the average of the local efficiencies of each node\(^1\).

Parameters G (networkx.Graph) – An undirected graph for which to compute the average local efficiency.

Returns The average local efficiency of the graph.

Return type float

Notes

Edge weights are ignored when computing the shortest path distances.

See also:

global_efficiency()

References

3.26.3 networkx.algorithms.efficiency_measures.global_efficiency

global_efficiency(G)

Returns the average global efficiency of the graph.

The efficiency of a pair of nodes in a graph is the multiplicative inverse of the shortest path distance between the nodes. The average global efficiency of a graph is the average efficiency of all pairs of nodes\(^1\).

Parameters G (networkx.Graph) – An undirected graph for which to compute the average global efficiency.

Returns The average global efficiency of the graph.

Return type float


Notes

Edge weights are ignored when computing the shortest path distances.

See also:

  `local_efficiency()`

References

3.27 Eulerian

Eulerian circuits and graphs.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>is_eulerian(G)</code></td>
<td>Returns True if and only if G is Eulerian.</td>
</tr>
<tr>
<td><code>eulerian_circuit(G[, source, keys])</code></td>
<td>Returns an iterator over the edges of an Eulerian circuit in G.</td>
</tr>
<tr>
<td><code>eulerize(G)</code></td>
<td>Transforms a graph into an Eulerian graph</td>
</tr>
<tr>
<td><code>is_semi_eulerian(G)</code></td>
<td>Return True iff G is semi-Eulerian.</td>
</tr>
<tr>
<td><code>has_eulerian_path(G)</code></td>
<td>Return True iff G has an Eulerian path.</td>
</tr>
<tr>
<td><code>eulerian_path(G[, source, keys])</code></td>
<td>Return an iterator over the edges of an Eulerian path in G.</td>
</tr>
</tbody>
</table>

3.27.1 networkx.algorithms.euler.is_eulerian

`is_eulerian(G)`

Returns True if and only if G is Eulerian.

A graph is Eulerian if it has an Eulerian circuit. An Eulerian circuit is a closed walk that includes each edge of a graph exactly once.

Parameters

- `G` (NetworkX graph) – A graph, either directed or undirected.

Examples

```python
>>> nx.is_eulerian(nx.DiGraph({0: [3], 1: [2], 2: [3], 3: [0, 1]}))
True
>>> nx.is_eulerian(nx.complete_graph(5))
True
>>> nx.is_eulerian(nx.petersen_graph())
False
```

Notes

If the graph is not connected (or not strongly connected, for directed graphs), this function returns False.

3.27.2 networkx.algorithms.euler.eulerian_circuit

`eulerian_circuit(G, source=None, keys=False)`

Returns an iterator over the edges of an Eulerian circuit in G.
An Eulerian circuit is a closed walk that includes each edge of a graph exactly once.

**Parameters**

- `G` (*NetworkX graph*) – A graph, either directed or undirected.
- `source` (*node, optional*) – Starting node for circuit.
- `keys` (*bool*) – If False, edges generated by this function will be of the form `(u, v)`. Otherwise, edges will be of the form `(u, v, k)`. This option is ignored unless `G` is a multigraph.

**Returns**

- `edges` – An iterator over edges in the Eulerian circuit.

**Return type**

- iterator

**Raises**

- `NetworkXError` – If the graph is not Eulerian.

**See also**

- `is_eulerian()`  

**Notes**

This is a linear time implementation of an algorithm adapted from\(^1\).

For general information about Euler tours, see\(^2\).

**References**

**Examples**

To get an Eulerian circuit in an undirected graph:

```python
>>> G = nx.complete_graph(3)
>>> list(nx.eulerian_circuit(G))
[(0, 2), (2, 1), (1, 0)]
>>> list(nx.eulerian_circuit(G, source=1))
[(1, 2), (2, 0), (0, 1)]
```

To get the sequence of vertices in an Eulerian circuit:

```python
>>> [u for u, v in nx.eulerian_circuit(G)]
[0, 2, 1]
```

### 3.27.3 networkx.algorithms.euler.eulerize

**eulerize** (*G*)

Transforms a graph into an Eulerian graph

**Parameters**

- `G` (*NetworkX graph*) – An undirected graph

**Returns**

- `G`

**Return type**

- NetworkX multigraph

**Raises**

- `NetworkXError` – If the graph is not connected.

---

See also:

\texttt{is_eulerian()}, \texttt{eulerian_circuit()}

References

Examples

```python
>>> G = nx.complete_graph(10)
>>> H = nx.eulerize(G)
>>> nx.is_eulerian(H)
True
```

3.27.4 \texttt{networkx.algorithms.euler.is_semi eulerian}

\texttt{is_semi eulerian}(\texttt{G})

Return True iff \texttt{G} is semi-Eulerian.

\texttt{G} is semi-Eulerian if it has an Eulerian path but no Eulerian circuit.

3.27.5 \texttt{networkx.algorithms.euler.has eulerian_path}

\texttt{has eulerian_path}(\texttt{G})

Return True iff \texttt{G} has an Eulerian path.

An Eulerian path is a path in a graph which uses each edge of a graph exactly once.

A directed graph has an Eulerian path iff:

- at most one vertex has out\_degree - in\_degree = 1,
- at most one vertex has in\_degree - out\_degree = 1,
- every other vertex has equal in\_degree and out\_degree,
- and all of its vertices with nonzero degree belong to a single connected component of the underlying undirected graph.

An undirected graph has an Eulerian path iff:

- exactly zero or two vertices have odd degree,
- and all of its vertices with nonzero degree belong to a single connected component.

Parameters \texttt{G} (\texttt{NetworkX Graph}) – The graph to find an euler path in.

Returns \texttt{Bool}

Return type True if \texttt{G} has an eulerian path.

See also:

\texttt{is_eulerian()}, \texttt{eulerian_path()}
3.27.6 `networkx.algorithms.euler.eulerian_path`

`eulerian_path(G, source=None, keys=False)`

Return an iterator over the edges of an Eulerian path in `G`.

Parameters

- `G` (*NetworkX Graph*) – The graph in which to look for an eulerian path.
- `source` (*node or None (default: None)*) – The node at which to start the search. None means search over all starting nodes.
- `keys` (*Bool (default: False)*) – Indicates whether to yield edge 3-tuples (u, v, edge_key). The default yields edge 2-tuples

Yields

- Edge tuples along the eulerian path.
- **Warning** (If `source` provided is not the start node of an Euler path)
  - will raise error even if an Euler Path exists.

3.28 Flows

3.28.1 Maximum Flow

`maximum_flow(flowG, _s, _t[, capacity, . . . ])` Find a maximum single-commodity flow.

Parameters

- `flowG` (*NetworkX graph*) – Edges of the graph are expected to have an attribute called ‘capacity’. If this attribute is not present, the edge is considered to have infinite capacity.
- `_s` (*node*) – Source node for the flow.
- `_t` (*node*) – Sink node for the flow.
- `capacity` (*string*) – Edges of the graph `G` are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: ‘capacity’.
- `flow_func` (*function*) – A function for computing the maximum flow among a pair of nodes in a capacitated graph. The function has to accept at least three parameters: a Graph or Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see Notes). If `flow_func` is None, the default maximum flow function (`preflow_push()`) is used. See below for alternative algorithms. The choice of the default function may change from version to version and should not be relied on. Default value: None.
• **kwargs** *(Any other keyword parameter is passed to the function that)* – computes the maximum flow.

**Returns**

• **flow_value** *(integer, float)* – Value of the maximum flow, i.e., net outflow from the source.

• **flow_dict** *(dict)* – A dictionary containing the value of the flow that went through each edge.

**Raises**

• **NetworkXError** – The algorithm does not support MultiGraph and MultiDiGraph. If the input graph is an instance of one of these two classes, a NetworkXError is raised.

• **NetworkXUnbounded** – If the graph has a path of infinite capacity, the value of a feasible flow on the graph is unbounded above and the function raises a NetworkXUnbounded.

**See also:**

- `maximum_flow_value()`, `minimum_cut()`, `minimum_cut_value()`, `edmonds_karp()`, `preflow_push()`, `shortest_augmenting_path()`

**Notes**

The function used in the `flow_func` parameter has to return a residual network that follows NetworkX conventions:

The residual network $R$ from an input graph $G$ has the same nodes as $G$. $R$ is a DiGraph that contains a pair of edges $(u, v)$ and $(v, u)$ iff $(u, v)$ is not a self-loop, and at least one of $(u, v)$ and $(v, u)$ exists in $G$.

For each edge $(u, v)$ in $R$, $R[u][v]['capacity']$ is equal to the capacity of $(u, v)$ in $G$ if it exists in $G$ or zero otherwise. If the capacity is infinite, $R[u][v]['capacity']$ will have a high arbitrary finite value that does not affect the solution of the problem. This value is stored in $R$.graph['inf']. For each edge $(u, v)$ in $R$, $R[u][v]['flow']$ represents the flow function of $(u, v)$ and satisfies $R[u][v]['flow'] == -R[v][u]['flow']$.

The flow value, defined as the total flow into $t$, the sink, is stored in $R$.graph['flow_value']. Reachability to $t$ using only edges $(u, v)$ such that $R[u][v]['flow'] < R[u][v]['capacity']$ induces a minimum $s$-$t$ cut.

Specific algorithms may store extra data in $R$.

The function should supports an optional boolean parameter `value_only`. When True, it can optionally terminate the algorithm as soon as the maximum flow value and the minimum cut can be determined.

**Examples**

```python
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_edge('x','a', capacity=3.0)
>>> G.add_edge('x','b', capacity=1.0)
>>> G.add_edge('a','c', capacity=3.0)
>>> G.add_edge('b','c', capacity=5.0)
>>> G.add_edge('b','d', capacity=4.0)
>>> G.add_edge('d','e', capacity=2.0)
>>> G.add_edge('c','y', capacity=2.0)
>>> G.add_edge('e','y', capacity=3.0)
```

maximum_flow returns both the value of the maximum flow and a dictionary with all flows.
>>> flow_value, flow_dict = nx.maximum_flow(G, 'x', 'y')
>>> flow_value
3.0
>>> print(flow_dict['x']['b'])
1.0

You can also use alternative algorithms for computing the maximum flow by using the flow_func parameter.

```python
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> flow_value == nx.maximum_flow(G, 'x', 'y', flow_func=shortest_augmenting_path)[0]
True
```

networkx.algorithms.flow.maximum_flow_value

maximum_flow_value (flowG, _s, _t, capacity='capacity', flow_func=None, **kwargs)

Find the value of maximum single-commodity flow.

Parameters

- **flowG** *(NetworkX graph)* - Edges of the graph are expected to have an attribute called ‘capacity’. If this attribute is not present, the edge is considered to have infinite capacity.
- **_s** *(node)* - Source node for the flow.
- **_t** *(node)* - Sink node for the flow.
- **capacity** *(string)* - Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: ‘capacity’.
- **flow_func** *(function)* - A function for computing the maximum flow among a pair of nodes in a capacitated graph. The function has to accept at least three parameters: a Graph or DiGraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see Notes). If flow_func is None, the default maximum flow function (preflow_push()) is used. See below for alternative algorithms. The choice of the default function may change from version to version and should not be relied on. Default value: None.
- **kwargs** *(Any other keyword parameter is passed to the function that)* - computes the maximum flow.

Returns **flow_value** - Value of the maximum flow, i.e., net outflow from the source.

Return type  integer, float

Raises

- **NetworkXError** - The algorithm does not support MultiGraph and MultiDiGraph. If the input graph is an instance of one of these two classes, a NetworkXError is raised.
- **NetworkXUnbounded** - If the graph has a path of infinite capacity, the value of a feasible flow on the graph is unbounded above and the function raises a NetworkXUnbounded.

See also:

maximum_flow(), minimum_cut(), minimum_cut_value(), edmonds_karp(), preflow_push(), shortest_augmenting_path()
Notes

The function used in the flow_func parameter has to return a residual network that follows NetworkX conventions:

The residual network $R$ from an input graph $G$ has the same nodes as $G$. $R$ is a DiGraph that contains a pair of edges $(u, v)$ and $(v, u)$ iff $(u, v)$ is not a self-loop, and at least one of $(u, v)$ and $(v, u)$ exists in $G$.

For each edge $(u, v)$ in $R$, $R[u][v]['capacity']$ is equal to the capacity of $(u, v)$ in $G$ if it exists in $G$ or zero otherwise. If the capacity is infinite, $R[u][v]['capacity']$ will have a high arbitrary finite value that does not affect the solution of the problem. This value is stored in $R$.graph['inf']. For each edge $(u, v)$ in $R$, $R[u][v]['flow']$ represents the flow function of $(u, v)$ and satisfies $R[u][v]['flow'] == -R[v][u]['flow']$.

The flow value, defined as the total flow into $t$, the sink, is stored in $R$.graph['flow_value']. Reachability to $t$ using only edges $(u, v)$ such that $R[u][v]['flow'] < R[u][v]['capacity']$ induces a minimum $s$-$t$ cut.

Specific algorithms may store extra data in $R$.

The function should supports an optional boolean parameter value_only. When True, it can optionally terminate the algorithm as soon as the maximum flow value and the minimum cut can be determined.

Examples

```python
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_edge('x','a', capacity=3.0)
>>> G.add_edge('x','b', capacity=1.0)
>>> G.add_edge('a','c', capacity=3.0)
>>> G.add_edge('b','c', capacity=5.0)
>>> G.add_edge('b','d', capacity=4.0)
>>> G.add_edge('d','e', capacity=2.0)
>>> G.add_edge('c','y', capacity=2.0)
>>> G.add_edge('e','y', capacity=3.0)
```

maximum_flow_value computes only the value of the maximum flow:

```python
>>> flow_value = nx.maximum_flow_value(G, 'x', 'y')
>>> flow_value
3.0
```

You can also use alternative algorithms for computing the maximum flow by using the flow_func parameter.

```python
>>> from networkx.algorithms.flow import shortest_augmenting_path
>>> flow_value == nx.maximum_flow_value(G, 'x', 'y', flow_func=shortest_augmenting_path)
True
```

networkx.algorithms.flow.minimum_cut

minimum_cut (flowG, s, t, capacity='capacity', flow_func=None, **kwargs)

Compute the value and the node partition of a minimum $(s, t)$-cut.

Use the max-flow min-cut theorem, i.e., the capacity of a minimum capacity cut is equal to the flow value of a maximum flow.

3.28. Flows
Parameters

- **flowG** (*NetworkX graph*) – Edges of the graph are expected to have an attribute called ‘capacity’. If this attribute is not present, the edge is considered to have infinite capacity.
- **s** (node) – Source node for the flow.
- **t** (node) – Sink node for the flow.
- **capacity** (string) – Edges of the graph \( G \) are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: ‘capacity’.
- **flow_func** (function) – A function for computing the maximum flow among a pair of nodes in a capacitated graph. The function has to accept at least three parameters: a Graph or DiGraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see Notes). If flow_func is None, the default maximum flow function (preflow_push()) is used. See below for alternative algorithms. The choice of the default function may change from version to version and should not be relied on. Default value: None.
- **kwargs** (Any other keyword parameter is passed to the function that) – computes the maximum flow.

Returns

- **cut_value** (integer, float) – Value of the minimum cut.
- **partition** (pair of node sets) – A partitioning of the nodes that defines a minimum cut.

Raises **NetworkXUnbounded** – If the graph has a path of infinite capacity, all cuts have infinite capacity and the function raises a NetworkXError.

See also: maximum_flow(), maximum_flow_value(), minimum_cut_value(), edmonds_karp(), preflow_push(), shortest_augmenting_path()

Notes

The function used in the flow_func parameter has to return a residual network that follows NetworkX conventions:

The residual network \( R \) from an input graph \( G \) has the same nodes as \( G \). \( R \) is a DiGraph that contains a pair of edges \((u, v)\) and \((v, u)\) iff \((u, v)\) is not a self-loop, and at least one of \((u, v)\) and \((v, u)\) exists in \( G \).

For each edge \((u, v)\) in \( R[u][v] \)['capacity'] is equal to the capacity of \((u, v)\) in \( G \) if it exists in \( G \) or zero otherwise. If the capacity is infinite, \( R[u][v] \)['capacity'] will have a high arbitrary finite value that does not affect the solution of the problem. This value is stored in \( R \).graph['inf']. For each edge \((u, v)\) in \( R[u][v] \)['flow'] represents the flow function of \((u, v)\) and satisfies \( R[u][v] \)['flow'] == \(-R[v][u] \)['flow'].

The flow value, defined as the total flow into \( t \), the sink, is stored in \( R \).graph['flow_value']. Reachability to \( t \) using only edges \((u, v)\) such that \( R[u][v] \)['flow'] < \( R[u][v] \)['capacity'] induces a minimum s-t cut.

Specific algorithms may store extra data in \( R \).

The function should supports an optional boolean parameter value_only. When True, it can optionally terminate the algorithm as soon as the maximum flow value and the minimum cut can be determined.
Examples

```python
>>> import networkx as nx

>>> G = nx.DiGraph()

>>> G.add_edge('x', 'a', capacity = 3.0)
>>> G.add_edge('x', 'b', capacity = 1.0)
>>> G.add_edge('a', 'c', capacity = 3.0)
>>> G.add_edge('b', 'c', capacity = 5.0)
>>> G.add_edge('b', 'd', capacity = 4.0)
>>> G.add_edge('d', 'e', capacity = 2.0)
>>> G.add_edge('c', 'y', capacity = 2.0)
>>> G.add_edge('e', 'y', capacity = 3.0)

minimum_cut computes both the value of the minimum cut and the node partition:

```python
>>> cut_value, partition = nx.minimum_cut(G, 'x', 'y')
>>> reachable, non_reachable = partition

'partition' here is a tuple with the two sets of nodes that define the minimum cut. You can compute the cut set of edges that induce the minimum cut as follows:

```python
>>> cutset = set()
>>> for u, nbrs in ((n, G[n]) for n in reachable):
...     cutset.update((u, v) for v in nbrs if v in non_reachable)
>>> print(sorted(cutset))
[('c', 'y'), ('x', 'b')]
>>> cut_value == sum(G.edges[u, v]['capacity'] for (u, v) in cutset)
True
```

You can also use alternative algorithms for computing the minimum cut by using the `flow_func` parameter.

```python
>>> from networkx.algorithms.flow import shortest_augmenting_path

>>> cut_value == nx.minimum_cut(G, 'x', 'y', flow_func=shortest_augmenting_path)[0]
True
```

networkx.algorithms.flow.minimum_cut_value

minimum_cut_value (flowG, _s, _t, capacity=’capacity’, flow_func=None, **kwargs)

Compute the value of a minimum (s, t)-cut.

Use the max-flow min-cut theorem, i.e., the capacity of a minimum capacity cut is equal to the flow value of a maximum flow.

Parameters

- flowG (NetworkX graph) – Edges of the graph are expected to have an attribute called ‘capacity’. If this attribute is not present, the edge is considered to have infinite capacity.
- _s (node) – Source node for the flow.
- _t (node) – Sink node for the flow.
- capacity (string) – Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: ‘capacity’.

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- **flow_func** *(function)* – A function for computing the maximum flow among a pair of nodes in a capacitated graph. The function has to accept at least three parameters: a Graph or Digraph, a source node, and a target node. And return a residual network that follows NetworkX conventions (see Notes). If flow_func is None, the default maximum flow function *(preflow_push())* is used. See below for alternative algorithms. The choice of the default function may change from version to version and should not be relied on. Default value: None.

- **kwargs** *(Any other keyword parameter is passed to the function that)* – computes the maximum flow.

**Returns** **cut_value** – Value of the minimum cut.

**Return type** integer, float

**Raises** *NetworkXUnbounded* – If the graph has a path of infinite capacity, all cuts have infinite capacity and the function raises a NetworkXError.

**See also:**

*maximum_flow(), maximum_flow_value(), minimum_cut(), edmonds_karp(), preflow_push(), shortest_augmenting_path()*

**Notes**

The function used in the flow_func parameter has to return a residual network that follows NetworkX conventions:

The residual network \( R \) from an input graph \( G \) has the same nodes as \( G \). \( R \) is a DiGraph that contains a pair of edges \((u, v)\) and \((v, u)\) iff \((u, v)\) is not a self-loop, and at least one of \((u, v)\) and \((v, u)\) exists in \( G \).

For each edge \((u, v)\) in \( R \), \( R[u][v]['capacity'] \) is equal to the capacity of \((u, v)\) in \( G \) if it exists in \( G \) or zero otherwise. If the capacity is infinite, \( R[u][v]['capacity'] \) will have a high arbitrary finite value that does not affect the solution of the problem. This value is stored in \( R\.graph['inf'] \). For each edge \((u, v)\) in \( R \), \( R[u][v]['flow'] \) represents the flow function of \((u, v)\) and satisfies \( R[u][v]['flow'] == -R[v][u]['flow'] \).

The flow value, defined as the total flow into \( t \), the sink, is stored in \( R\.graph['flow_value'] \). Reachability to \( t \) using only edges \((u, v)\) such that \( R[u][v]['flow'] < R[u][v]['capacity'] \) induces a minimum s-t cut.

Specific algorithms may store extra data in \( R \).

The function should supports an optional boolean parameter value_only. When True, it can optionally terminate the algorithm as soon as the maximum flow value and the minimum cut can be determined.

**Examples**

```python
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_edge('x','a', capacity = 3.0)
>>> G.add_edge('x','b', capacity = 1.0)
>>> G.add_edge('a','c', capacity = 3.0)
>>> G.add_edge('b','c', capacity = 5.0)
>>> G.add_edge('b','d', capacity = 4.0)
>>> G.add_edge('d','e', capacity = 2.0)
```
minimum_cut_value computes only the value of the minimum cut:

```python
>>> cut_value = nx.minimum_cut_value(G, 'x', 'y')
>>> cut_value
3.0
```

You can also use alternative algorithms for computing the minimum cut by using the flow_func parameter.

```python
>>> from networkx.algorithms.flow import shortest_augmenting_path
```

```python
>>> cut_value == nx.minimum_cut_value(G, 'x', 'y', flow_func=shortest_augmenting_path)
True
```

### 3.28.2 Edmonds-Karp

`edmonds_karp(G, s, t[, capacity, residual, ...])` Find a maximum single-commodity flow using the Edmonds-Karp algorithm.

```python
networkx.algorithms.flow.edmonds_karp
```

`edmonds_karp` *(G, s, t, capacity=’capacity’, residual=None, value_only=False, cutoff=None)*

Find a maximum single-commodity flow using the Edmonds-Karp algorithm.

This function returns the residual network resulting after computing the maximum flow. See below for details about the conventions NetworkX uses for defining residual networks.

This algorithm has a running time of $O(nm^2)$ for $n$ nodes and $m$ edges.

**Parameters**

- **G** *(NetworkX graph)* – Edges of the graph are expected to have an attribute called ‘capacity’. If this attribute is not present, the edge is considered to have infinite capacity.

- **s** *(node)* – Source node for the flow.

- **t** *(node)* – Sink node for the flow.

- **capacity** *(string)* – Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: ‘capacity’.

- **residual** *(NetworkX graph)* – Residual network on which the algorithm is to be executed. If None, a new residual network is created. Default value: None.

- **value_only** *(bool)* – If True compute only the value of the maximum flow. This parameter will be ignored by this algorithm because it is not applicable.

- **cutoff** *(integer, float)* – If specified, the algorithm will terminate when the flow value reaches or exceeds the cutoff. In this case, it may be unable to immediately determine a minimum cut. Default value: None.

**Returns** *R* – Residual network after computing the maximum flow.

**Return type** *NetworkX DiGraph*
Raises

- **NetworkXError** – The algorithm does not support MultiGraph and MultiDiGraph. If the input graph is an instance of one of these two classes, a NetworkXError is raised.

- **NetworkXUnbounded** – If the graph has a path of infinite capacity, the value of a feasible flow on the graph is unbounded above and the function raises a NetworkXUnbounded.

See also:

maximum_flow(), minimum_cut(), preflow_push(), shortest_augmenting_path()

Notes

The residual network \( R \) from an input graph \( G \) has the same nodes as \( G \). \( R \) is a DiGraph that contains a pair of edges \((u, v)\) and \((v, u)\) iff \((u, v)\) is not a self-loop, and at least one of \((u, v)\) and \((v, u)\) exists in \( G \).

For each edge \((u, v)\) in \( R[u][v][\text{capacity}] \) is equal to the capacity of \((u, v)\) in \( G \) if it exists in \( G \) or zero otherwise. If the capacity is infinite, \( R[u][v][\text{capacity}] \) will have a high arbitrary finite value that does not affect the solution of the problem. This value is stored in \( R\.graph[\text{inf}] \). For each edge \((u, v)\) in \( R[u][v][\text{flow}] \) represents the flow function of \((u, v)\) and satisfies \( R[u][v][\text{flow}] == -R[v][u][\text{flow}] \).

The flow value, defined as the total flow into \( t \), the sink, is stored in \( R\.graph[\text{flow_value}] \). If cutoff is not specified, reachability to \( t \) using only edges \((u, v)\) such that \( R[u][v][\text{flow}] < R[u][v][\text{capacity}] \) induces a minimum \( s \)-\( t \) cut.

Examples

```python
>>> import networkx as nx
>>> from networkx.algorithms.flow import edmonds_karp

The functions that implement flow algorithms and output a residual network, such as this one, are not imported to the base NetworkX namespace, so you have to explicitly import them from the flow package.

```python
>>> G = nx.DiGraph()
>>> G.add_edge('x', 'a', capacity=3.0)
>>> G.add_edge('x', 'b', capacity=1.0)
>>> G.add_edge('a', 'c', capacity=3.0)
>>> G.add_edge('b', 'c', capacity=5.0)
>>> G.add_edge('b', 'd', capacity=4.0)
>>> G.add_edge('d', 'e', capacity=2.0)
>>> G.add_edge('c', 'y', capacity=2.0)
>>> G.add_edge('e', 'y', capacity=3.0)
>>> R = edmonds_karp(G, 'x', 'y')
>>> flow_value = nx.maximum_flow_value(G, 'x', 'y')
>>> flow_value
3.0
>>> flow_value == R.graph[\'flow_value\']
True
```

3.28.3 Shortest Augmenting Path
NetworkX Reference, Release 2.5rc1.dev20191214022633

**shortest_augmenting_path**(G, s, t[, ...]) Find a maximum single-commodity flow using the shortest augmenting path algorithm.

**networkx.algorithms.flow.shortest_augmenting_path**

**shortest_augmenting_path** *(G, s, t, capacity='capacity', residual=None, value_only=False, two_phase=False, cutoff=None)*

Find a maximum single-commodity flow using the shortest augmenting path algorithm.

This function returns the residual network resulting after computing the maximum flow. See below for details about the conventions NetworkX uses for defining residual networks.

This algorithm has a running time of $O(n^2m)$ for $n$ nodes and $m$ edges.

**Parameters**

- **G** *(NetworkX graph)* – Edges of the graph are expected to have an attribute called ‘capacity’. If this attribute is not present, the edge is considered to have infinite capacity.
- **s** *(node)* – Source node for the flow.
- **t** *(node)* – Sink node for the flow.
- **capacity** *(string)* – Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: ‘capacity’.
- **residual** *(NetworkX graph)* – Residual network on which the algorithm is to be executed. If None, a new residual network is created. Default value: None.
- **value_only** *(bool)* – If True compute only the value of the maximum flow. This parameter will be ignored by this algorithm because it is not applicable.
- **two_phase** *(bool)* – If True, a two-phase variant is used. The two-phase variant improves the running time on unit-capacity networks from $O(nm)$ to $O(\min(n^{2/3}, m^{1/2})m)$. Default value: False.
- **cutoff** *(integer, float)* – If specified, the algorithm will terminate when the flow value reaches or exceeds the cutoff. In this case, it may be unable to immediately determine a minimum cut. Default value: None.

**Returns** **R** – Residual network after computing the maximum flow.

**Return type** NetworkX DiGraph

**Raises**

- **NetworkXError** – The algorithm does not support MultiGraph and MultiDiGraph. If the input graph is an instance of one of these two classes, a NetworkXError is raised.
- **NetworkXUnbounded** – If the graph has a path of infinite capacity, the value of a feasible flow on the graph is unbounded above and the function raises a NetworkXUnbounded.

**See also:**

*maximum_flow*, *minimum_cut*, *edmonds_karp*, *.preflow_push*

**Notes**

The residual network $R$ from an input graph $G$ has the same nodes as $G$. $R$ is a DiGraph that contains a pair of edges $(u, v)$ and $(v, u)$ iff $(u, v)$ is not a self-loop, and at least one of $(u, v)$ and $(v, u)$ exists in $G$. 3.28. Flows 335
For each edge \((u,v)\) in \(R[u][v][\text{capacity}]\) is equal to the capacity of \((u,v)\) in \(G\) if it exists in \(G\) or zero otherwise. If the capacity is infinite, \(R[u][v][\text{capacity}]\) will have a high arbitrary finite value that does not affect the solution of the problem. This value is stored in \(R\.\text{graph}[\text{inf}]\). For each edge \((u,v)\) in \(R\), \(R[u][v][\text{flow}]\) represents the flow function of \((u,v)\) and satisfies \(R[u][v][\text{flow}] == -R[v][u][\text{flow}]\).

The flow value, defined as the total flow into \(t\), the sink, is stored in \(R\.\text{graph}[\text{flow_value}]\). If \(cutoff\) is not specified, reachability to \(t\) using only edges \((u,v)\) such that \(R[u][v][\text{flow}] < R[u][v][\text{capacity}]\) induces a minimum \(s\)-\(t\) cut.

**Examples**

```python
>>> import networkx as nx
>>> from networkx.algorithms.flow import shortest_augmenting_path

>>> G = nx.DiGraph()
>>> G.add_edge('x','a', capacity=3.0)
>>> G.add_edge('x','b', capacity=1.0)
>>> G.add_edge('a','c', capacity=3.0)
>>> G.add_edge('b','c', capacity=5.0)
>>> G.add_edge('b','d', capacity=4.0)
>>> G.add_edge('d','e', capacity=2.0)
>>> G.add_edge('c','y', capacity=2.0)
>>> G.add_edge('e','y', capacity=3.0)
>>> R = shortest_augmenting_path(G, 'x', 'y')
>>> flow_value = nx.maximum_flow_value(G, 'x', 'y')
>>> flow_value
3.0
>>> flow_value == R\.\text{graph}[\text{flow_value}]
True
```

### 3.28.4 Preflow-Push

`preflow_push(G, s, t[, capacity, residual, ...])`  
Find a maximum single-commodity flow using the highest-label preflow-push algorithm.

**Examples**

```python
>>> G = nx.DiGraph()
>>> G.add_edge('x','a', capacity=3.0)
>>> G.add_edge('x','b', capacity=1.0)
>>> G.add_edge('a','c', capacity=3.0)
>>> G.add_edge('b','c', capacity=5.0)
>>> G.add_edge('b','d', capacity=4.0)
>>> G.add_edge('d','e', capacity=2.0)
>>> G.add_edge('c','y', capacity=2.0)
>>> G.add_edge('e','y', capacity=3.0)
>>> R = shortest_augmenting_path(G, 'x', 'y')
>>> flow_value = nx.maximum_flow_value(G, 'x', 'y')
>>> flow_value
3.0
>>> flow_value == R\.\text{graph}[\text{flow_value}]
True
```
• \texttt{t (node)} – Sink node for the flow.

• \texttt{capacity (string)} – Edges of the graph \(G\) are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: ‘capacity’.

• \texttt{residual (NetworkX graph)} – Residual network on which the algorithm is to be executed. If None, a new residual network is created. Default value: None.

• \texttt{global_relabel_freq (integer, float)} – Relative frequency of applying the global relabeling heuristic to speed up the algorithm. If it is None, the heuristic is disabled. Default value: 1.

• \texttt{value_only (bool)} – If False, compute a maximum flow; otherwise, compute a maximum preflow which is enough for computing the maximum flow value. Default value: False.

Returns \(R\) – Residual network after computing the maximum flow.

Return type NetworkX DiGraph

Raises

• \texttt{NetworkXError} – The algorithm does not support MultiGraph and MultiDiGraph. If the input graph is an instance of one of these two classes, a NetworkXError is raised.

• \texttt{NetworkXUnbounded} – If the graph has a path of infinite capacity, the value of a feasible flow on the graph is unbounded above and the function raises a NetworkXUnbounded.

See also:

\(\text{maximum\_flow()}, \text{minimum\_cut()}, \text{edmonds\_karp()}, \text{shortest\_augmenting\_path()}\)

Notes

The residual network \(R\) from an input graph \(G\) has the same nodes as \(G\). \(R\) is a DiGraph that contains a pair of edges \((u, v)\) and \((v, u)\) iff \((u, v)\) is not a self-loop, and at least one of \((u, v)\) and \((v, u)\) exists in \(G\). For each node \(u\) in \(R\), \(R\).nodes[\(u\)]['excess'] represents the difference between flow into \(u\) and flow out of \(u\).

For each edge \((u, v)\) in \(R, R[u][v]['capacity']\) is equal to the capacity of \((u, v)\) in \(G\) if it exists in \(G\) or zero otherwise. If the capacity is infinite, \(R[u][v]['capacity']\) will have a high arbitrary finite value that does not affect the solution of the problem. This value is stored in \(R\.graph['inf']\). For each edge \((u, v)\) in \(R, R[u][v]['flow']\) represents the flow function of \((u, v)\) and satisfies \(R[u][v]['flow'] == -R[v][u]['flow']\).

The flow value, defined as the total flow into \(t\), the sink, is stored in \(R\.graph['flow\_value']\). Reachability to \(t\) using only edges \((u, v)\) such that \(R[u][v]['flow'] < R[u][v]['capacity']\) induces a minimum s-t cut.

Examples

```python
>>> import networkx as nx
>>> from networkx.algorithms.flow import preflow_push
```

The functions that implement flow algorithms and output a residual network, such as this one, are not imported to the base NetworkX namespace, so you have to explicitly import them from the flow package.
```python
>>> G = nx.DiGraph()
>>> G.add_edge('x', 'a', capacity=3.0)
>>> G.add_edge('x', 'b', capacity=1.0)
>>> G.add_edge('a', 'c', capacity=3.0)
>>> G.add_edge('b', 'c', capacity=5.0)
>>> G.add_edge('b', 'd', capacity=4.0)
>>> G.add_edge('d', 'e', capacity=2.0)
>>> G.add_edge('c', 'y', capacity=2.0)
>>> G.add_edge('e', 'y', capacity=3.0)
>>> R = preflow_push(G, 'x', 'y')
>>> flow_value = nx.maximum_flow_value(G, 'x', 'y')
>>> flow_value == R.graph['flow_value']
True
>>> # preflow_push also stores the maximum flow value
>>> # in the excess attribute of the sink node t
>>> flow_value == R.nodes['y']['excess']
True
>>> # For some problems, you might only want to compute a
>>> # maximum preflow.
>>> R = preflow_push(G, 'x', 'y', value_only=True)
>>> flow_value == R.graph['flow_value']
True
>>> flow_value == R.nodes['y']['excess']
True
```

### 3.28.5 Dinitz

*dinitz*(G, s, t[, capacity, residual,…])

Find a maximum single-commodity flow using Dinitz’ algorithm.

**networkx.algorithms.flow.dinitz**

**dinitz** (G, s, t, capacity='capacity', residual=None, value_only=False, cutoff=None)

Find a maximum single-commodity flow using Dinitz’ algorithm.

This function returns the residual network resulting after computing the maximum flow. See below for details about the conventions NetworkX uses for defining residual networks.

This algorithm has a running time of \(O(n^2m)\) for \(n\) nodes and \(m\) edges\(^1\).

**Parameters**

- **G** (*NetworkX graph*) – Edges of the graph are expected to have an attribute called ‘capacity’. If this attribute is not present, the edge is considered to have infinite capacity.
- **s** (*node*) – Source node for the flow.
- **t** (*node*) – Sink node for the flow.
- **capacity** (*string*) – Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: ‘capacity’.
- **residual** (*NetworkX graph*) – Residual network on which the algorithm is to be executed. If None, a new residual network is created. Default value: None.

---

• **value_only** (bool) – If True compute only the value of the maximum flow. This parameter will be ignored by this algorithm because it is not applicable.

• **cutoff** (integer, float) – If specified, the algorithm will terminate when the flow value reaches or exceeds the cutoff. In this case, it may be unable to immediately determine a minimum cut. Default value: None.

**Returns** R – Residual network after computing the maximum flow.

**Return type** NetworkX DiGraph

**Raises**

• **NetworkXError** – The algorithm does not support MultiGraph and MultiDiGraph. If the input graph is an instance of one of these two classes, a NetworkXError is raised.

• **NetworkXUnbounded** – If the graph has a path of infinite capacity, the value of a feasible flow on the graph is unbounded above and the function raises a NetworkXUnbounded.

See also:

`maximum_flow()`, `minimum_cut()`, `preflow_push()`, `shortest_augmenting_path()`

**Notes**

The residual network \( R \) from an input graph \( G \) has the same nodes as \( G \). \( R \) is a DiGraph that contains a pair of edges \((u, v)\) and \((v, u)\) iff \((u, v)\) is not a self-loop, and at least one of \((u, v)\) and \((v, u)\) exists in \( G \).

For each edge \((u, v)\) in \( R \), \( R[u][v]['capacity'] \) is equal to the capacity of \((u, v)\) in \( G \) if it exists in \( G \) or zero otherwise. If the capacity is infinite, \( R[u][v]['capacity'] \) will have a high arbitrary finite value that does not affect the solution of the problem. This value is stored in \( R.graph['inf'] \). For each edge \((u, v)\) in \( R \), \( R[u][v]['flow'] \) represents the flow function of \((u, v)\) and satisfies \( R[u][v]['flow'] = -R[v][u]['flow'] \).

The flow value, defined as the total flow into \( t \), the sink, is stored in \( R.graph['flow_value'] \). If cutoff is not specified, reachability to \( t \) using only edges \((u, v)\) such that \( R[u][v]['flow'] < R[u][v]['capacity'] \) induces a minimum s-t cut.

**Examples**

```python
>>> import networkx as nx
>>> from networkx.algorithms.flow import dinitz
```

The functions that implement flow algorithms and output a residual network, such as this one, are not imported to the base NetworkX namespace, so you have to explicitly import them from the flow package.

```python
>>> G = nx.DiGraph()
>>> G.add_edge('x','a', capacity=3.0)
>>> G.add_edge('x','b', capacity=1.0)
>>> G.add_edge('a','c', capacity=3.0)
>>> G.add_edge('b','c', capacity=5.0)
>>> G.add_edge('b','d', capacity=4.0)
>>> G.add_edge('d','e', capacity=2.0)
>>> G.add_edge('c','y', capacity=2.0)
>>> G.add_edge('e','y', capacity=3.0)
>>> R = dinitz(G, 'x', 'y')
>>> flow_value = nx.maximum_flow_value(G, 'x', 'y')
```

(continues on next page)
>>> flow_value
3.0
>>> flow_value == R.graph['flow_value']
True

References

3.28.6 Boykov-Kolmogorov

`boykov_kolmogorov(G, s, t[, capacity, ...])`

Find a maximum single-commodity flow using Boykov-Kolmogorov algorithm.

Parameters

- `G (NetworkX graph)` – Edges of the graph are expected to have an attribute called 'capacity'. If this attribute is not present, the edge is considered to have infinite capacity.
- `s (node)` – Source node for the flow.
- `t (node)` – Sink node for the flow.
- `capacity (string)` – Edges of the graph `G` are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: ‘capacity’.
- `residual (NetworkX graph)` – Residual network on which the algorithm is to be executed. If None, a new residual network is created. Default value: None.
- `value_only (bool)` – If True compute only the value of the maximum flow. This parameter will be ignored by this algorithm because it is not applicable.
- `cutoff (integer, float)` – If specified, the algorithm will terminate when the flow value reaches or exceeds the cutoff. In this case, it may be unable to immediately determine a minimum cut. Default value: None.

Returns `R` – Residual network after computing the maximum flow.

Return type NetworkX DiGraph

Raises

• **NetworkXError** – The algorithm does not support MultiGraph and MultiDiGraph. If the input graph is an instance of one of these two classes, a NetworkXError is raised.

• **NetworkXUnbounded** – If the graph has a path of infinite capacity, the value of a feasible flow on the graph is unbounded above and the function raises a NetworkXUnbounded.

**See also:**

`maximum_flow()`, `minimum_cut()`, `preflow_push()`, `shortest_augmenting_path()`

**Notes**

The residual network \( R \) from an input graph \( G \) has the same nodes as \( G \). \( R \) is a DiGraph that contains a pair of edges \((u, v)\) and \((v, u)\) iff \((u, v)\) is not a self-loop, and at least one of \((u, v)\) and \((v, u)\) exists in \( G \).

For each edge \((u, v)\) in \( R[u][v][\text{'capacity'}] \) is equal to the capacity of \((u, v)\) in \( G \) if it exists in \( G \) or zero otherwise. If the capacity is infinite, \( R[u][v][\text{'capacity'}] \) will have a high arbitrary finite value that does not affect the solution of the problem. This value is stored in \( R\.graph[\text{inf}] \). For each edge \((u, v)\) in \( R, R[u][v][\text{'flow'}] \) represents the flow function of \((u, v)\) and satisfies \( R[u][v][\text{'flow'}] = -R[v][u][\text{'flow'}] \).

The flow value, defined as the total flow into \( t \), the sink, is stored in \( R\.graph[\text{flow_value}] \). If \( \text{cutoff} \) is not specified, reachability to \( t \) using only edges \((u, v)\) such that \( R[u][v][\text{'flow'}] < R[u][v][\text{'capacity'}] \) induces a minimum \( s-t \) cut.

**Examples**

```python
>>> import networkx as nx
>>> from networkx.algorithms.flow import boykov_kolmogorov

The functions that implement flow algorithms and output a residual network, such as this one, are not imported to the base NetworkX namespace, so you have to explicitly import them from the flow package.

```python
>>> G = nx.DiGraph()
>>> G.add_edge('x','a', capacity=3.0)
>>> G.add_edge('x','b', capacity=1.0)
>>> G.add_edge('a','c', capacity=3.0)
>>> G.add_edge('b','c', capacity=5.0)
>>> G.add_edge('b','d', capacity=4.0)
>>> G.add_edge('d','e', capacity=2.0)
>>> G.add_edge('c','y', capacity=2.0)
>>> G.add_edge('e','y', capacity=3.0)
>>> R = boykov_kolmogorov(G, 'x', 'y')
>>> flow_value = nx.maximum_flow_value(G, 'x', 'y')
>>> flow_value
3.0
>>> flow_value == R.graph['flow_value']
True
```

A nice feature of the Boykov-Kolmogorov algorithm is that a partition of the nodes that defines a minimum cut can be easily computed based on the search trees used during the algorithm. These trees are stored in the graph attribute `trees` of the residual network.

```python
>>> source_tree, target_tree = R.graph['trees']
>>> partition = (set(source_tree), set(G) - set(source_tree))
```
Or equivalently:

```python
>>> partition = (set(G) - set(target_tree), set(target_tree))
```

References

3.28.7 Gomory-Hu Tree

**gomory_hu_tree** *(G[, capacity, flow_func])*  
Returns the Gomory-Hu tree of an undirected graph G.

A Gomory-Hu tree of an undirected graph with capacities is a weighted tree that represents the minimum s-t cuts for all s-t pairs in the graph.

It only requires \( n-1 \) minimum cut computations instead of the obvious \( n(n-1)/2 \). The tree represents all s-t cuts as the minimum cut value among any pair of nodes is the minimum edge weight in the shortest path between the two nodes in the Gomory-Hu tree.

The Gomory-Hu tree also has the property that removing the edge with the minimum weight in the shortest path between any two nodes leaves two connected components that form a partition of the nodes in G that defines the minimum s-t cut.

See Examples section below for details.

**Parameters**

- **G** *(NetworkX graph)* – Undirected graph
- **capacity** *(string)* – Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: ‘capacity’.
- **flow_func** *(function)* – Function to perform the underlying flow computations. Default value `edmonds_karp()`. This function performs better in sparse graphs with right tailed degree distributions. `shortest_augmenting_path()` will perform better in denser graphs.

**Returns**  
Tree – A NetworkX graph representing the Gomory-Hu tree of the input graph.

**Return type** NetworkX graph

**Raises**

- **NetworkXNotImplemented** – Raised if the input graph is directed.
- **NetworkXError** – Raised if the input graph is an empty Graph.

**Examples**

```python
>>> G = nx.karate_club_graph()
>>> nx.set_edge_attributes(G, 1, 'capacity')
>>> T = nx.gomory_hu_tree(G)
>>> # The value of the minimum cut between any pair
... # of nodes in G is the minimum edge weight in the
```

(continues on next page)
... # shortest path between the two nodes in the
... # Gomory-Hu tree.
... def minimum_edge_weight_in_shortest_path(T, u, v):
...     path = nx.shortest_path(T, u, v, weight='weight')
...     return min((T[u][v]['weight'], (u,v)) for (u, v) in zip(path, path[1:]))
>>> u, v = 0, 33
>>> cut_value, edge = minimum_edge_weight_in_shortest_path(T, u, v)
>>> cut_value
10
>>> nx.minimum_cut_value(G, u, v)
10
>>> u, v = 0, 33
>>> cut_value, edge = minimum_edge_weight_in_shortest_path(T, u, v)
>>> T.remove_edge(*edge)
>>> U, V = list(nx.connected_components(T))
>>> # Thus U and V form a partition that defines a minimum cut
... # between u and v in G. You can compute the edge cut set,
... # that is, the set of edges that if removed from G will
... # disconnect u from v in G, with this information:
... cutset = set()
>>> for x, nbrs in ((n, G[n]) for n in U):
...     cutset.update((x, y) for y in nbrs if y in V)
>>> # Because we have set the capacities of all edges to 1
... # the cutset contains ten edges
... len(cutset)
10
>>> # You can use any maximum flow algorithm for the underlying
... # flow computations using the argument flow_func
... from networkx.algorithms import flow
>>> T = nx.gomory_hu_tree(G, flow_func=flow.boykov_kolmogorov)
>>> cut_value, edge = minimum_edge_weight_in_shortest_path(T, u, v)
>>> cut_value
10
>>> nx.minimum_cut_value(G, u, v, flow_func=flow.boykov_kolmogorov)
10

Notes

This implementation is based on Gusfield approach\(^1\) to compute Comory-Hu trees, which does not require node contractions and has the same computational complexity than the original method.

See also:

minimum_cut(), maximum_flow()

References

3.28.8 Utils

build_residual_network(G, capacity)  
Build a residual network and initialize a zero flow.

networkx.algorithms.flow.build_residual_network

build_residual_network (G, capacity)

Build a residual network and initialize a zero flow.

The residual network R from an input graph G has the same nodes as G. R is a DiGraph that contains a pair of edges (u, v) and (v, u) iff (u, v) is not a self-loop, and at least one of (u, v) and (v, u) exists in G.

For each edge (u, v) in R, R[u][v]['capacity'] is equal to the capacity of (u, v) in G if it exists in G or zero otherwise. If the capacity is infinite, R[u][v]['capacity'] will have a high arbitrary finite value that does not affect the solution of the problem. This value is stored in R.graph['inf']. For each edge (u, v) in R, R[u][v]['flow'] represents the flow function of (u, v) and satisfies R[u][v]['flow'] == -R[v][u]['flow'].

The flow value, defined as the total flow into t, the sink, is stored in R.graph['flow_value']. If cutoff is not specified, reachability to t using only edges (u, v) such that R[u][v]['flow'] < R[u][v]['capacity'] induces a minimum s-t cut.

3.28.9 Network Simplex

network_simplex(G[, demand, capacity, weight])  
Find a minimum cost flow satisfying all demands in digraph G.

This is a primal network simplex algorithm that uses the leaving arc rule to prevent cycling.

G is a digraph with edge costs and capacities and in which nodes have demand, i.e., they want to send or receive some amount of flow. A negative demand means that the node wants to send flow, a positive demand means that the node want to receive flow. A flow on the digraph G satisfies all demand if the net flow into each node is equal to the demand of that node.

Parameters

- G (NetworkX graph) – DiGraph on which a minimum cost flow satisfying all demands is to be found.
- demand (string) – Nodes of the graph G are expected to have an attribute demand that indicates how much flow a node wants to send (negative demand) or receive (positive demand). Note that the sum of the demands should be 0 otherwise the problem in not feasible. If this attribute is not present, a node is considered to have 0 demand. Default value: ‘demand’.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
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<tbody>
<tr>
<td>network_simplex</td>
<td>Find a minimum cost flow satisfying all demands in digraph G.</td>
</tr>
<tr>
<td>min_cost_flow_cost</td>
<td>Find the cost of a minimum cost flow satisfying all demands in digraph G.</td>
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<td>Returns a minimum cost flow satisfying all demands in digraph G.</td>
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<td>cost_of_flow</td>
<td>Compute the cost of the flow given by flowDict on graph G.</td>
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<td>max_flow_min_cost</td>
<td>Returns a maximum (s, t)-flow of minimum cost.</td>
</tr>
</tbody>
</table>

networkx.algorithms.min_cost_flow

network_simplex (G, demand='demand', capacity='capacity', weight='weight')  
Find a minimum cost flow satisfying all demands in digraph G.

This is a primal network simplex algorithm that uses the leaving arc rule to prevent cycling.

G is a digraph with edge costs and capacities and in which nodes have demand, i.e., they want to send or receive some amount of flow. A negative demand means that the node wants to send flow, a positive demand means that the node want to receive flow. A flow on the digraph G satisfies all demand if the net flow into each node is equal to the demand of that node.

Parameters

- G (NetworkX graph) – DiGraph on which a minimum cost flow satisfying all demands is to be found.
- demand (string) – Nodes of the graph G are expected to have an attribute demand that indicates how much flow a node wants to send (negative demand) or receive (positive demand). Note that the sum of the demands should be 0 otherwise the problem in not feasible. If this attribute is not present, a node is considered to have 0 demand. Default value: ‘demand’.
• **capacity** *(string)* – Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: ‘capacity’.

• **weight** *(string)* – Edges of the graph G are expected to have an attribute weight that indicates the cost incurred by sending one unit of flow on that edge. If not present, the weight is considered to be 0. Default value: ‘weight’.

**Returns**

• **flowCost** *(integer, float)* – Cost of a minimum cost flow satisfying all demands.

• **flowDict** *(dictionary)* – Dictionary of dictionaries keyed by nodes such that flowDict[u][v] is the flow edge (u, v).

**Raises**

• **NetworkXError** – This exception is raised if the input graph is not directed, not connected or is a multigraph.

• **NetworkXUnfeasible** – This exception is raised in the following situations:
  
  – The sum of the demands is not zero. Then, there is no flow satisfying all demands.
  
  – There is no flow satisfying all demand.

• **NetworkXUnbounded** – This exception is raised if the digraph G has a cycle of negative cost and infinite capacity. Then, the cost of a flow satisfying all demands is unbounded below.

**Notes**

This algorithm is not guaranteed to work if edge weights or demands are floating point numbers (overflows and roundoff errors can cause problems). As a workaround you can use integer numbers by multiplying the relevant edge attributes by a convenient constant factor (eg 100).

**See also:**

cost_of_flow(), max_flow_min_cost(), min_cost_flow(), min_cost_flow_cost()

**Examples**

A simple example of a min cost flow problem.

```python
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_node('a', demand=-5)
>>> G.add_node('d', demand=5)
>>> G.add_edge('a', 'b', weight=3, capacity=4)
>>> G.add_edge('a', 'c', weight=6, capacity=10)
>>> G.add_edge('b', 'd', weight=1, capacity=9)
>>> G.add_edge('c', 'd', weight=2, capacity=5)
>>> flowCost, flowDict = nx.network_simplex(G)
>>> flowCost
24
>>> flowDict
{'a': {'c': 1, 'b': 4}, 'c': {'d': 1}, 'b': {'d': 4}, 'd': {}}
```
The mincost flow algorithm can also be used to solve shortest path problems. To find the shortest path between two nodes u and v, give all edges an infinite capacity, give node u a demand of -1 and node v a demand a 1. Then run the network simplex. The value of a min cost flow will be the distance between u and v and edges carrying positive flow will indicate the path.

```python
>>> G=nx.DiGraph()
>>> G.add_weighted_edges_from([('s', 'u' ,10), ('s' ,'x' ,5),
... ('u', 'v' ,1), ('u' ,'x' ,2),
... ('v', 'y' ,1), ('x' ,u' ,3),
... ('x', 'v' ,5), ('x' ,y' ,2),
... ('y', 's' ,7), ('y' ,'v' ,6)])
>>> G.add_node('s', demand = -1)
>>> G.add_node('v', demand = 1)
>>> flowCost, flowDict = nx.network_simplex(G)
>>> flowCost == nx.shortest_path_length(G, 's', 'v', weight='weight')
True
>>> sorted([(u, v) for u in flowDict for v in flowDict[u] if flowDict[u][v] > 0])
[('s', 'x'), ('u', 'v'), ('x', 'u')]
>>> nx.shortest_path(G, 's', 'v', weight = 'weight')
['s', 'x', 'u', 'v']
```

It is possible to change the name of the attributes used for the algorithm.

```python
>>> G = nx.DiGraph()
>>> G.add_node('p', spam=-4)
>>> G.add_node('q', spam=2)
>>> G.add_node('a', spam=-2)
>>> G.add_node('d', spam=-1)
>>> G.add_node('t', spam=2)
>>> G.add_node('w', spam=3)
>>> G.add_edge('p', 'q', cost=7, vacancies=5)
>>> G.add_edge('p', 'a', cost=1, vacancies=4)
>>> G.add_edge('q', 'd', cost=2, vacancies=3)
>>> G.add_edge('t', 'q', cost=1, vacancies=2)
>>> G.add_edge('a', 't', cost=2, vacancies=4)
>>> G.add_edge('d', 'w', cost=3, vacancies=4)
>>> G.add_edge('t', 'w', cost=4, vacancies=1)
>>> flowCost, flowDict = nx.network_simplex(G, demand='spam',
... capacity='vacancies',
... weight='cost')
>>> flowCost
37
>>> flowDict
{'a': {'t': 4}, 'd': {'w': 2}, 'q': {'d': 1}, 'p': {'q': 2, 'a': 2}, 't': {'q': 1, 'w': 1}, 'w': {}}
```

References

networkx.algorithms.flow.min_cost_flow

```
min_cost_flow_cost (G, demand='demand', capacity='capacity', weight='weight')
```

Find the cost of a minimum cost flow satisfying all demands in digraph G.

G is a digraph with edge costs and capacities and in which nodes have demand, i.e., they want to send or receive some amount of flow. A negative demand means that the node wants to send flow, a positive demand means that the node want to receive flow. A flow on the digraph G satisfies all demand if the net flow into each node is equal to the demand of that node.
Parameters

- **G** (*NetworkX graph*) – DiGraph on which a minimum cost flow satisfying all demands is to be found.
- **demand** (*string*) – Nodes of the graph G are expected to have an attribute demand that indicates how much flow a node wants to send (negative demand) or receive (positive demand). Note that the sum of the demands should be 0 otherwise the problem in not feasible. If this attribute is not present, a node is considered to have 0 demand. Default value: ‘demand’.
- **capacity** (*string*) – Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: ‘capacity’.
- **weight** (*string*) – Edges of the graph G are expected to have an attribute weight that indicates the cost incurred by sending one unit of flow on that edge. If not present, the weight is considered to be 0. Default value: ‘weight’.

Returns **flowCost** – Cost of a minimum cost flow satisfying all demands.

Return type integer, float

Raises

- **NetworkXError** – This exception is raised if the input graph is not directed or not connected.
- **NetworkXUnfeasible** – This exception is raised in the following situations:
  - The sum of the demands is not zero. Then, there is no flow satisfying all demands.
  - There is no flow satisfying all demand.
- **NetworkXUnbounded** – This exception is raised if the digraph G has a cycle of negative cost and infinite capacity. Then, the cost of a flow satisfying all demands is unbounded below.

See also:

cost_of_flow(), max_flow_min_cost(), min_cost_flow(), network_simplex()

Notes

This algorithm is not guaranteed to work if edge weights or demands are floating point numbers (overflows and roundoff errors can cause problems). As a workaround you can use integer numbers by multiplying the relevant edge attributes by a convenient constant factor (eg 100).

Examples

A simple example of a min cost flow problem.

```python
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_node('a', demand = -5)
>>> G.add_node('d', demand = 5)
>>> G.add_edge('a', 'b', weight = 3, capacity = 4)
>>> G.add_edge('a', 'c', weight = 6, capacity = 10)
>>> G.add_edge('b', 'd', weight = 1, capacity = 9)
>>> G.add_edge('c', 'd', weight = 2, capacity = 5)
>>> flowCost = nx.min_cost_flow_cost(G)
```

(continues on next page)
networkx.algorithms.flow.min_cost_flow

**min_cost_flow** \((G, \text{demand}='\text{demand}', \text{capacity}='\text{capacity}', \text{weight}='\text{weight}')\)

Returns a minimum cost flow satisfying all demands in digraph \(G\).

\(G\) is a digraph with edge costs and capacities and in which nodes have demand, i.e., they want to send or receive some amount of flow. A negative demand means that the node wants to send flow, a positive demand means that the node want to receive flow. A flow on the digraph \(G\) satisfies all demand if the net flow into each node is equal to the demand of that node.

**Parameters**

- **\(G\) (NetworkX graph)** – DiGraph on which a minimum cost flow satisfying all demands is to be found.
- **\text{demand} (string)\)** – Nodes of the graph \(G\) are expected to have an attribute demand that indicates how much flow a node wants to send (negative demand) or receive (positive demand). Note that the sum of the demands should be 0 otherwise the problem in not feasible. If this attribute is not present, a node is considered to have 0 demand. Default value: ‘demand’.
- **\text{capacity} (string)\)** – Edges of the graph \(G\) are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: ‘capacity’.
- **\text{weight} (string)\)** – Edges of the graph \(G\) are expected to have an attribute weight that indicates the cost incurred by sending one unit of flow on that edge. If not present, the weight is considered to be 0. Default value: ‘weight’.

**Returns** **flowDict** – Dictionary of dictionaries keyed by nodes such that flowDict[u][v] is the flow edge \((u, v)\).

**Return type** dictionary

**Raises**

- **NetworkXError** – This exception is raised if the input graph is not directed or not connected.
- **NetworkXUnfeasible** – This exception is raised in the following situations:
  - The sum of the demands is not zero. Then, there is no flow satisfying all demands.
  - There is no flow satisfying all demand.
- **NetworkXUnbounded** – This exception is raised if the digraph \(G\) has a cycle of negative cost and infinite capacity. Then, the cost of a flow satisfying all demands is unbounded below.

**See also:**

- cost_of_flow(), max_flow_min_cost(), min_cost_flow_cost(), network_simplex()

**Notes**

This algorithm is not guaranteed to work if edge weights or demands are floating point numbers (overflows and roundoff errors can cause problems). As a workaround you can use integer numbers by multiplying the relevant...
edge attributes by a convenient constant factor (eg 100).

**Examples**

A simple example of a min cost flow problem.

```python
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_node('a', demand = -5)
>>> G.add_node('d', demand = 5)
>>> G.add_edge('a', 'b', weight = 3, capacity = 4)
>>> G.add_edge('a', 'c', weight = 6, capacity = 10)
>>> G.add_edge('b', 'd', weight = 1, capacity = 9)
>>> G.add_edge('c', 'd', weight = 2, capacity = 5)
>>> flowDict = nx.min_cost_flow(G)
```

**networkx.algorithms.flow.cost_of_flow**

```python
networkx.algorithms.flow.cost_of_flow(G, flowDict, weight='weight')
```

Compute the cost of the flow given by flowDict on graph G.

Note that this function does not check for the validity of the flow flowDict. This function will fail if the graph G and the flow don’t have the same edge set.

**Parameters**

- **G** *(NetworkX graph)* – DiGraph on which a minimum cost flow satisfying all demands is to be found.

- **weight** *(string)* – Edges of the graph G are expected to have an attribute weight that indicates the cost incurred by sending one unit of flow on that edge. If not present, the weight is considered to be 0. Default value: ‘weight’.

- **flowDict** *(dictionary)* – Dictionary of dictionaries keyed by nodes such that flowDict[u][v] is the flow edge (u, v).

**Returns**

- **cost** – The total cost of the flow. This is given by the sum over all edges of the product of the edge’s flow and the edge’s weight.

**Return type**

- Integer, float

**See also:**

- `max_flow_min_cost()`, `min_cost_flow()`, `min_cost_flow_cost()`, `network_simplex()`

**Notes**

This algorithm is not guaranteed to work if edge weights or demands are floating point numbers (overflows and roundoff errors can cause problems). As a workaround you can use integer numbers by multiplying the relevant edge attributes by a convenient constant factor (eg 100).
G is a digraph with edge costs and capacities. There is a source node s and a sink node t. This function finds a maximum flow from s to t whose total cost is minimized.

**Parameters**

- **G** (*NetworkX* graph) – DiGraph on which a minimum cost flow satisfying all demands is to be found.
- **s** (node label) – Source of the flow.
- **t** (node label) – Destination of the flow.
- **capacity** (*string*) – Edges of the graph G are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: ‘capacity’.
- **weight** (*string*) – Edges of the graph G are expected to have an attribute weight that indicates the cost incurred by sending one unit of flow on that edge. If not present, the weight is considered to be 0. Default value: ‘weight’.

**Returns**

- **flowDict** – Dictionary of dictionaries keyed by nodes such that flowDict[u][v] is the flow edge (u, v).

**Return type** dictionary

** Raises**

- **NetworkXError** – This exception is raised if the input graph is not directed or not connected.
- **NetworkXUnbounded** – This exception is raised if there is an infinite capacity path from s to t in G. In this case there is no maximum flow. This exception is also raised if the digraph G has a cycle of negative cost and infinite capacity. Then, the cost of a flow is unbounded below.

**See also:**

- `cost_of_flow()`, `min_cost_flow()`, `min_cost_flow_cost()`, `network_simplex()`

**Notes**

This algorithm is not guaranteed to work if edge weights or demands are floating point numbers (overflows and roundoff errors can cause problems). As a workaround you can use integer numbers by multiplying the relevant edge attributes by a convenient constant factor (e.g., 100).

**Examples**

```python
>>> G = nx.DiGraph()
>>> G.add_edges_from([(1, 2, {'capacity': 12, 'weight': 4}),
                    (1, 3, {'capacity': 20, 'weight': 6}),
                    (2, 3, {'capacity': 6, 'weight': -3}),
                    (2, 6, {'capacity': 14, 'weight': 1}),
                    (3, 4, {'weight': 9}),
                    (3, 5, {'capacity': 10, 'weight': 5}),
                    (4, 2, {'capacity': 19, 'weight': 13}),
                    (4, 5, {'capacity': 4, 'weight': 0}),
                    (5, 7, {'capacity': 28, 'weight': 2}),
                    (6, 5, {'capacity': 11, 'weight': 1}),
                    (6, 7, {'weight': 8})],
```
... (7, 4, {'capacity': 6, 'weight': 6}))

```python
>>> mincostFlow = nx.max_flow_min_cost(G, 1, 7)
>>> mincost = nx.cost_of_flow(G, mincostFlow)
>>> mincost
373
>>> from networkx.algorithms.flow import maximum_flow
>>> maxFlow = maximum_flow(G, 1, 7)[1]
>>> nx.cost_of_flow(G, maxFlow) >= mincost
True
>>> mincostFlowValue = (sum((mincostFlow[u][7] for u in G.predecessors(7)))
... - sum((mincostFlow[7][v] for v in G.successors(7))))
>>> mincostFlowValue == nx.maximum_flow_value(G, 1, 7)
True
```

### 3.28.10 Capacity Scaling Minimum Cost Flow

`capacity_scaling(G[, demand, capacity, ...])` Find a minimum cost flow satisfying all demands in digraph `G`.

**networkx.algorithms.flow.capacity_scaling**

`capacity_scaling(G, demand='demand', capacity='capacity', weight='weight', heap=<class 'networkx.utils.heaps.BinaryHeap'>)`

Find a minimum cost flow satisfying all demands in digraph `G`.

This is a capacity scaling successive shortest augmenting path algorithm.

`G` is a digraph with edge costs and capacities and in which nodes have demand, i.e., they want to send or receive some amount of flow. A negative demand means that the node wants to send flow, a positive demand means that the node want to receive flow. A flow on the digraph `G` satisfies all demand if the net flow into each node is equal to the demand of that node.

**Parameters**

- `G` (*NetworkX graph*) – DiGraph or MultiDiGraph on which a minimum cost flow satisfying all demands is to be found.
- `demand` (*string*) – Nodes of the graph `G` are expected to have an attribute demand that indicates how much flow a node wants to send (negative demand) or receive (positive demand). Note that the sum of the demands should be 0 otherwise the problem in not feasible. If this attribute is not present, a node is considered to have 0 demand. Default value: ‘demand’.
- `capacity` (*string*) – Edges of the graph `G` are expected to have an attribute capacity that indicates how much flow the edge can support. If this attribute is not present, the edge is considered to have infinite capacity. Default value: ‘capacity’.
- `weight` (*string*) – Edges of the graph `G` are expected to have an attribute weight that indicates the cost incurred by sending one unit of flow on that edge. If not present, the weight is considered to be 0. Default value: ‘weight’.
- `heap` (*class*) – Type of heap to be used in the algorithm. It should be a subclass of `MinHeap` or implement a compatible interface.

If a stock heap implementation is to be used, `BinaryHeap` is recommended over `PairingHeap` for Python implementations without optimized attribute accesses (e.g.,...
CPython) despite a slower asymptotic running time. For Python implementations with optimized attribute accesses (e.g., PyPy), PairingHeap provides better performance. Default value: BinaryHeap.

Returns

- `flowCost (integer)` – Cost of a minimum cost flow satisfying all demands.
- `flowDict (dictionary)` – If G is a digraph, a dict-of-dicts keyed by nodes such that flowDict[u][v] is the flow on edge (u, v). If G is a MultiDiGraph, a dict-of-dicts-of-dicts keyed by nodes so that flowDict[u][v][key] is the flow on edge (u, v, key).

Raises

- `NetworkXError` – This exception is raised if the input graph is not directed, not connected.
- `NetworkXUnfeasible` – This exception is raised in the following situations:
  - The sum of the demands is not zero. Then, there is no flow satisfying all demands.
  - There is no flow satisfying all demand.
- `NetworkXUnbounded` – This exception is raised if the digraph G has a cycle of negative cost and infinite capacity. Then, the cost of a flow satisfying all demands is unbounded below.

Notes

This algorithm does not work if edge weights are floating-point numbers.

See also:

*network_simplex()*

Examples

A simple example of a min cost flow problem.

```python
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_node('a', demand = -5)
>>> G.add_node('d', demand = 5)
>>> G.add_edge('a', 'b', weight = 3, capacity = 4)
>>> G.add_edge('a', 'c', weight = 6, capacity = 10)
>>> G.add_edge('b', 'd', weight = 1, capacity = 9)
>>> G.add_edge('c', 'd', weight = 2, capacity = 5)
>>> flowCost, flowDict = nx.capacity_scaling(G)
>>> flowCost
24
>>> flowDict
{'a': {'c': 1, 'b': 4}, 'c': {'d': 1}, 'b': {'d': 4}, 'd': {}}
```

It is possible to change the name of the attributes used for the algorithm.

```python
>>> G = nx.DiGraph()
>>> G.add_node('p', spam = -4)
>>> G.add_node('q', spam = 2)
>>> G.add_node('a', spam = -2)
```

(continues on next page)
>>> G.add_node('d', spam = -1)
>>> G.add_node('t', spam = 2)
>>> G.add_node('w', spam = 3)
>>> G.add_edge('p', 'q', cost = 7, vacancies = 5)
>>> G.add_edge('p', 'a', cost = 1, vacancies = 4)
>>> G.add_edge('q', 'd', cost = 2, vacancies = 3)
>>> G.add_edge('t', 'q', cost = 1, vacancies = 2)
>>> G.add_edge('a', 't', cost = 2, vacancies = 4)
>>> G.add_edge('d', 'w', cost = 3, vacancies = 4)
>>> G.add_edge('t', 'w', cost = 4, vacancies = 1)

>>> flowCost, flowDict = nx.capacity_scaling(G, demand = 'spam', 
... capacity = 'vacancies', 
... weight = 'cost')

>>> flowCost
37

>>> flowDict
{'a': {'t': 4}, 'd': {'w': 2}, 'q': {'d': 1}, 'p': {'q': 2, 'a': 2}, 't': {'q': 1, 
→ 'w': 1}, 'w': {}}

## 3.29 Graphical degree sequence

Test sequences for graphiness.

### is_graphical(sequence[, method])

Returns True if sequence is a valid degree sequence. A degree sequence is valid if some graph can realize it.

**Parameters**

- `sequence` *(list or iterable container)* – A sequence of integer node degrees
- `method` *("eg" | "hh" (default: "eg")) – The method used to validate the degree sequence. “eg” corresponds to the Erdős-Gallai algorithm, and “hh” to the Havel-Hakimi algorithm.

**Returns** `true` – True if the sequence is a valid degree sequence and False if not.

**Return type** `bool`
Examples

```python
>>> G = nx.path_graph(4)
>>> sequence = (d for n, d in G.degree())
>>> nx.is_graphical(sequence)
True
```
3.29.4 networkx.algorithms.graphical.is_pseudographical

**is_pseudographical** *(sequence)*

Returns True if some pseudograph can realize the sequence.

Every nonnegative integer sequence with an even sum is pseudographical (see\(^1\)).

**Parameters**
- **sequence** *(list or iterable container)* – A sequence of integer node degrees

**Returns**
- **valid** – True if the sequence is a pseudographic degree sequence and False if not.

**Return type**
- bool

**Notes**

The worst-case run time is \(O(n)\) where \(n\) is the length of the sequence.

**References**

3.29.5 networkx.algorithms.graphical.is_valid_degree_sequence_havel_hakimi

**is_valid_degree_sequence_havel_hakimi** *(deg_sequence)*

Returns True if deg_sequence can be realized by a simple graph.

The validation proceeds using the Havel-Hakimi theorem. Worst-case run time is \(O(s)\) where \(s\) is the sum of the sequence.

**Parameters**
- **deg_sequence** *(list)* – A list of integers where each element specifies the degree of a node in a graph.

**Returns**
- **valid** – True if deg_sequence is graphical and False if not.

**Return type**
- bool

**Notes**

The ZZ condition says that for the sequence \(d\) if

\[
|d| \geq \frac{(\max(d) + \min(d) + 1)^2}{4 \times \min(d)}
\]

then \(d\) is graphical. This was shown in Theorem 6 in\(^1\).

**References**

[?], [?], [?]


3.29.6 networkx.algorithms.graphical.is_valid_degree_sequence_erdos_gallai

is_valid_degree_sequence_erdos_gallai(deg_sequence)

Returns True if deg_sequence can be realized by a simple graph.

The validation is done using the Erdős-Gallai theorem [?].

Parameters deg_sequence (list) – A list of integers

Returns valid – True if deg_sequence is graphical and False if not.

Return type bool

Notes

This implementation uses an equivalent form of the Erdős-Gallai criterion. Worst-case run time is \(O(n)\) where \(n\) is the length of the sequence.

Specifically, a sequence \(d\) is graphical if and only if the sum of the sequence is even and for all strong indices \(k\) in the sequence,

\[
\sum_{i=1}^{k} d_i \leq k(k - 1) + \sum_{j=k+1}^{n} \min(d_i, k) = k(n - 1) - (k \sum_{j=0}^{k-1} n_j - \sum_{j=0}^{k-1} jn_j)
\]

A strong index \(k\) is any index where \(d_k \geq k\) and the value \(n_j\) is the number of occurrences of \(j\) in \(d\). The maximal strong index is called the Durfee index.

This particular rearrangement comes from the proof of Theorem 3 in \(^2\).

The ZZ condition says that for the sequence \(d\) if

\[
|d| \geq \frac{(\max(d) + \min(d) + 1)^2}{4 \cdot \min(d)}
\]

then \(d\) is graphical. This was shown in Theorem 6 in \(^2\).

References

[?]. [?]

3.30 Hierarchy

Flow Hierarchy.

flow_hierarchy(G[, weight])

Returns the flow hierarchy of a directed network.

---

3.30.1 networkx.algorithms.hierarchy.flow_hierarchy

**flow_hierarchy** *(G, weight=None)*  
Returns the flow hierarchy of a directed network.

Flow hierarchy is defined as the fraction of edges not participating in cycles in a directed graph\(^1\).

**Parameters**

- **G** *(DiGraph or MultiDiGraph)* – A directed graph
- **weight** *(key, optional (default=None))* – Attribute to use for node weights. If None the weight defaults to 1.

**Returns**  
**h** – Flow hierarchy value

**Return type**  
**float**

**Notes**

The algorithm described in\(^1\) computes the flow hierarchy through exponentiation of the adjacency matrix. This function implements an alternative approach that finds strongly connected components. An edge is in a cycle if and only if it is in a strongly connected component, which can be found in \(O(m)\) time using Tarjan’s algorithm.

**References**

3.31 Hybrid

Provides functions for finding and testing for locally \((k, l)\)-connected graphs.

| **kl_connected_subgraph** *(G, k, l[, low_memory, ...])* | Returns the maximum locally \((k, l)\)-connected subgraph of G. |
| **is_kl_connected** *(G, k, l[, low_memory])* | Returns True if and only if G is locally \((k, l)\)-connected. |

3.31.1 networkx.algorithms.hybrid.kl_connected_subgraph

**kl_connected_subgraph** *(G, k, l, low_memory=True, same_as_graph=False)*  
Returns the maximum locally \((k, l)\)-connected subgraph of G.

A graph is locally \((k, l)\)-connected if for each edge \((u, v)\) in the graph there are at least \(l\) edge-disjoint paths of length at most \(k\) joining \(u\) to \(v\).

**Parameters**

- **G** *(NetworkX graph)* – The graph in which to find a maximum locally \((k, l)\)-connected subgraph.
- **k** *(integer)* – The maximum length of paths to consider. A higher number means a looser connectivity requirement.
- **l** *(integer)* – The number of edge-disjoint paths. A higher number means a stricter connectivity requirement.

---

• **low_memory** *(bool)* – If this is True, this function uses an algorithm that uses slightly more time but less memory.

• **same_as_graph** *(bool)* – If True then return a tuple of the form *(H, is_same)*, where H is the maximum locally \((k, l)\)-connected subgraph and is_same is a Boolean representing whether G is locally \((k, l)\)-connected (and hence, whether H is simply a copy of the input graph G).

**Returns** If same_as_graph is True, then this function returns a two-tuple as described above. Otherwise, it returns only the maximum locally \((k, l)\)-connected subgraph.

**Return type** NetworkX graph or two-tuple

**See also:**

`is_kl_connected()`

**References**

### 3.31.2 networkx.algorithms.hybrid.is_kl_connected

`is_kl_connected(G, k, l, low_memory=False)`

Returns True if and only if \(G\) is locally \((k, l)\)-connected.

A graph is locally \((k, l)\)-connected if for each edge \((u, v)\) in the graph there are at least \(l\) edge-disjoint paths of length at most \(k\) joining \(u\) to \(v\).

**Parameters**

- G *(NetworkX graph)* – The graph to test for local \((k, l)\)-connectedness.
- k *(integer)* – The maximum length of paths to consider. A higher number means a looser connectivity requirement.
- l *(integer)* – The number of edge-disjoint paths. A higher number means a stricter connectivity requirement.
- low_memory *(bool)* – If this is True, this function uses an algorithm that uses slightly more time but less memory.

**Returns** Whether the graph is locally \((k, l)\)-connected subgraph.

**Return type** bool

**See also:**

`kl_connected_subgraph()`

**References**

### 3.32 Isolates

Functions for identifying isolate (degree zero) nodes.

<table>
<thead>
<tr>
<th><strong>is_isolate(G, n)</strong></th>
<th>Determines whether a node is an isolate.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>isolates(G)</strong></td>
<td>Iterator over isolates in the graph.</td>
</tr>
<tr>
<td><strong>number_of_isolates(G)</strong></td>
<td>Returns the number of isolates in the graph.</td>
</tr>
</tbody>
</table>
### 3.32.1 networkx.algorithms.isolate.is_isolate

**is_isolate** *(G, n)*  
Determines whether a node is an isolate.

An *isolate* is a node with no neighbors (that is, with degree zero). For directed graphs, this means no in-neighbors and no out-neighbors.

**Parameters**  
- **G** *(NetworkX graph)*  
- **n** *(node)* – A node in G.

**Returns**  
*is_isolate* – True if and only if n has no neighbors.

**Return type**  
`bool`

**Examples**

```python
>>> G=nx.Graph()
>>> G.add_edge(1,2)
>>> G.add_node(3)
>>> nx.is_isolate(G,2)
False
>>> nx.is_isolate(G,3)
True
```

### 3.32.2 networkx.algorithms.isolate.isolates

**isolates** *(G)*  
Iterator over isolates in the graph.

An *isolate* is a node with no neighbors (that is, with degree zero). For directed graphs, this means no in-neighbors and no out-neighbors.

**Parameters**  
- **G** *(NetworkX graph)*

**Returns**  
An iterator over the isolates of G.

**Return type**  
`iterator`

**Examples**

To get a list of all isolates of a graph, use the list constructor:

```python
>>> G = nx.Graph()
>>> G.add_edge(1, 2)
>>> G.add_node(3)
>>> list(nx.isolates(G))
[3]
```

To remove all isolates in the graph, first create a list of the isolates, then use `Graph.remove_nodes_from()`:

```python
```
For digraphs, isolates have zero in-degree and zero out_degree:

```python
 >>> G = nx.DiGraph([(0, 1), (1, 2)])
 >>> G.add_node(3)
 >>> list(nx.isolates(G))
 [3]
```

### 3.32.3 networkx.algorithms.isolate.number_of_isolates

**number_of_isolates** *(G)*

Returns the number of isolates in the graph.

An *isolate* is a node with no neighbors (that is, with degree zero). For directed graphs, this means no in-neighbors and no out-neighbors.

**Parameters**

- **G** *(NetworkX graph)*

**Returns**

The number of degree zero nodes in the graph *G*.

**Return type**

`int`

### 3.33 Isomorphism

<table>
<thead>
<tr>
<th>is_isomorphic(G1, G2[, node_match, edge_match])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns True if the graphs G1 and G2 are isomorphic and False otherwise.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>could_be_isomorphic(G1, G2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns False if graphs are definitely not isomorphic.</td>
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</table>

<table>
<thead>
<tr>
<th>fast_could_be_isomorphic(G1, G2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns False if graphs are definitely not isomorphic.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>faster_could_be_isomorphic(G1, G2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns False if graphs are definitely not isomorphic.</td>
</tr>
</tbody>
</table>

### 3.33.1 networkx.algorithms.isomorphism.is_isomorphic

**is_isomorphic** *(G1, G2, node_match=None, edge_match=None)*

Returns True if the graphs G1 and G2 are isomorphic and False otherwise.

**Parameters**

- **G1, G2** *(graphs)* – The two graphs G1 and G2 must be the same type.
- **node_match** *(callable)* – A function that returns True if node n1 in G1 and n2 in G2 should be considered equal during the isomorphism test. If node_match is not specified then node attributes are not considered.
  
The function will be called like
  
  ```python
  node_match(G1.nodes[n1], G2.nodes[n2]).
  ```
  
  That is, the function will receive the node attribute dictionaries for n1 and n2 as inputs.
- **edge_match** *(callable)* – A function that returns True if the edge attribute dictionary for the pair of nodes (u1, v1) in G1 and (u2, v2) in G2 should be considered equal during the isomorphism test. If edge_match is not specified then edge attributes are not considered.
The function will be called like

```
edge_match(G1[u1][v1], G2[u2][v2]).
```

That is, the function will receive the edge attribute dictionaries of the edges under consideration.

**Notes**

Uses the vf2 algorithm\(^1\).

**Examples**

```python
>>> import networkx.algorithms.isomorphism as iso
```

For digraphs G1 and G2, using ‘weight’ edge attribute (default: 1)

```python
>>> G1 = nx.DiGraph()
>>> G2 = nx.DiGraph()
>>> nx.add_path(G1, [1,2,3,4], weight=1)
>>> nx.add_path(G2, [10,20,30,40], weight=2)
>>> em = iso.numerical_edge_match('weight', 1)
>>> nx.is_isomorphic(G1, G2)  # no weights considered
True
>>> nx.is_isomorphic(G1, G2, edge_match=em)  # match weights
False
```

For multidigraphs G1 and G2, using ‘fill’ node attribute (default: ‘’)

```python
>>> G1 = nx.MultiDiGraph()
>>> G2 = nx.MultiDiGraph()
>>> G1.add_nodes_from([1,2,3], fill='red')
>>> G2.add_nodes_from([10,20,30,40], fill='red')
>>> nx.add_path(G1, [1,2,3,4], weight=3, linewidth=2.5)
>>> nx.add_path(G2, [10,20,30,40], weight=3)
>>> nm = iso.categorical_node_match('fill', 'red')
>>> nx.is_isomorphic(G1, G2, node_match=nm)
True
```

For multidigraphs G1 and G2, using ‘weight’ edge attribute (default: 7)

```python
>>> G1.add_edge(1,2, weight=7)
1
>>> G2.add_edge(10,20)
1
>>> em = iso.numerical_multiedge_match('weight', 7, rtol=1e-6)
>>> nx.is_isomorphic(G1, G2, edge_match=em)
True
```

For multigraphs G1 and G2, using ‘weight’ and ‘linewidth’ edge attributes with default values 7 and 2.5. Also using ‘fill’ node attribute with default value ‘red’.

```
>>> em = iso.numerical_multiedge_match(['weight', 'linewidth'], [7, 2.5])
>>> nm = iso.categorical_node_match('fill', 'red')
>>> nx.is_isomorphic(G1, G2, edge_match=em, node_match=nm)
True
```

See also:

- `numerical_node_match()`,
- `numerical_edge_match()`,
- `numerical_multiedge_match()`,
- `categorical_node_match()`,
- `categorical_edge_match()`,
- `categorical_multiedge_match()`

References

### 3.3.2 `networkx.algorithms.isomorphism.could_be_isomorphic`

`could_be_isomorphic(G1, G2)`

Returns False if graphs are definitely not isomorphic. True does NOT guarantee isomorphism.

**Parameters**

- `G1`, `G2` (graphs) – The two graphs G1 and G2 must be the same type.

**Notes**

Checks for matching degree, triangle, and number of cliques sequences.

### 3.3.3 `networkx.algorithms.isomorphism.fast_could_be_isomorphic`

`fast_could_be_isomorphic(G1, G2)`

Returns False if graphs are definitely not isomorphic.

True does NOT guarantee isomorphism.

**Parameters**

- `G1`, `G2` (graphs) – The two graphs G1 and G2 must be the same type.

**Notes**

Checks for matching degree and triangle sequences.

### 3.3.4 `networkx.algorithms.isomorphism.faster_could_be_isomorphic`

`faster_could_be_isomorphic(G1, G2)`

Returns False if graphs are definitely not isomorphic.

True does NOT guarantee isomorphism.

**Parameters**

- `G1`, `G2` (graphs) – The two graphs G1 and G2 must be the same type.

**Notes**

Checks for matching degree sequences.
3.33.5 Advanced Interfaces

VF2 Algorithm

VF2 Algorithm

An implementation of VF2 algorithm for graph isomorphism testing.
The simplest interface to use this module is to call networkx.is_isomorphic().

Introduction

The GraphMatcher and DiGraphMatcher are responsible for matching graphs or directed graphs in a predetermined manner. This usually means a check for an isomorphism, though other checks are also possible. For example, a subgraph of one graph can be checked for isomorphism to a second graph.

Matching is done via syntactic feasibility. It is also possible to check for semantic feasibility. Feasibility, then, is defined as the logical AND of the two functions.

To include a semantic check, the (Di)GraphMatcher class should be subclassed, and the semantic_feasibility() function should be redefined. By default, the semantic feasibility function always returns True. The effect of this is that semantics are not considered in the matching of G1 and G2.

Examples

Suppose G1 and G2 are isomorphic graphs. Verification is as follows:

```python
>>> from networkx.algorithms import isomorphism
>>> G1 = nx.path_graph(4)
>>> G2 = nx.path_graph(4)
>>> GM = isomorphism.GraphMatcher(G1,G2)
>>> GM.is_isomorphic()
True
```

GM.mapping stores the isomorphism mapping from G1 to G2.

```python
>>> GM.mapping
{0: 0, 1: 1, 2: 2, 3: 3}
```

Suppose G1 and G2 are isomorphic directed graphs graphs. Verification is as follows:

```python
>>> G1 = nx.path_graph(4, create_using=nx.DiGraph())
>>> G2 = nx.path_graph(4, create_using=nx.DiGraph())
>>> DiGM = isomorphism.DiGraphMatcher(G1,G2)
>>> DiGM.is_isomorphic()
True
```

DiGM.mapping stores the isomorphism mapping from G1 to G2.

```python
>>> DiGM.mapping
{0: 0, 1: 1, 2: 2, 3: 3}
```
Subgraph Isomorphism

Graph theory literature can be ambiguous about the meaning of the above statement, and we seek to clarify it now.

In the VF2 literature, a mapping $M$ is said to be a graph-subgraph isomorphism iff $M$ is an isomorphism between $G_2$ and a subgraph of $G_1$. Thus, to say that $G_1$ and $G_2$ are graph-subgraph isomorphic is to say that a subgraph of $G_1$ is isomorphic to $G_2$.

Other literature uses the phrase ‘subgraph isomorphic’ as in ‘$G_1$ does not have a subgraph isomorphic to $G_2$’. Another use is as an in adverb for isomorphic. Thus, to say that $G_1$ and $G_2$ are subgraph isomorphic is to say that a subgraph of $G_1$ is isomorphic to $G_2$.

Finally, the term ‘subgraph’ can have multiple meanings. In this context, ‘subgraph’ always means a ‘node-induced subgraph’. Edge-induced subgraph isomorphisms are not directly supported, but one should be able to perform the check by making use of `nx.line_graph()`. For subgraphs which are not induced, the term ‘monomorphism’ is preferred over ‘isomorphism’.

Let $G=(N,E)$ be a graph with a set of nodes $N$ and set of edges $E$.

**If $G'=(N',E')$ is a subgraph, then:** $N'$ is a subset of $N$ E' is a subset of E

**If $G'=(N',E')$ is a node-induced subgraph, then:** $N'$ is a subset of $N$ E’ is the subset of edges in $E$ relating nodes in $N'$

**If $G'=(N',E')$ is an edge-induced subgraph, then:** $N'$ is the subset of nodes in $N$ related by edges in $E'$ E’ is a subset of E

**If $G'=(N',E')$ is a monomorphism, then:** $N'$ is a subset of $N$ E’ is a subset of the set of edges in $E$ relating nodes in $N'$

Note that if $G'$ is a node-induced subgraph of $G$, then it is always a subgraph monomorphism of $G$, but the opposite is not always true, as a monomorphism can have fewer edges.

References


See also:
syntactic_feasibility, semantic_feasibility

Notes

The implementation handles both directed and undirected graphs as well as multigraphs.

In general, the subgraph isomorphism problem is NP-complete whereas the graph isomorphism problem is most likely not NP-complete (although no polynomial-time algorithm is known to exist).

Graph Matcher
<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
</table>
| `GraphMatcher.__init__(G1, G2[, node_match, ...
| Initialize graph matcher.                   |
| `GraphMatcher.initialize()`                | Reinitializes the state of the algorithm.                                   |
| `GraphMatcher.is_isomorphic()`              | Returns True if G1 and G2 are isomorphic graphs.                            |
| `GraphMatcher.subgraph_is_isomorphic()`     | Returns True if a subgraph of G1 is isomorphic to G2.                       |
| `GraphMatcher.isomorphisms_iter()`         | Generator over isomorphisms between G1 and G2.                              |
| `GraphMatcher.subgraph_isomorphisms_iter()`| Generator over isomorphisms between a subgraph of G1 and G2.                |
| `GraphMatcher.candidate_pairs_iter()`      | Iterator over candidate pairs of nodes in G1 and G2.                        |
| `GraphMatcher.match()`                      | Extends the isomorphism mapping.                                            |
| `GraphMatcher.semantic_feasibility(G1_node,...)` | Returns True if mapping G1_node to G2_node is semantically feasible.       |
| `GraphMatcher.syntactic_feasibility(G1_node,...)` | Returns True if adding (G1_node, G2_node) is syntactically feasible.    |

**networkx.algorithms.isomorphism.GraphMatcher.__init__**

GraphMatcher.__init__(G1, G2, node_match=None, edge_match=None)

Initialize graph matcher.

**Parameters**

- **G1, G2 (graph)** – The graphs to be tested.
- **node_match (callable)** – A function that returns True iff node n1 in G1 and n2 in G2 should be considered equal during the isomorphism test. The function will be called like:

  ```python
  node_match(G1.nodes[n1], G2.nodes[n2])
  ```

  That is, the function will receive the node attribute dictionaries of the nodes under consideration. If None, then no attributes are considered when testing for an isomorphism.

- **edge_match (callable)** – A function that returns True iff the edge attribute dictionary for the pair of nodes (u1, v1) in G1 and (u2, v2) in G2 should be considered equal during the isomorphism test. The function will be called like:

  ```python
  edge_match(G1[u1][v1], G2[u2][v2])
  ```

  That is, the function will receive the edge attribute dictionaries of the edges under consideration. If None, then no attributes are considered when testing for an isomorphism.

**networkx.algorithms.isomorphism.GraphMatcher.initialize**

GraphMatcher.initialize()

Reinitializes the state of the algorithm.

This method should be redefined if using something other than GMState. If only subclassing GraphMatcher, a redefinition is not necessary.

**networkx.algorithms.isomorphism.GraphMatcher.is_isomorphic**

GraphMatcher.is_isomorphic()

Returns True if G1 and G2 are isomorphic graphs.
networkx.algorithms.isomorphism.GraphMatcher.subgraph_is_isomorphic

GraphMatcher.subgraph_is_isomorphic()
   Returns True if a subgraph of G1 is isomorphic to G2.

networkx.algorithms.isomorphism.GraphMatcher.isomorphisms_iter

GraphMatcher.isomorphisms_iter()
   Generator over isomorphisms between G1 and G2.

networkx.algorithms.isomorphism.GraphMatcher.subgraph_isomorphisms_iter

GraphMatcher.subgraph_isomorphisms_iter()
   Generator over isomorphisms between a subgraph of G1 and G2.

networkx.algorithms.isomorphism.GraphMatcher.candidate_pairs_iter

GraphMatcher.candidate_pairs_iter()
   Iterator over candidate pairs of nodes in G1 and G2.

networkx.algorithms.isomorphism.GraphMatcher.match

GraphMatcher.match()
   Extends the isomorphism mapping.
   This function is called recursively to determine if a complete isomorphism can be found between G1 and G2. It
   cleans up the class variables after each recursive call. If an isomorphism is found, we yield the mapping.

networkx.algorithms.isomorphism.GraphMatcher.semantic_feasibility

GraphMatcher.semantic_feasibility(G1_node, G2_node)
   Returns True if mapping G1_node to G2_node is semantically feasible.

networkx.algorithms.isomorphism.GraphMatcher.syntactic_feasibility

GraphMatcher.syntactic_feasibility(G1_node, G2_node)
   Returns True if adding (G1_node, G2_node) is syntactically feasible.
   This function returns True if it is adding the candidate pair to the current partial isomorphism/monomorphism
   mapping is allowable. The addition is allowable if the inclusion of the candidate pair does not make it impossible
   for an isomorphism/monomorphism to be found.

DiGraph Matcher

DiGraphMatcher.__init__(G1, G2[, ...])
   Initialize graph matcher.

DiGraphMatcher.initialize()
   Reinitializes the state of the algorithm.

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<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DiGraphMatcher.is_isomorphic()</td>
<td>Returns True if G1 and G2 are isomorphic graphs.</td>
</tr>
<tr>
<td>DiGraphMatcher.subgraph_is_isomorphic()</td>
<td>Returns True if a subgraph of G1 is isomorphic to G2.</td>
</tr>
<tr>
<td>DiGraphMatcher.isomorphisms_iter()</td>
<td>Generator over isomorphisms between G1 and G2.</td>
</tr>
<tr>
<td>DiGraphMatcher.subgraph_isomorphisms_iter()</td>
<td>Generator over isomorphisms between a subgraph of G1 and G2.</td>
</tr>
<tr>
<td>DiGraphMatcher.candidate_pairs_iter()</td>
<td>Iterator over candidate pairs of nodes in G1 and G2.</td>
</tr>
<tr>
<td>DiGraphMatcher.match()</td>
<td>Extends the isomorphism mapping.</td>
</tr>
<tr>
<td>DiGraphMatcher.semantic_feasibility(G1_node,...)</td>
<td>Returns True if mapping G1_node to G2_node is semantically feasible.</td>
</tr>
<tr>
<td>DiGraphMatcher.syntactic_feasibility(...</td>
<td>Returns True if adding (G1_node, G2_node) is syntactically feasible.</td>
</tr>
</tbody>
</table>

**networkx.algorithms.isomorphism.DiGraphMatcher.__init__**

DiGraphMatcher.__init__(G1, G2, node_match=None, edge_match=None)

Initialize graph matcher.

**Parameters**

- G1, G2 (graph) – The graphs to be tested.
- node_match (callable) – A function that returns True iff node n1 in G1 and n2 in G2 should be considered equal during the isomorphism test. The function will be called like:
  
  ```python
  node_match(G1.nodes[n1], G2.nodes[n2])
  ```

That is, the function will receive the node attribute dictionaries of the nodes under consideration. If None, then no attributes are considered when testing for an isomorphism.

- edge_match (callable) – A function that returns True iff the edge attribute dictionary for the pair of nodes (u1, v1) in G1 and (u2, v2) in G2 should be considered equal during the isomorphism test. The function will be called like:
  
  ```python
  edge_match(G1[u1][v1], G2[u2][v2])
  ```

That is, the function will receive the edge attribute dictionaries of the edges under consideration. If None, then no attributes are considered when testing for an isomorphism.

**networkx.algorithms.isomorphism.DiGraphMatcher.initialize**

DiGraphMatcher.initialize()

Reinitializes the state of the algorithm.

This method should be redefined if using something other than DiGMState. If only subclassing GraphMatcher, a redefinition is not necessary.

**networkx.algorithms.isomorphism.DiGraphMatcher.is_isomorphic**

DiGraphMatcher.is_isomorphic()

Returns True if G1 and G2 are isomorphic graphs.
networkx.algorithms.isomorphism.DiGraphMatcher.subgraph_is_isomorphic

DiGraphMatcher.subgraph_is_isomorphic()
Returns True if a subgraph of G1 is isomorphic to G2.

networkx.algorithms.isomorphism.DiGraphMatcher.isomorphisms_iter

DiGraphMatcher.isomorphisms_iter()
Generator over isomorphisms between G1 and G2.

networkx.algorithms.isomorphism.DiGraphMatcher.subgraph_isomorphisms_iter

DiGraphMatcher.subgraph_isomorphisms_iter()
Generator over isomorphisms between a subgraph of G1 and G2.

networkx.algorithms.isomorphism.DiGraphMatcher.candidate_pairs_iter

DiGraphMatcher.candidate_pairs_iter()
Iterator over candidate pairs of nodes in G1 and G2.

networkx.algorithms.isomorphism.DiGraphMatcher.match

DiGraphMatcher.match()
Extends the isomorphism mapping.
This function is called recursively to determine if a complete isomorphism can be found between G1 and G2. It cleans up the class variables after each recursive call. If an isomorphism is found, we yield the mapping.

networkx.algorithms.isomorphism.DiGraphMatcher.semantic_feasibility

DiGraphMatcher.semantic_feasibility(G1_node, G2_node)
Returns True if mapping G1_node to G2_node is semantically feasible.

networkx.algorithms.isomorphism.DiGraphMatcher.syntactic_feasibility

DiGraphMatcher.syntactic_feasibility(G1_node, G2_node)
Returns True if adding (G1_node, G2_node) is syntactically feasible.
This function returns True if it is adding the candidate pair to the current partial isomorphism/monomorphism mapping is allowable. The addition is allowable if the inclusion of the candidate pair does not make it impossible for an isomorphism/monomorphism to be found.

Match helpers
**categorical_node_match** *(attr, default)*

Returns a comparison function for a categorical node attribute.

The value(s) of the attr(s) must be hashable and comparable via the == operator since they are placed into a set([]) object. If the sets from G1 and G2 are the same, then the constructed function returns True.

**Parameters**

- **attr** *(string | list)* – The categorical node attribute to compare, or a list of categorical node attributes to compare.

- **default** *(value | list)* – The default value for the categorical node attribute, or a list of default values for the categorical node attributes.

**Returns** match – The customized, categorical node_match function.

**Return type** function

**Examples**

```python
>>> import networkx.algorithms.isomorphism as iso
>>> nm = iso.categorical_node_match('size', 1)
>>> nm = iso.categorical_node_match(['color', 'size'], ['red', 2])
```

**categorical_edge_match** *(attr, default)*

Returns a comparison function for a categorical edge attribute.

The value(s) of the attr(s) must be hashable and comparable via the == operator since they are placed into a set([]) object. If the sets from G1 and G2 are the same, then the constructed function returns True.

**Parameters**

- **attr** *(string | list)* – The categorical edge attribute to compare, or a list of categorical edge attributes to compare.

- **default** *(value | list)* – The default value for the categorical edge attribute, or a list of default values for the categorical edge attributes.

**Returns** match – The customized, categorical edge_match function.

**networkx.algorithms.isomorphism.categorical_node_match**

**categorical_node_match** *(attr, default)*

Returns a comparison function for a categorical node attribute.

The value(s) of the attr(s) must be hashable and comparable via the == operator since they are placed into a set([]) object. If the sets from G1 and G2 are the same, then the constructed function returns True.

**Parameters**

- **attr** *(string | list)* – The categorical node attribute to compare, or a list of categorical node attributes to compare.

- **default** *(value | list)* – The default value for the categorical node attribute, or a list of default values for the categorical node attributes.

**Returns** match – The customized, categorical node_match function.

**Return type** function

**Examples**

```python
>>> import networkx.algorithms.isomorphism as iso
>>> nm = iso.categorical_node_match('size', 1)
>>> nm = iso.categorical_node_match(['color', 'size'], ['red', 2])
```

**categorical_edge_match** *(attr, default)*

Returns a comparison function for a categorical edge attribute.

The value(s) of the attr(s) must be hashable and comparable via the == operator since they are placed into a set([]) object. If the sets from G1 and G2 are the same, then the constructed function returns True.

**Parameters**

- **attr** *(string | list)* – The categorical edge attribute to compare, or a list of categorical edge attributes to compare.

- **default** *(value | list)* – The default value for the categorical edge attribute, or a list of default values for the categorical edge attributes.

**Returns** match – The customized, categorical edge_match function.
networkx.algorithms.isomorphism.categorical_edge_match

categorical_edge_match (attr, default)

Returns a comparison function for a categorical edge attribute.

The value(s) of the attr(s) must be hashable and comparable via the == operator since they are placed into a set([]) object. If the sets from G1 and G2 are the same, then the constructed function returns True.

Parameters

• attr (string | list) – The categorical edge attribute to compare, or a list of categorical edge attributes to compare.

• default (value | list) – The default value for the categorical edge attribute, or a list of default values for the categorical edge attributes.

Returns match – The customized, categorical edge_match function.

Return type function

Examples

```python
>>> import networkx.algorithms.isomorphism as iso
>>> nm = iso.categorical_edge_match('size', 1)
>>> nm = iso.categorical_edge_match(['color', 'size'], ['red', 2])
```

networkx.algorithms.isomorphism.categorical_multiedge_match

categorical_multiedge_match (attr, default)

Returns a comparison function for a categorical edge attribute.

The value(s) of the attr(s) must be hashable and comparable via the == operator since they are placed into a set([]) object. If the sets from G1 and G2 are the same, then the constructed function returns True.

Parameters

• attr (string | list) – The categorical edge attribute to compare, or a list of categorical edge attributes to compare.

• default (value | list) – The default value for the categorical edge attribute, or a list of default values for the categorical edge attributes.

Returns match – The customized, categorical edge_match function.

Return type function

Examples

```python
>>> import networkx.algorithms.isomorphism as iso
>>> nm = iso.categorical_multiedge_match('size', 1)
>>> nm = iso.categorical_multiedge_match(['color', 'size'], ['red', 2])
```

networkx.algorithms.isomorphism.numerical_node_match

numerical_node_match (attr, default, rtol=1e-05, atol=1e-08)

Returns a comparison function for a numerical node attribute.

The value(s) of the attr(s) must be numerical and sortable. If the sorted list of values from G1 and G2 are the same within some tolerance, then the constructed function returns True.

Parameters

• attr (string | list) – The numerical node attribute to compare, or a list of numerical node attributes to compare.

• default (value | list) – The default value for the numerical node attribute, or a list of default values for the numerical node attributes.

• rtol (float) – The relative error tolerance.

Examples

```python
>>> import networkx.algorithms.isomorphism as iso
>>> nm = iso.numerical_node_match('size', 1)
>>> nm = iso.numerical_node_match(['color', 'size'], ['red', 2])
```
• **atol (float)** – The absolute error tolerance.

**Returns** match – The customized, numerical node_match function.

**Return type** function

**Examples**

```python
>>> import networkx.algorithms.isomorphism as iso
>>> nm = iso.numerical_node_match('weight', 1.0)
>>> nm = iso.numerical_node_match(['weight', 'linewidth'], [.25, .5])
```

**networkx.algorithms.isomorphism.numerical_edge_match**

`numerical_edge_match (attr, default, rtol=1e-05, atol=1e-08)`

Returns a comparison function for a numerical edge attribute.

The value(s) of the attr(s) must be numerical and sortable. If the sorted list of values from G1 and G2 are the same within some tolerance, then the constructed function returns True.

**Parameters**

- **attr (string | list)** – The numerical edge attribute to compare, or a list of numerical edge attributes to compare.
- **default (value | list)** – The default value for the numerical edge attribute, or a list of default values for the numerical edge attributes.
- **rtol (float)** – The relative error tolerance.
- **atol (float)** – The absolute error tolerance.

**Returns** match – The customized, numerical edge_match function.

**Return type** function

**Examples**

```python
>>> import networkx.algorithms.isomorphism as iso
>>> nm = iso.numerical_edge_match('weight', 1.0)
>>> nm = iso.numerical_edge_match(['weight', 'linewidth'], [.25, .5])
```

**networkx.algorithms.isomorphism.numerical_multiedge_match**

`numerical_multiedge_match (attr, default, rtol=1e-05, atol=1e-08)`

Returns a comparison function for a numerical edge attribute.

The value(s) of the attr(s) must be numerical and sortable. If the sorted list of values from G1 and G2 are the same within some tolerance, then the constructed function returns True.

**Parameters**

- **attr (string | list)** – The numerical edge attribute to compare, or a list of numerical edge attributes to compare.
- **default (value | list)** – The default value for the numerical edge attribute, or a list of default values for the numerical edge attributes.

**Examples**

```python
>>> import networkx.algorithms.isomorphism as iso
>>> nm = iso.numerical_multiedge_match('weight', 1.0)
>>> nm = iso.numerical_multiedge_match(['weight', 'linewidth'], [.25, .5])
```
- **rtol** (*float*) – The relative error tolerance.
- **atol** (*float*) – The absolute error tolerance.

**Returns** `match` – The customized, numerical `edge_match` function.

**Return type** function

**Examples**

```python
codeblock
>>> import networkx.algorithms.isomorphism as iso
>>> nm = iso.numerical_multiedge_match('weight', 1.0)
>>> nm = iso.numerical_multiedge_match(['weight', 'linewidth'], [.25, .5])
```

**networkx.algorithms.isomorphism.generic_node_match**

`generic_node_match` (*attr, default, op*)

Returns a comparison function for a generic attribute.

The value(s) of the `attr(s)` are compared using the specified operators. If all the attributes are equal, then the constructed function returns True.

**Parameters**

- **attr** (*string | list*) – The node attribute to compare, or a list of node attributes to compare.
- **default** (*value | list*) – The default value for the node attribute, or a list of default values for the node attributes.
- **op** (*callable | list*) – The operator to use when comparing attribute values, or a list of operators to use when comparing values for each attribute.

**Returns** `match` – The customized, generic `node_match` function.

**Return type** function

**Examples**

```python
codeblock
>>> from operator import eq
>>> from networkx.algorithms.isomorphism.matchhelpers import close
>>> from networkx.algorithms.isomorphism import generic_node_match
>>> nm = generic_node_match('weight', 1.0, close)
>>> nm = generic_node_match('color', 'red', eq)
>>> nm = generic_node_match(['weight', 'color'], [1.0, 'red'], [close, eq])
```

**networkx.algorithms.isomorphism.generic_edge_match**

`generic_edge_match` (*attr, default, op*)

Returns a comparison function for a generic attribute.

The value(s) of the `attr(s)` are compared using the specified operators. If all the attributes are equal, then the constructed function returns True.

**Parameters**

- **attr** (*string | list*) – The edge attribute to compare, or a list of edge attributes to compare.

**Examples**

```python
codeblock
```
NetworkX Reference, Release 2.5rc1.dev20191214022633

- **default** (value | list) – The default value for the edge attribute, or a list of default values for the edge attributes.
- **op** (callable | list) – The operator to use when comparing attribute values, or a list of operators to use when comparing values for each attribute.

**Returns** match – The customized, generic edge_match function.

**Return type** function

**Examples**

```python
>>> from operator import eq
>>> from networkx.algorithms.isomorphism.matchhelpers import close
>>> from networkx.algorithms.isomorphism import generic_edge_match

>>> nm = generic_edge_match('weight', 1.0, close)
>>> nm = generic_edge_match('color', 'red', eq)
>>> nm = generic_edge_match(['weight', 'color'], [1.0, 'red'], [close, eq])
```

networkx.algorithms.isomorphism.generic_multiedge_match

**generic_multiedge_match** (attr, default, op)

Returns a comparison function for a generic attribute.

The value(s) of the attr(s) are compared using the specified operators. If all the attributes are equal, then the constructed function returns True. Potentially, the constructed edge_match function can be slow since it must verify that no isomorphism exists between the multiedges before it returns False.

**Parameters**

- **attr** (string | list) – The edge attribute to compare, or a list of node attributes to compare.
- **default** (value | list) – The default value for the edge attribute, or a list of default values for the edge attributes.
- **op** (callable | list) – The operator to use when comparing attribute values, or a list of operators to use when comparing values for each attribute.

**Returns** match – The customized, generic edge_match function.

**Return type** function

**Examples**

```python
>>> from operator import eq
>>> from networkx.algorithms.isomorphism.matchhelpers import close
>>> from networkx.algorithms.isomorphism import generic_node_match

>>> nm = generic_node_match('weight', 1.0, close)
>>> nm = generic_node_match('color', 'red', eq)
>>> nm = generic_node_match(['weight', 'color'], [1.0, 'red'], [close, eq])
```
ISMAGS Algorithm

ISMAGS Algorithm

Provides a Python implementation of the ISMAGS algorithm.¹

It is capable of finding (subgraph) isomorphisms between two graphs, taking the symmetry of the subgraph into account. In most cases the VF2 algorithm is faster (at least on small graphs) than this implementation, but in some cases there is an exponential number of isomorphisms that are symmetrically equivalent. In that case, the ISMAGS algorithm will provide only one solution per symmetry group.

```python
>>> import networkx as nx
>>> petersen = nx.petersen_graph()
>>> ismags = nx.isomorphism.ISMAGS(petersen, petersen)
>>> isomorphisms = list(ismags.isomorphisms_iter(symmetry=False))
>>> len(isomorphisms)
120
>>> isomorphisms = list(ismags.isomorphisms_iter(symmetry=True))
>>> answer = [{0: 0, 1: 1, 2: 2, 3: 3, 4: 4, 5: 5, 6: 6, 7:7, 8: 8, 9: 9}]
>>> answer == isomorphisms
True
```

In addition, this implementation also provides an interface to find the largest common induced subgraph² between any two graphs, again taking symmetry into account. Given `graph` and `subgraph` the algorithm will remove nodes from the `subgraph` until `subgraph` is isomorphic to a subgraph of `graph`. Since only the symmetry of `subgraph` is taken into account it is worth thinking about how you provide your graphs:

```python
>>> graph1 = nx.path_graph(4)
>>> graph2 = nx.star_graph(3)
>>> ismags = nx.isomorphism.ISMAGS(graph1, graph2)
>>> ismags.is_isomorphic()
False
>>> largest_common_subgraph = list(ismags.largest_common_subgraph())
>>> answer = [
... {1: 0, 0: 1, 2: 2},
... {2: 0, 1: 1, 3: 2}
... ]
>>> answer == largest_common_subgraph
True
>>> ismags2 = nx.isomorphism.ISMAGS(graph2, graph1)
>>> largest_common_subgraph = list(ismags2.largest_common_subgraph())
>>> answer = [
... {1: 0, 0: 1, 2: 2},
... {1: 0, 0: 1, 3: 2},
... {2: 0, 0: 1, 1: 2},
... {2: 0, 0: 1, 3: 2},
... {3: 0, 0: 1, 1: 2},
... {3: 0, 0: 1, 2: 2}
... ]
>>> answer == largest_common_subgraph
True
```

However, when not taking symmetry into account, it doesn’t matter:

² https://en.wikipedia.org/wiki/Maximum_common_induced_subgraph
```python
>>> largest_common_subgraph = list(ismags.largest_common_subgraph(symmetry=False))
>>> answer = [...
...     {1: 0, 0: 1, 2: 2},
...     {1: 0, 0: 1, 3: 2},
...     {2: 0, 0: 1, 3: 2},
...     {2: 0, 0: 1, 3: 2},
...     {3: 0, 0: 1, 3: 2},
...     {1: 1, 0: 2, 2: 3},
...     {1: 1, 0: 2, 3: 3},
...     {2: 1, 0: 2, 3: 3},
...     {3: 1, 0: 2, 3: 3},
...     {2: 0, 0: 1, 3: 2},
...     {2: 0, 0: 1, 3: 2},
...     {3: 0, 0: 1, 3: 2},
...     {1: 1, 0: 2, 2: 3},
...     {1: 1, 0: 2, 3: 3},
...     {2: 1, 0: 2, 3: 3},
...     {3: 1, 0: 2, 3: 3},
...]
>>> answer == largest_common_subgraph
True
>>> largest_common_subgraph = list(ismags2.largest_common_subgraph(symmetry=False))
>>> answer = [...
...     {1: 0, 0: 1, 2: 2},
...     {1: 0, 0: 1, 3: 2},
...     {2: 0, 0: 1, 3: 2},
...     {2: 0, 0: 1, 3: 2},
...     {3: 0, 0: 1, 3: 2},
...     {1: 1, 0: 2, 2: 3},
...     {1: 1, 0: 2, 3: 3},
...     {2: 1, 0: 2, 3: 3},
...     {3: 1, 0: 2, 3: 3},
...     {2: 0, 0: 1, 3: 2},
...     {2: 0, 0: 1, 3: 2},
...     {3: 0, 0: 1, 3: 2},
...     {1: 1, 0: 2, 2: 3},
...     {1: 1, 0: 2, 3: 3},
...     {2: 1, 0: 2, 3: 3},
...     {3: 1, 0: 2, 3: 3},
...]
>>> answer == largest_common_subgraph
True
```

**Notes**

- The current implementation works for undirected graphs only. The algorithm in general should work for directed graphs as well though.
- Node keys for both provided graphs need to be fully orderable as well as hashable.
- Node and edge equality is assumed to be transitive: if A is equal to B, and B is equal to C, then A is equal to C.

**References**

**ISMAGS object**

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td><code>ISMAGS(graph, subgraph[, node_match, ...])</code></td>
<td>Implements the ISMAGS subgraph matching algorithm.</td>
</tr>
<tr>
<td><code>ISMAGS.analyze_symmetry(graph,...)</code></td>
<td>Find a minimal set of permutations and corresponding co-sets that describe the symmetry of subgraph.</td>
</tr>
<tr>
<td><code>ISMAGS.is_isomorphic([symmetry])</code></td>
<td>Returns True if graph is isomorphic to subgraph and False otherwise.</td>
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<tr>
<th>Method</th>
<th>Description</th>
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<tbody>
<tr>
<td><code>ISMAGS.subgraph_is_isomorphic()</code></td>
<td>Returns True if a subgraph of <code>graph</code> is isomorphic to <code>subgraph</code> and False otherwise.</td>
</tr>
<tr>
<td><code>ISMAGS.isomorphisms_iter()</code></td>
<td>Does the same as <code>find_isomorphisms()</code> if <code>graph</code> and <code>subgraph</code> have the same number of nodes.</td>
</tr>
<tr>
<td><code>ISMAGS.subgraph_isomorphisms_iter()</code></td>
<td>Alternative name for <code>find_isomorphisms()</code>.</td>
</tr>
<tr>
<td><code>ISMAGS.largest_common_subgraph()</code></td>
<td>Find the largest common induced subgraphs between <code>subgraph</code> and <code>graph</code>.</td>
</tr>
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</table>

`networkx.algorithms.isomorphism.ISMAGS`

class `ISMAGS` (graph, subgraph, node_match=None, edge_match=None, cache=None)

Implements the ISMAGS subgraph matching algorithm. ISMAGS stands for “Index-based Subgraph Matching Algorithm with General Symmetries”. As the name implies, it is symmetry aware and will only generate non-symmetric isomorphisms.

**Notes**

The implementation imposes additional conditions compared to the VF2 algorithm on the graphs provided and the comparison functions (node_equality and edge_equality):

- Node keys in both graphs must be orderable as well as hashable.
- Equality must be transitive: if A is equal to B, and B is equal to C, then A must be equal to C.

**graph**

Type `networkx.Graph`

**subgraph**

Type `networkx.Graph`

**node_equality**

The function called to see if two nodes should be considered equal. It’s signature looks like this: `f(graph1: networkx.Graph, node1, graph2: networkx.Graph, node2) -> bool`. `node1` is a node in `graph1`, and `node2` a node in `graph2`. Constructed from the argument `node_match`.

Type `collections.abc.Callable`

**edge_equality**

The function called to see if two edges should be considered equal. It’s signature looks like this: `f(graph1: networkx.Graph, edge1, graph2: networkx.Graph, edge2) -> bool`. `edge1` is an edge in `graph1`, and `edge2` an edge in `graph2`. Constructed from the argument `edge_match`.

Type `collections.abc.Callable`

**References**

Parameters

- **graph** (*networkx.Graph*)
- **subgraph** (*networkx.Graph*)
- **node_match** (*collections.abc.Callable or None*) – Function used to determine whether two nodes are equivalent. Its signature should look like \(f(n1: \text{dict}, n2: \text{dict}) \rightarrow \text{bool}\), with \(n1\) and \(n2\) node property dicts. See also `categorical_node_match()` and friends. If `None`, all nodes are considered equal.
- **edge_match** (*collections.abc.Callable or None*) – Function used to determine whether two edges are equivalent. Its signature should look like \(f(e1: \text{dict}, e2: \text{dict}) \rightarrow \text{bool}\), with \(e1\) and \(e2\) edge property dicts. See also `categorical_edge_match()` and friends. If `None`, all edges are considered equal.
- **cache** (*collections.abc.Mapping*) – A cache used for caching graph symmetries.

Methods

```python
__init__(graph, subgraph[, node_match, ...])

analyze_symmetry(graph, node_partitions, ...)
```

**Parameters**

- **graph** (*networkx.Graph*)

**analyze_symmetry**

Find a minimal set of permutations and corresponding co-sets that describe the symmetry of `subgraph`.

```python
find_isomorphisms([symmetry])
```

Find all subgraph isomorphisms between `subgraph` and `graph`.

```python
is_isomorphic([symmetry])
```

Returns True if `graph` is isomorphic to `subgraph` and False otherwise.

```python
isomorphisms_iter([symmetry])
```

Does the same as `find_isomorphisms()` if `graph` and `subgraph` have the same number of nodes.

```python
largest_common_subgraph([symmetry])
```

Find the largest common induced subgraphs between `subgraph` and `graph`.

```python
subgraph_is_isomorphic([symmetry])
```

Returns True if a subgraph of `graph` is isomorphic to `subgraph` and False otherwise.

```python
subgraph_isomorphisms_iter([symmetry])
```

Alternative name for `find_isomorphisms()`.

networkx.algorithms.isomorphism.ISMAGS.analyze_symmetry

**ISMAGS.analyze_symmetry**

```
ISMAGS.analyze_symmetry (graph, node_partitions, edge_colors)
```

Find a minimal set of permutations and corresponding co-sets that describe the symmetry of `subgraph`.

**Returns**

- **set[frozenset]** – The found permutations. This is a set of frozenset of pairs of node keys which can be exchanged without changing `subgraph`.
- **dict[collections.abc.Hashable, set[collections.abc.Hashable]]** – The found co-sets. The co-sets is a dictionary of \{ node key: set of node keys \}. Every key-value pair describes which values can be interchanged without changing nodes less than key.

3.33. Isomorphism
networkx.algorithms.isomorphism.ISMAGS.is_isomorphic

ISMAGS.is_isomorphic(symmetry=False)
Returns True if graph is isomorphic to subgraph and False otherwise.
Returns
Return type bool

networkx.algorithms.isomorphism.ISMAGS.subgraph_is_isomorphic

ISMAGS.subgraph_is_isomorphic(symmetry=False)
Returns True if a subgraph of graph is isomorphic to subgraph and False otherwise.
Returns
Return type bool

networkx.algorithms.isomorphism.ISMAGS.isomorphisms_iter

ISMAGS.isomorphisms_iter(symmetry=True)
Does the same as find_isomorphisms() if graph and subgraph have the same number of nodes.

networkx.algorithms.isomorphism.ISMAGS.subgraph_isomorphisms_iter

ISMAGS.subgraph_isomorphisms_iter(symmetry=True)
Alternative name for find_isomorphisms().

networkx.algorithms.isomorphism.ISMAGS.largest_common_subgraph

ISMAGS.largest_common_subgraph(symmetry=True)
Find the largest common induced subgraphs between subgraph and graph.

Parameters symmetry (bool) – Whether symmetry should be taken into account. If False, found largest common subgraphs may be symmetrically equivalent.

Yields dict – The found isomorphism mappings of {graph_node: subgraph_node}.

3.34 Link Analysis

3.34.1 PageRank

PageRank analysis of graph structure.

<table>
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<tr>
<th>Function</th>
<th>Description</th>
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<tr>
<td>pagerank(G[, alpha, personalization, ...])</td>
<td>Returns the PageRank of the nodes in the graph.</td>
</tr>
<tr>
<td>pagerank_numpy(G[, alpha, personalization, ...])</td>
<td>Returns the PageRank of the nodes in the graph.</td>
</tr>
<tr>
<td>pagerank_scipy(G[, alpha, personalization, ...])</td>
<td>Returns the PageRank of the nodes in the graph.</td>
</tr>
<tr>
<td>google_matrix(G[, alpha, personalization, ...])</td>
<td>Returns the Google matrix of the graph.</td>
</tr>
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</table>
networkx.algorithms.link_analysis.pagerank_alg.pagerank

**pagerank** *(G, alpha=0.85, personalization=None, max_iter=100, tol=1e-06, nstart=None, weight='weight', dangling=None)*

Returns the PageRank of the nodes in the graph.

PageRank computes a ranking of the nodes in the graph G based on the structure of the incoming links. It was originally designed as an algorithm to rank web pages.

**Parameters**

- **G** *(graph)* – A NetworkX graph. Undirected graphs will be converted to a directed graph with two directed edges for each undirected edge.
- **alpha** *(float, optional)* – Damping parameter for PageRank, default=0.85.
- **personalization** *(dict, optional)* – The “personalization vector” consisting of a dictionary with a key some subset of graph nodes and personalization value each of those. At least one personalization value must be non-zero. If not specified, a nodes personalization value will be zero. By default, a uniform distribution is used.
- **max_iter** *(integer, optional)* – Maximum number of iterations in power method eigenvalue solver.
- **tol** *(float, optional)* – Error tolerance used to check convergence in power method solver.
- **nstart** *(dictionary, optional)* – Starting value of PageRank iteration for each node.
- **weight** *(key, optional)* – Edge data key to use as weight. If None weights are set to 1.
- **dangling** *(dict, optional)* – The outedges to be assigned to any “dangling” nodes, i.e., nodes without any outedges. The dict key is the node the outedge points to and the dict value is the weight of that outedge. By default, dangling nodes are given outedges according to the personalization vector (uniform if not specified). This must be selected to result in an irreducible transition matrix (see notes under google_matrix). It may be common to have the dangling dict to be the same as the personalization dict.

**Returns** **pagerank** – Dictionary of nodes with PageRank as value

**Return type** dictionary

**Examples**

```python
g = nx.DiGraph(nx.path_graph(4))
pr = nx.pagerank(g, alpha=0.9)
```

**Notes**

The eigenvector calculation is done by the power iteration method and has no guarantee of convergence. The iteration will stop after an error tolerance of \( \text{len}(G) \times \text{tol} \) has been reached. If the number of iterations exceed max_iter, a networkx.exception.PowerIterationFailedConvergence exception is raised.

The PageRank algorithm was designed for directed graphs but this algorithm does not check if the input graph is directed and will execute on undirected graphs by converting each edge in the directed graph to two edges.

**See also:**

- pagerank_numpy()
- pagerank_scipy()
- google_matrix()
Raises `PowerIterationFailedConvergence` – If the algorithm fails to converge to the specified tolerance within the specified number of iterations of the power iteration method.

References

networkx.algorithms.link_analysis.pagerank_alg.pagerank_numpy

`pagerank_numpy(G, alpha=0.85, personalization=None, weight='weight', dangling=None)`

Returns the PageRank of the nodes in the graph.

PageRank computes a ranking of the nodes in the graph G based on the structure of the incoming links. It was originally designed as an algorithm to rank web pages.

Parameters

- `G (graph)` – A NetworkX graph. Undirected graphs will be converted to a directed graph with two directed edges for each undirected edge.
- `alpha (float, optional)` – Damping parameter for PageRank, default=0.85.
- `personalization (dict, optional)` – The “personalization vector” consisting of a dictionary with a key some subset of graph nodes and personalization value each of those. At least one personalization value must be non-zero. If not specified, a nodes personalization value will be zero. By default, a uniform distribution is used.
- `weight (key, optional)` – Edge data key to use as weight. If None weights are set to 1.
- `dangling (dict, optional)` – The outedges to be assigned to any “dangling” nodes, i.e., nodes without any outedges. The dict key is the node the outedge points to and the dict value is the weight of that outedge. By default, dangling nodes are given outedges according to the personalization vector (uniform if not specified) This must be selected to result in an irreducible transition matrix (see notes under google_matrix). It may be common to have the dangling dict to be the same as the personalization dict.

Returns `pagerank` – Dictionary of nodes with PageRank as value.

Return type: dictionary

Examples

```python
>>> G = nx.DiGraph(nx.path_graph(4))
>>> pr = nx.pagerank_numpy(G, alpha=0.9)
```

Notes

The eigenvector calculation uses NumPy’s interface to the LAPACK eigenvalue solvers. This will be the fastest and most accurate for small graphs.

This implementation works with Multi(Di)Graphs. For multigraphs the weight between two nodes is set to be the sum of all edge weights between those nodes.

See also:

`pagerank(), pagerank_scipy(), google_matrix()`
PageRank computes a ranking of the nodes in the graph $G$ based on the structure of the incoming links. It was originally designed as an algorithm to rank web pages.

**Parameters**

- $G$ (*graph*) – A NetworkX graph. Undirected graphs will be converted to a directed graph with two directed edges for each undirected edge.
- alpha (*float, optional*) – Damping parameter for PageRank, default=0.85.
- personalization (*dict, optional*) – The “personalization vector” consisting of a dictionary with a key some subset of graph nodes and personalization value each of those. At least one personalization value must be non-zero. If not specified, a nodes personalization value will be zero. By default, a uniform distribution is used.
- max_iter (*integer, optional*) – Maximum number of iterations in power method eigenvalue solver.
- tol (*float, optional*) – Error tolerance used to check convergence in power method solver.
- nstart (*dictionary, optional*) – Starting value of PageRank iteration for each node.
- weight (*key, optional*) – Edge data key to use as weight. If None weights are set to 1.
- dangling (*dict, optional*) – The outedges to be assigned to any “dangling” nodes, i.e., nodes without any outedges. The dict key is the node the outedge points to and the dict value is the weight of that outedge. By default, dangling nodes are given outedges according to the personalization vector (uniform if not specified) This must be selected to result in an irreducible transition matrix (see notes under google_matrix). It may be common to have the dangling dict to be the same as the personalization dict.

**Returns** pagerank – Dictionary of nodes with PageRank as value

**Return type** dictionary

**Examples**

```python
>>> G = nx.DiGraph(nx.path_graph(4))
>>> pr = nx.pagerank_scipy(G, alpha=0.9)
```

**Notes**

The eigenvector calculation uses power iteration with a SciPy sparse matrix representation.

This implementation works with Multi(Di)Graphs. For multigraphs the weight between two nodes is set to be the sum of all edge weights between those nodes.

**See also:**

`pagerank()`, `pagerank_numpy()`, `google_matrix()`
Raises `PowerIterationFailedConvergence` – If the algorithm fails to converge to the specified tolerance within the specified number of iterations of the power iteration method.

References

networkx.algorithms.link_analysis.pagerank_alg.google_matrix

google_matrix(G, alpha=0.85, personalization=None, nodelist=None, weight='weight', dangling=None)

Returns the Google matrix of the graph.

Parameters

- **G** *(graph)* – A NetworkX graph. Undirected graphs will be converted to a directed graph with two directed edges for each undirected edge.
- **alpha** *(float)* – The damping factor.
- **personalization** *(dict, optional)* – The “personalization vector” consisting of a dictionary with a key some subset of graph nodes and personalization value each of those. At least one personalization value must be non-zero. If not specified, a nodes personalization value will be zero. By default, a uniform distribution is used.
- **nodelist** *(list, optional)* – The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- **weight** *(key, optional)* – Edge data key to use as weight. If None weights are set to 1.
- **dangling** *(dict, optional)* – The outedges to be assigned to any “dangling” nodes, i.e., nodes without any outedges. The dict key is the node the outedge points to and the dict value is the weight of that outedge. By default, dangling nodes are given outedges according to the personalization vector (uniform if not specified) This must be selected to result in an irreducible transition matrix (see notes below). It may be common to have the dangling dict to be the same as the personalization dict.

Returns **A** – Google matrix of the graph

Return type NumPy matrix

Notes

The matrix returned represents the transition matrix that describes the Markov chain used in PageRank. For PageRank to converge to a unique solution (i.e., a unique stationary distribution in a Markov chain), the transition matrix must be irreducible. In other words, it must be that there exists a path between every pair of nodes in the graph, or else there is the potential of “rank sinks.”

This implementation works with Multi(Di)Graphs. For multigraphs the weight between two nodes is set to be the sum of all edge weights between those nodes.

See also:

`pagerank()`, `pagerank_numpy()`, `pagerank_scipy()`

3.34.2 Hits

Hubs and authorities analysis of graph structure.
### NetworkX Reference, Release 2.5rc1.dev20191214022633

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>hits(G[, max_iter, tol, nstart, normalized])</code></td>
<td>Returns HITS hubs and authorities values for nodes.</td>
</tr>
<tr>
<td><code>hits_numpy(G[, normalized])</code></td>
<td>Returns HITS hubs and authorities values for nodes.</td>
</tr>
<tr>
<td><code>hits_scipy(G[, max_iter, tol, normalized])</code></td>
<td>Returns HITS hubs and authorities values for nodes.</td>
</tr>
<tr>
<td><code>hub_matrix(G[, nodelist])</code></td>
<td>Returns the HITS hub matrix.</td>
</tr>
<tr>
<td><code>authority_matrix(G[, nodelist])</code></td>
<td>Returns the HITS authority matrix.</td>
</tr>
</tbody>
</table>

#### `networkx.algorithms.link_analysis.hits_alg.hits`

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>hits(G, max_iter=100, tol=1e-08, nstart=None, normalized=True)</code></td>
<td>Returns HITS hubs and authorities values for nodes.</td>
</tr>
</tbody>
</table>

The HITS algorithm computes two numbers for a node. Authorities estimates the node value based on the incoming links. Hubs estimates the node value based on outgoing links.

**Parameters**

- `G (graph)` – A NetworkX graph
- `max_iter (integer, optional)` – Maximum number of iterations in power method.
- `tol (float, optional)` – Error tolerance used to check convergence in power method iteration.
- `nstart (dictionary, optional)` – Starting value of each node for power method iteration.
- `normalized (bool (default=True))` – Normalize results by the sum of all of the values.

**Returns**

- `hubs` and `authorities` – Two dictionaries keyed by node containing the hub and authority values.

**Return type**

two-tuple of dictionaries

**Raises** `PowerIterationFailedConvergence` – If the algorithm fails to converge to the specified tolerance within the specified number of iterations of the power iteration method.

#### Examples

```python
>>> G=nx.path_graph(4)
>>> h,a=nx.hits(G)
```

#### Notes

The eigenvector calculation is done by the power iteration method and has no guarantee of convergence. The iteration will stop after max_iter iterations or an error tolerance of number_of_nodes(G)*tol has been reached.

The HITS algorithm was designed for directed graphs but this algorithm does not check if the input graph is directed and will execute on undirected graphs.

#### References

`networkx.algorithms.link_analysis.hits_alg.hits_numpy`

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>hits_numpy(G, normalized=True)</code></td>
<td>Returns HITS hubs and authorities values for nodes.</td>
</tr>
</tbody>
</table>

3.34. Link Analysis
The HITS algorithm computes two numbers for a node. Authorities estimates the node value based on the incoming links. Hubs estimates the node value based on outgoing links.

**Parameters**
- **G (graph)** – A NetworkX graph
- **normalized (bool (default=True))** – Normalize results by the sum of all of the values.

**Returns** (hubs,authorities) – Two dictionaries keyed by node containing the hub and authority values.

**Return type** two-tuple of dictionaries

**Examples**

```python
>>> G=nx.path_graph(4)
>>> h,a=nx.hits(G)
```

**Notes**
The eigenvector calculation uses NumPy’s interface to LAPACK.

The HITS algorithm was designed for directed graphs but this algorithm does not check if the input graph is directed and will execute on undirected graphs.

**References**

networkx.algorithms.link_analysis.hits_alg.hits_scipy

**hits_scipy** (G, max_iter=100, tol=1e-06, normalized=True)

Returns HITS hubs and authorities values for nodes.

The HITS algorithm computes two numbers for a node. Authorities estimates the node value based on the incoming links. Hubs estimates the node value based on outgoing links.

**Parameters**
- **G (graph)** – A NetworkX graph
- **max_iter (integer, optional)** – Maximum number of iterations in power method.
- **tol (float, optional)** – Error tolerance used to check convergence in power method iteration.
- **nstart (dictionary, optional)** – Starting value of each node for power method iteration.
- **normalized (bool (default=True))** – Normalize results by the sum of all of the values.

**Returns** (hubs,authorities) – Two dictionaries keyed by node containing the hub and authority values.

**Return type** two-tuple of dictionaries

**Examples**

```python
>>> G=nx.path_graph(4)
>>> h,a=nx.hits(G)
```
Notes

This implementation uses SciPy sparse matrices.

The eigenvector calculation is done by the power iteration method and has no guarantee of convergence. The iteration will stop after max_iter iterations or an error tolerance of number_of_nodes(G)*tol has been reached.

The HITS algorithm was designed for directed graphs but this algorithm does not check if the input graph is directed and will execute on undirected graphs.

Raises **PowerIterationFailedConvergence** – If the algorithm fails to converge to the specified tolerance within the specified number of iterations of the power iteration method.

References

networkx.algorithms.link_analysis.hits_alg.hub_matrix

hub_matrix(G, nodelist=None)

Returns the HITS hub matrix.

networkx.algorithms.link_analysis.hits_alg.authority_matrix

authority_matrix(G, nodelist=None)

Returns the HITS authority matrix.

3.35 Link Prediction

Link prediction algorithms.

<table>
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<tr>
<th>Function</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>resource_allocation_index(G[, ebunch])</td>
<td>Compute the resource allocation index of all node pairs in ebunch.</td>
</tr>
<tr>
<td>jaccard_coefficient(G[, ebunch])</td>
<td>Compute the Jaccard coefficient of all node pairs in ebunch.</td>
</tr>
<tr>
<td>adamic_adar_index(G[, ebunch])</td>
<td>Compute the Adamic-Adar index of all node pairs in ebunch.</td>
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<tr>
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<td>Compute the preferential attachment score of all node pairs in ebunch.</td>
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<tr>
<td>cn_soundarajan_hopcroft(G[, ebunch, community])</td>
<td>Count the number of common neighbors of all node pairs in ebunch.</td>
</tr>
<tr>
<td>ra_index_soundarajan_hopcroft(G[, ebunch, ...])</td>
<td>Compute the resource allocation index of all node pairs in ebunch using community information.</td>
</tr>
<tr>
<td>within_inter_cluster(G[, ebunch, delta, ...])</td>
<td>Compute the ratio of within- and inter-cluster common neighbors of all node pairs in ebunch.</td>
</tr>
</tbody>
</table>

3.35.1 networkx.algorithms.link_prediction.resource_allocation_index

resource_allocation_index(G, ebunch=None)

Compute the resource allocation index of all node pairs in ebunch.
Resource allocation index of \( u \) and \( v \) is defined as
\[
\sum_{w \in \Gamma(u) \cap \Gamma(v)} \frac{1}{|\Gamma(w)|}
\]
where \( \Gamma(u) \) denotes the set of neighbors of \( u \).

**Parameters**

- **G** *(graph)* – A NetworkX undirected graph.
- **ebunch** *(iterable of node pairs, optional (default = None))* – Resource allocation index will be computed for each pair of nodes given in the iterable. The pairs must be given as 2-tuples \((u, v)\) where \( u \) and \( v \) are nodes in the graph. If ebunch is None then all non-existent edges in the graph will be used. Default value: None.

**Returns**

- **piter** – An iterator of 3-tuples in the form \((u, v, p)\) where \((u, v)\) is a pair of nodes and \( p \) is their resource allocation index.

**Return type** iterator

**Examples**

```python
>>> import networkx as nx
>>> G = nx.complete_graph(5)
>>> preds = nx.resource_allocation_index(G, [(0, 1), (2, 3)])
>>> for u, v, p in preds:
...     print('(%d, %d) -> %.8f' % (u, v, p))
... '(0, 1) -> 0.75000000'
'(%d, %d) -> 0.75000000' % (u, v, p)
'(%d, %d) -> 0.75000000' % (u, v, p)
```

**References**

3.35.2 networkx.algorithms.link_prediction.jaccard_coefficient

**jaccard_coefficient** *(G, ebunch=None)*

Compute the Jaccard coefficient of all node pairs in ebunch.

Jaccard coefficient of nodes \( u \) and \( v \) is defined as
\[
\frac{|\Gamma(u) \cap \Gamma(v)|}{|\Gamma(u) \cup \Gamma(v)|}
\]
where \( \Gamma(u) \) denotes the set of neighbors of \( u \).

**Parameters**

- **G** *(graph)* – A NetworkX undirected graph.
- **ebunch** *(iterable of node pairs, optional (default = None))* – Jaccard coefficient will be computed for each pair of nodes given in the iterable. The pairs must be given as 2-tuples \((u, v)\) where \( u \) and \( v \) are nodes in the graph. If ebunch is None then all non-existent edges in the graph will be used. Default value: None.

**Returns**

- **piter** – An iterator of 3-tuples in the form \((u, v, p)\) where \((u, v)\) is a pair of nodes and \( p \) is their Jaccard coefficient.

**Return type** iterator
Examples

```python
>>> import networkx as nx
>>> G = nx.complete_graph(5)
>>> preds = nx.jaccard_coefficient(G, [(0, 1), (2, 3)])
>>> for u, v, p in preds:
...     '(%d, %d) -> %.8f' % (u, v, p)
...     '(0, 1) -> 0.60000000'
     '(2, 3) -> 0.60000000'
```

References

3.35.3 networkx.algorithms.link_prediction.adamic_adar_index

**adamic_adar_index** *(G, ebunch=None)*

Compute the Adamic-Adar index of all node pairs in ebunch.

Adamic-Adar index of \(u\) and \(v\) is defined as

\[
\frac{1}{\log |\Gamma(u) \cap \Gamma(v)|}
\]

where \(\Gamma(u)\) denotes the set of neighbors of \(u\). This index leads to zero-division for nodes only connected via self-loops. It is intended to be used when no self-loops are present.

Parameters

- \(G\) (*graph*) – NetworkX undirected graph.
- \(ebunch\) (*iterable of node pairs, optional (default = None)*) – Adamic-Adar index will be computed for each pair of nodes given in the iterable. The pairs must be given as 2-tuples \((u, v)\) where \(u\) and \(v\) are nodes in the graph. If \(ebunch\) is None then all non-existent edges in the graph will be used. Default value: None.

Returns

- \(piter\) – An iterator of 3-tuples in the form \((u, v, p)\) where \((u, v)\) is a pair of nodes and \(p\) is their Adamic-Adar index.

Return type

- *iterator*

Examples

```python
>>> import networkx as nx
>>> G = nx.complete_graph(5)
>>> preds = nx.adamic_adar_index(G, [(0, 1), (2, 3)])
>>> for u, v, p in preds:
...     '(%d, %d) -> %.8f' % (u, v, p)
...     '(0, 1) -> 2.16404256'
     '(2, 3) -> 2.16404256'
```
References

3.35.4 networkx.algorithms.link_prediction.preferential_attachment

preferential_attachment (G, ebunch=None)
Compute the preferential attachment score of all node pairs in ebunch.

Preferential attachment score of \( u \) and \( v \) is defined as

\[ |\Gamma(u)||\Gamma(v)| \]

where \( \Gamma(u) \) denotes the set of neighbors of \( u \).

Parameters

- \( G \) (graph) – NetworkX undirected graph.
- \( ebunch \) (iterable of node pairs, optional (default = None)) – Preferential attachment score will be computed for each pair of nodes given in the iterable. The pairs must be given as 2-tuples \((u, v)\) where \( u \) and \( v \) are nodes in the graph. If \( ebunch \) is None then all non-existent edges in the graph will be used. Default value: None.

Returns

- \( piter \) – An iterator of 3-tuples in the form \((u, v, p)\) where \((u, v)\) is a pair of nodes and \( p \) is their preferential attachment score.

Return type

iterator

Examples

```python
>>> import networkx as nx
>>> G = nx.complete_graph(5)
>>> preds = nx.preferential_attachment(G, [(0, 1), (2, 3)])
>>> for u, v, p in preds:
...    print('(%d, %d) -> %d' % (u, v, p))
...    print('0, 1) -> 16'
...    print('(2, 3) -> 16'
```

References

3.35.5 networkx.algorithms.link_prediction.cn_soundarajan_hopcroft

cn_soundarajan_hopcroft (G, ebunch=None, community='community')

Count the number of common neighbors of all node pairs in ebunch using community information.

For two nodes \( u \) and \( v \), this function computes the number of common neighbors and bonus one for each common neighbor belonging to the same community as \( u \) and \( v \). Mathematically,

\[ |
\Gamma(u) \cap \Gamma(v) | + \sum_{w \in \Gamma(u) \cap \Gamma(v)} f(w) \]

where \( f(w) \) equals 1 if \( w \) belongs to the same community as \( u \) and \( v \) or 0 otherwise and \( \Gamma(u) \) denotes the set of neighbors of \( u \).

Parameters

- \( G \) (graph) – A NetworkX undirected graph.
• **ebunch** *(iterable of node pairs, optional (default = None))* – The score will be computed for each pair of nodes given in the iterable. The pairs must be given as 2-tuples (u, v) where u and v are nodes in the graph. If ebunch is None then all non-existent edges in the graph will be used. Default value: None.

• **community** *(string, optional (default = 'community'))* – Nodes attribute name containing the community information. \(G[u][\text{community}]\) identifies which community u belongs to. Each node belongs to at most one community. Default value: ‘community’.

**Returns** `piter` – An iterator of 3-tuples in the form (u, v, p) where (u, v) is a pair of nodes and p is their score.

**Return type** iterator

**Examples**

```python
>>> import networkx as nx
>>> G = nx.path_graph(3)
>>> G.nodes[0]['community'] = 0
>>> G.nodes[1]['community'] = 0
>>> G.nodes[2]['community'] = 0
>>> preds = nx.cn_soundarajan_hopcroft(G, [(0, 2)])
>>> for u, v, p in preds:
...     print('%d, %d' % (u, v) + ' -> ' + str(p))
(0, 2) -> 2
```

**References**

3.35.6 networkx.algorithms.link_prediction.ra_index_soundarajan_hopcroft

```
ra_index_soundarajan_hopcroft (G, ebunch=None, community='community')
Compute the resource allocation index of all node pairs in ebunch using community information.
For two nodes \(u\) and \(v\), this function computes the resource allocation index considering only common neighbors belonging to the same community as \(u\) and \(v\). Mathematically,
\[
\sum_{w \in \Gamma(u) \cap \Gamma(v)} f(w) / |\Gamma(w)|
\]

where \(f(w)\) equals 1 if \(w\) belongs to the same community as \(u\) and \(v\) or 0 otherwise and \(\Gamma(u)\) denotes the set of neighbors of \(u\).

**Parameters**

- **G** *(graph)* – A NetworkX undirected graph.
- **ebunch** *(iterable of node pairs, optional (default = None))* – The score will be computed for each pair of nodes given in the iterable. The pairs must be given as 2-tuples (u, v) where u and v are nodes in the graph. If ebunch is None then all non-existent edges in the graph will be used. Default value: None.
- **community** *(string, optional (default = 'community'))* – Nodes attribute name containing the community information. \(G[u][\text{community}]\) identifies which community u belongs to. Each node belongs to at most one community. Default value: ‘community’.

**Returns** `piter` – An iterator of 3-tuples in the form (u, v, p) where (u, v) is a pair of nodes and p is their score.
```
Return type  iterator

Examples

```python
>>> import networkx as nx
>>> G = nx.Graph()
>>> G.add_edges_from([(0, 1), (0, 2), (1, 3), (2, 3)])
>>> G.nodes[0]['community'] = 0
>>> G.nodes[1]['community'] = 0
>>> G.nodes[2]['community'] = 1
>>> G.nodes[3]['community'] = 0
>>> preds = nx.ra_index_soundarajan_hopcroft(G, [(0, 3)])
>>> for u, v, p in preds:
...     print(u, v, p)
'(0, 3) -> 0.50000000'
```

References

3.35.7 networkx.algorithms.link_prediction.within_inter_cluster

within_inter_cluster (G, ebunch=None, delta=0.001, community='community')

Compute the ratio of within- and inter-cluster common neighbors of all node pairs in ebunch.

For two nodes u and v, if a common neighbor w belongs to the same community as them, w is considered as within-cluster common neighbor of u and v. Otherwise, it is considered as inter-cluster common neighbor of u and v. The ratio between the size of the set of within- and inter-cluster common neighbors is defined as the WIC measure.\(^1\)

Parameters

- **G** (graph) – A NetworkX undirected graph.
- **ebunch** (iterable of node pairs, optional (default = None)) – The WIC measure will be computed for each pair of nodes given in the iterable. The pairs must be given as 2-tuples (u, v) where u and v are nodes in the graph. If ebunch is None then all non-existent edges in the graph will be used. Default value: None.
- **delta** (float, optional (default = 0.001)) – Value to prevent division by zero in case there is no inter-cluster common neighbor between two nodes. See\(^1\) for details. Default value: 0.001.
- **community** (string, optional (default = ‘community’)) – Nodes attribute name containing the community information. G[u][community] identifies which community u belongs to. Each node belongs to at most one community. Default value: ‘community’.

Returns piter – An iterator of 3-tuples in the form (u, v, p) where (u, v) is a pair of nodes and p is their WIC measure.

Return type  iterator

---

\(^1\) Jorge Carlos Valverde-Rebaza and Alneu de Andrade Lopes. Link prediction in complex networks based on cluster information. In Proceedings of the 21st Brazilian conference on Advances in Artificial Intelligence (SBIA’12) https://doi.org/10.1007/978-3-642-34459-6_10
Examples

```python
>>> import networkx as nx
>>> G = nx.Graph()
>>> G.add_edges_from([(0, 1), (0, 2), (0, 3), (1, 4), (2, 4), (3, 4)])
>>> G.nodes[0]['community'] = 0
>>> G.nodes[1]['community'] = 1
>>> G.nodes[2]['community'] = 0
>>> G.nodes[3]['community'] = 0
>>> G.nodes[4]['community'] = 0
>>> preds = nx.within_inter_cluster(G, [(0, 4)])
>>> for u, v, p in preds:
...    print('({}, {}) -> {:.8f}'.format(u, v, p))
...    
...    (0, 4) -> 1.99800200
>>> preds = nx.within_inter_cluster(G, [(0, 4)], delta=0.5)
>>> for u, v, p in preds:
...    print('({}, {}) -> {:.8f}'.format(u, v, p))
...    
...    (0, 4) -> 1.33333333
```

References

3.36 Lowest Common Ancestor

Algorithms for finding the lowest common ancestor of trees and DAGs.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>all_pairs_lowest_common_ancestor(G[, pairs])</code></td>
<td>Compute the lowest common ancestor for pairs of nodes.</td>
</tr>
<tr>
<td><code>tree_all_pairs_lowest_common_ancestor(G)</code></td>
<td>Yield the lowest common ancestor for sets of pairs in a tree.</td>
</tr>
<tr>
<td><code>lowest_common_ancestor(G, node1, node2[, ...])</code></td>
<td>Compute the lowest common ancestor of the given pair of nodes.</td>
</tr>
</tbody>
</table>

3.36.1 `networkx.algorithms.lowest_common_ancestors.all_pairs_lowest_common_ancestor`

`all_pairs_lowest_common_ancestor(G, pairs=None)`

Compute the lowest common ancestor for pairs of nodes.

Parameters

- `G` (NetworkX directed graph)
- `pairs` (iterable of pairs of nodes, optional (default: all pairs)) – The pairs of nodes of interest. If None, will find the LCA of all pairs of nodes.

Returns

- An iterator over ((node1, node2), lca) where (node1, node2) are the pairs specified and lca is a lowest common ancestor of the pair.
- Note that for the default of all pairs in G, we consider unordered pairs, e.g. you will not get both (b, a) and (a, b).
Notes

Only defined on non-null directed acyclic graphs.


See also:

`tree_all_pairs_lowest_common_ancestor()`, `lowest_common_ancestor()`

3.36.2 networkx.algorithms.lowest_common_ancestors.tree_all_pairs_lowest_common_ancestor

`tree_all_pairs_lowest_common_ancestor(G, root=None, pairs=None)`

Yield the lowest common ancestor for sets of pairs in a tree.

Parameters

- `G` (*NetworkX directed graph (must be a tree)*)
- `root` (*node, optional (default: None)*) – The root of the subtree to operate on. If None, assume the entire graph has exactly one source and use that.
- `pairs` (*iterable or iterator of pairs of nodes, optional (default: None)*) – The pairs of interest. If None, Defaults to all pairs of nodes under `root` that have a lowest common ancestor.

Returns `lcas` – in `pairs` and `lca` is their lowest common ancestor.

Return type generator of tuples `((u, v), lca)` where `u` and `v` are nodes

Notes

Only defined on non-null trees represented with directed edges from parents to children. Uses Tarjan’s off-line lowest-common-ancestors algorithm. Runs in time $O(4 \times (V + E + P))$ time, where 4 is the largest value of the inverse Ackermann function likely to ever come up in actual use, and $P$ is the number of pairs requested (or $V^2$ if all are needed).


See also:

`all_pairs_lowest_common_ancestor()`, `lowest_common_ancestor()`

3.36.3 networkx.algorithms.lowest_common_ancestors.lowest_common_ancestor

`lowest_common_ancestor(G, node1, node2, default=None)`

Compute the lowest common ancestor of the given pair of nodes.

Parameters

- `G` (*NetworkX directed graph*)
- `node1, node2` (*nodes in the graph.*)
- `default` (*object*) – Returned if no common ancestor between `node1` and `node2`

Returns
• The lowest common ancestor of node1 and node2,
• or default if they have no common ancestors.

Notes

Only defined on non-null directed acyclic graphs. Takes n log(n) time in the size of the graph. See all_pairs_lowest_common_ancestor when you have more than one pair of nodes of interest.

See also:

tree_all_pairs_lowest_common_ancestor(), all_pairs_lowest_common_ancestor()

### 3.37 Matching

Functions for computing and verifying matchings in a graph.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>is_matching(G, matching)</td>
<td>Decides whether the given set or dictionary represents a valid matching in G.</td>
</tr>
<tr>
<td>is_maximal_matching(G, matching)</td>
<td>Decides whether the given set or dictionary represents a valid maximal matching in G.</td>
</tr>
<tr>
<td>is_perfect_matching(G, matching)</td>
<td>Decides whether the given set represents a valid perfect matching in G.</td>
</tr>
<tr>
<td>maximal_matching(G)</td>
<td>Find a maximal matching in the graph.</td>
</tr>
<tr>
<td>max_weight_matching(G[, maxcardinality, weight])</td>
<td>Compute a maximum-weighted matching of G.</td>
</tr>
</tbody>
</table>

#### 3.37.1 networkx.algorithms.matching.is_matching

**is_matching** (G, matching)

Decides whether the given set or dictionary represents a valid matching in G.

A matching in a graph is a set of edges in which no two distinct edges share a common endpoint.

**Parameters**

- **G** (NetworkX graph)
- **matching** (dict or set) – A dictionary or set representing a matching. If a dictionary, it must have matching[u] == v and matching[v] == u for each edge (u, v) in the matching. If a set, it must have elements of the form (u, v), where (u, v) is an edge in the matching.

**Returns** Whether the given set or dictionary represents a valid matching in the graph.

**Return type** bool

#### 3.37.2 networkx.algorithms.matching.is_maximal_matching

**is_maximal_matching** (G, matching)

Decides whether the given set or dictionary represents a valid maximal matching in G.

A maximal matching in a graph is a matching in which adding any edge would cause the set to no longer be a valid matching.
3.37.3 networkx.algorithms.matching.is_perfect_matching

is_perfect_matching(G, matching)
Decides whether the given set represents a valid perfect matching in G.

A perfect matching in a graph is a matching in which exactly one edge is incident upon each vertex.

Parameters

- G (NetworkX graph)
- matching (dict or set) – A dictionary or set representing a matching. If a dictionary, it must have matching[u] == v and matching[v] == u for each edge (u, v) in the matching. If a set, it must have elements of the form (u, v), where (u, v) is an edge in the matching.

Returns Whether the given set or dictionary represents a valid perfect matching in the graph.

Return type bool

3.37.4 networkx.algorithms.matching.maximal_matching

maximal_matching(G)
Find a maximal matching in the graph.

A matching is a subset of edges in which no node occurs more than once. A maximal matching cannot add more edges and still be a matching.

Parameters G (NetworkX graph) – Undirected graph

Returns matching – A maximal matching of the graph.

Return type set

Notes

The algorithm greedily selects a maximal matching M of the graph G (i.e. no superset of M exists). It runs in $O(|E|)$ time.

3.37.5 networkx.algorithms.matching.max_weight_matching

max_weight_matching(G, maxcardinality=False, weight='weight')
Compute a maximum-weighted matching of G.
A matching is a subset of edges in which no node occurs more than once. The weight of a matching is the sum of the weights of its edges. A maximal matching cannot add more edges and still be a matching. The cardinality of a matching is the number of matched edges.

**Parameters**

- `G` (*NetworkX graph*) – Undirected graph
- `maxcardinality` (*bool, optional (default=False)*) – If maxcardinality is True, compute the maximum-cardinality matching with maximum weight among all maximum-cardinality matchings.
- `weight` (*string, optional (default='weight')*) – Edge data key corresponding to the edge weight. If key not found, uses 1 as weight.

**Returns**

- `matching` – A maximal matching of the graph.

**Return type** *set*

**Notes**

If `G` has edges with weight attributes the edge data are used as weight values else the weights are assumed to be 1.

This function takes time $O(number\_of\_nodes ** 3)$.

If all edge weights are integers, the algorithm uses only integer computations. If floating point weights are used, the algorithm could return a slightly suboptimal matching due to numeric precision errors.

This method is based on the “blossom” method for finding augmenting paths and the “primal-dual” method for finding a matching of maximum weight, both methods invented by Jack Edmonds\(^1\).

Bipartite graphs can also be matched using the functions present in `networkx.algorithms.bipartite.matching`.

**References**

3.38 Minors

Provides functions for computing minors of a graph.

| `contracted_edge(G, edge[, self_loops])` | Returns the graph that results from contracting the specified edge. |
| `contracted_nodes(G, u, v[, self_loops, copy])` | Returns the graph that results from contracting `u` and `v`. |
| `identified_nodes(G, u, v[, self_loops, copy])` | Returns the graph that results from contracting `u` and `v`. |
| `quotient_graph(G, partition[, ...])` | Returns the quotient graph of `G` under the specified equivalence relation on nodes. |

3.38.1 networkx.algorithms.minors.contracted_edge

`contracted_edge (G, edge, self_loops=True)`

Returns the graph that results from contracting the specified edge.

Edge contraction identifies the two endpoints of the edge as a single node incident to any edge that was incident

---

to the original two nodes. A graph that results from edge contraction is called a minor of the original graph.

**Parameters**
- `G` (*NetworkX graph*) – The graph whose edge will be contracted.
- `edge` (*tuple*) – Must be a pair of nodes in `G`.
- `self_loops` (*Boolean*) – If this is True, any edges (including `edge`) joining the endpoints of `edge` in `G` become self-loops on the new node in the returned graph.

**Returns** A new graph object of the same type as `G` (leaving `G` unmodified) with endpoints of `edge` identified in a single node. The right node of `edge` will be merged into the left one, so only the left one will appear in the returned graph.

**Return type** NetworkX graph

**Raises** `ValueError` – If `edge` is not an edge in `G`.

### Examples

Attempting to contract two nonadjacent nodes yields an error:

```python
>>> import networkx as nx
>>> G = nx.cycle_graph(4)
>>> nx.contracted_edge(G, (1, 3))
Traceback (most recent call last):
  ...ValueError: Edge (1, 3) does not exist in graph G; cannot contract it
```

Contracting two adjacent nodes in the cycle graph on `n` nodes yields the cycle graph on `n - 1` nodes:

```python
>>> import networkx as nx
>>> C5 = nx.cycle_graph(5)
>>> C4 = nx.cycle_graph(4)
>>> M = nx.contracted_edge(C5, (0, 1), self_loops=False)
>>> nx.is_isomorphic(M, C4)
True
```

**See also:**
- `contracted_nodes()`, `quotient_graph()`

### 3.38.2 networkx.algorithms.minors.contracted_nodes

`contracted_nodes(G, u, v, self_loops=True, copy=True)`

Returns the graph that results from contracting `u` and `v`.

Node contraction identifies the two nodes as a single node incident to any edge that was incident to the original two nodes.

**Parameters**
- `G` (*NetworkX graph*) – The graph whose nodes will be contracted.
- `u, v` (*nodes*) – Must be nodes in `G`.
- `self_loops` (*Boolean*) – If this is True, any edges joining `u` and `v` in `G` become self-loops on the new node in the returned graph.
• **copy** (*Boolean*) – If this is True (default True), make a copy of \( G \) and return that instead of directly changing \( G \).

**Returns** If Copy is True: A new graph object of the same type as \( G \) (leaving \( G \) unmodified) with \( u \) and \( v \) identified in a single node. The right node \( v \) will be merged into the node \( u \), so only \( u \) will appear in the returned graph. If Copy is False: Modifies \( G \) with \( u \) and \( v \) identified in a single node. The right node \( v \) will be merged into the node \( u \), so only \( u \) will appear in the returned graph.

**Return type** Networkx graph

**Notes**

For multigraphs, the edge keys for the realigned edges may not be the same as the edge keys for the old edges. This is natural because edge keys are unique only within each pair of nodes.

**Examples**

Contracting two nonadjacent nodes of the cycle graph on four nodes \( C_4 \) yields the path graph (ignoring parallel edges):

```python
>>> G = nx.cycle_graph(4)
>>> M = nx.contracted_nodes(G, 1, 3)
>>> P3 = nx.path_graph(3)
>>> nx.is_isomorphic(M, P3)
True

>>> G = nx.MultiGraph(P3)
>>> M = nx.contracted_nodes(G, 0, 2)
>>> M.edges
MultiEdgeView([(0, 1, 0), (0, 1, 1)])

>>> G = nx.Graph([(1,2), (2,2)])
>>> H = nx.contracted_nodes(G, 1, 2, self_loops=False)
>>> list(H.nodes())
[1]
>>> list(H.edges())
[(1, 1)]
```

**See also:**

`contracted_edge()`, `quotient_graph()`

**Notes**

This function is also available as `identified_nodes`.

### 3.38.3 networkx.algorithms.minors.identified_nodes

`identified_nodes(G, u, v, self_loops=True, copy=True)`

Returns the graph that results from contracting \( u \) and \( v \).

Node contraction identifies the two nodes as a single node incident to any edge that was incident to the original two nodes.
Parameters

- **G** (*NetworkX graph*) – The graph whose nodes will be contracted.
- **u, v** (*nodes*) – Must be nodes in *G*.
- **self_loops** (*Boolean*) – If this is True, any edges joining *u* and *v* in *G* become self-loops on the new node in the returned graph.
- **copy** (*Boolean*) – If this is True (default True), make a copy of *G* and return that instead of directly changing *G*.

Returns

If Copy is True: A new graph object of the same type as *G* (leaving *G* unmodified) with *u* and *v* identified in a single node. The right node *v* will be merged into the node *u*, so only *u* will appear in the returned graph. If Copy is False: Modifies *G* with *u* and *v* identified in a single node. The right node *v* will be merged into the node *u*, so only *u* will appear in the returned graph.

Return type Networkx graph

Notes

For multigraphs, the edge keys for the realigned edges may not be the same as the edge keys for the old edges. This is natural because edge keys are unique only within each pair of nodes.

Examples

Contracting two nonadjacent nodes of the cycle graph on four nodes $C_4$ yields the path graph (ignoring parallel edges):

```python
>>> G = nx.cycle_graph(4)
>>> M = nx.contracted_nodes(G, 1, 3)
>>> P3 = nx.path_graph(3)
>>> nx.is_isomorphic(M, P3)
True

>>> G = nx.MultiGraph(P3)
>>> M = nx.contracted_nodes(G, 0, 2)
>>> M.edges
MultiEdgeView([(0, 1, 0), (0, 1, 1)])

>>> G = nx.Graph([(1,2), (2,2)])
>>> H = nx.contracted_nodes(G, 1, 2, self_loops=False)
>>> list(H.nodes())
[1]
>>> list(H.edges())
[(1, 1)]
```

See also:

`contracted_edge()`, `quotient_graph()`

Notes

This function is also available as `identified_nodes`. 
3.38.4 networkx.algorithms.minors.quotient_graph

quotient_graph(G, partition, edge_relation=None, node_data=None, edge_data=None, relabel=False, create_using=None)

Returns the quotient graph of G under the specified equivalence relation on nodes.

Parameters

- **G** *(NetworkX graph)* – The graph for which to return the quotient graph with the specified node relation.
- **partition** *(function or list of sets)* – If a function, this function must represent an equivalence relation on the nodes of G. It must take two arguments u and v and return True exactly when u and v are in the same equivalence class. The equivalence classes form the nodes in the returned graph.
- **edge_relation** *(Boolean function with two arguments)* – This function must represent an edge relation on the blocks of G in the partition induced by node_relation. It must take two arguments, B and C, each one a set of nodes, and return True exactly when there should be an edge joining block B to block C in the returned graph.
- **edge_data** *(function)* – This function takes two arguments, B and C, each one a set of nodes, and must return a dictionary representing the edge data attributes to set on the edge joining B and C, should there be an edge joining B and C in the quotient graph (if no such edge occurs in the quotient graph as determined by edge_relation, then the output of this function is ignored).
- **node_data** *(function)* – This function takes one argument, B, a set of nodes in G, and must return a dictionary representing the node data attributes to set on the node representing B in the quotient graph. If None, the following node attributes will be set:
  - ‘graph’, the subgraph of the graph G that this block represents,
  - ‘nnodes’, the number of nodes in this block,
  - ‘nedges’, the number of edges within this block,
  - ‘density’, the density of the subgraph of G that this block represents.
- **relabel** *(bool)* – If True, relabel the nodes of the quotient graph to be nonnegative integers. Otherwise, the nodes are identified with frozenset instances representing the blocks given in partition.
- **create_using** *(NetworkX graph constructor, optional (default=nx.Graph))* – Graph type to create. If graph instance, then cleared before populated.

Returns The quotient graph of G under the equivalence relation specified by partition. If the partition were given as a list of set instances and relabel is False, each node will be a frozenset corresponding to the same set.

Return type NetworkX graph

Raises NetworkXException – If the given partition is not a valid partition of the nodes of G.
Examples

The quotient graph of the complete bipartite graph under the “same neighbors” equivalence relation is $K_2$. Under this relation, two nodes are equivalent if they are not adjacent but have the same neighbor set:

```python
>>> import networkx as nx
>>> G = nx.complete_bipartite_graph(2, 3)
>>> same_neighbors = lambda u, v: (u not in G[v] and v not in G[u] and G[u] == G[v])
>>> Q = nx.quotient_graph(G, same_neighbors)
>>> K2 = nx.complete_graph(2)
>>> nx.is_isomorphic(Q, K2)
True
```

The quotient graph of a directed graph under the “same strongly connected component” equivalence relation is the condensation of the graph (see `condensation()`). This example comes from the Wikipedia article ‘Strongly connected component’:

```python
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> edges = ['ab', 'be', 'bf', 'bc', 'cd', 'dh', 'ea',
          'ef', 'fg', 'hd', 'hf']
>>> G.add_edges_from(tuple(x) for x in edges)
>>> components = list(nx.strongly_connected_components(G))
>>> sorted(sorted(component) for component in components)
[['a', 'b', 'e'], ['c', 'd', 'h'], ['f', 'g']]
>>> C = nx.condensation(G, components)
>>> component_of = C.graph['mapping']
>>> same_component = lambda u, v: component_of[u] == component_of[v]
>>> Q = nx.quotient_graph(G, same_component)
>>> nx.is_isomorphic(C, Q)
True
```

Node identification can be represented as the quotient of a graph under the equivalence relation that places the two nodes in one block and each other node in its own singleton block:

```python
>>> import networkx as nx
>>> K24 = nx.complete_bipartite_graph(2, 4)
>>> K34 = nx.complete_bipartite_graph(3, 4)
>>> C = nx.contracted_nodes(K34, 1, 2)
>>> nodes = {1, 2}
>>> is_contracted = lambda u, v: u in nodes and v in nodes
>>> Q = nx.quotient_graph(K34, is_contracted)
>>> nx.is_isomorphic(Q, C)
True
>>> nx.is_isomorphic(Q, K24)
True
```

The blockmodeling technique described in\(^1\) can be implemented as a quotient graph:

```python
>>> G = nx.path_graph(6)
>>> partition = [{0, 1}, {2, 3}, {4, 5}]
>>> M = nx.quotient_graph(G, partition, relabel=True)
>>> list(M.edges())
[(0, 1), (1, 2)]
```

3.39 Maximal independent set

Algorithm to find a maximal (not maximum) independent set.

maximal_independent_set(G[, nodes, seed]) Returns a random maximal independent set guaranteed to contain a given set of nodes.

3.39.1 networkx.algorithms.mis.maximal_independent_set

maximal_independent_set (G, nodes=None, seed=None) Returns a random maximal independent set guaranteed to contain a given set of nodes.

An independent set is a set of nodes such that the subgraph of G induced by these nodes contains no edges. A maximal independent set is an independent set such that it is not possible to add a new node and still get an independent set.

Parameters

- G (NetworkX graph)
- nodes (list or iterable) – Nodes that must be part of the independent set. This set of nodes must be independent.
- seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns indep_nodes – List of nodes that are part of a maximal independent set.

Return type list

Raises

- NetworkXUnfeasible – If the nodes in the provided list are not part of the graph or do not form an independent set, an exception is raised.
- NetworkXNotImplemented – If G is directed.

Examples

>>> G = nx.path_graph(5)
>>> nx.maximal_independent_set(G)
[4, 0, 2]
>>> nx.maximal_independent_set(G, [1])
[1, 3]

Notes

This algorithm does not solve the maximum independent set problem.
3.40 non-randomness

Computation of graph non-randomness

\( \text{non_randomness}(G[, k]) \) Compute the non-randomness of graph G.

3.40.1 networkx.algorithms.non_randomness.non_randomness

\texttt{non_randomness}(G, k=None)

Compute the non-randomness of graph G.

The first returned value \( \text{nr} \) is the sum of non-randomness values of all edges within the graph (where the non-randomness of an edge tends to be small when the two nodes linked by that edge are from two different communities).

The second computed value \( \text{nr}_\text{rd} \) is a relative measure that indicates to what extent graph G is different from random graphs in terms of probability. When it is close to 0, the graph tends to be more likely generated by an Erdos Renyi model.

\textbf{Parameters}

- \( G \) (NetworkX graph) – Graph must be binary, symmetric, connected, and without self-loops.
- \( k \) (int) – The number of communities in G. If \( k \) is not set, the function will use a default community detection algorithm to set it.

\textbf{Returns} non-randomness – Non-randomness, Relative non-randomness w.r.t. Erdos Renyi random graphs.

\textbf{Return type} (float, float) tuple

\textbf{Examples}

```python
>>> G = nx.karate_club_graph()
>>> nr, nr_rd = nx.non_randomness(G, 2)
```

\textbf{Notes}

This computes Eq. (4.4) and (4.5) in Ref.\textsuperscript{1}.

\textbf{References}

3.41 Moral

Function for computing the moral graph of a directed graph.

\texttt{moral_graph}(G) Return the Moral Graph

\textsuperscript{1} Xiaowei Ying and Xintao Wu, On Randomness Measures for Social Networks, SIAM International Conference on Data Mining, 2009
3.41.1 networkx.algorithms.moral.moral_graph

moral_graph\((G)\)

Return the Moral Graph

Returns the moralized graph of a given directed graph.

**Parameters**

- \(G\) (*NetworkX graph*) – Directed graph

**Returns**

- \(H\) – The undirected moralized graph of \(G\)

**Return type**

- NetworkX graph

**Notes**

A moral graph is an undirected graph \(H = (V, E)\) generated from a directed Graph, where if a node has more than one parent node, edges between these parent nodes are inserted and all directed edges become undirected. 
https://en.wikipedia.org/wiki/Moral_graph

**References**

3.42 Node Classification

This module provides the functions for node classification problem.

The functions in this module are not imported into the top level networkx namespace. You can access these functions by importing the networkx.algorithms.node_classification modules, then accessing the functions as attributes of node_classification. For example:

```python
>>> import networkx as nx
>>> from networkx.algorithms import node_classification

>>> G = nx.path_graph(4)
>>> G.edges()
EdgeView([(0, 1), (1, 2), (2, 3)])
>>> G.nodes[0]['label'] = 'A'
>>> G.nodes[3]['label'] = 'B'
>>> node_classification.harmonic_function(G)
['A', 'A', 'B', 'B']
```

3.42.1 Harmonic Function

Function for computing Harmonic function algorithm by Zhu et al.

**References**


```python
harmonic_function(G[, max_iter, label_name])  # Node classification by Harmonic function
```
Node classification by Harmonic function

Parameters
- \( G \) (NetworkX Graph)
- \( \text{max_iter} \) (int) – maximum number of iterations allowed
- \( \text{label_name} \) (string) – name of target labels to predict

Returns \( \text{predicted} \) – Array of predicted labels

Return type array, shape = [n_samples]

Raises \text{NetworkXError} – If no nodes on \( G \) has \( \text{label_name} \).

Examples

```python
>>> from networkx.algorithms import node_classification
>>> G = nx.path_graph(4)
>>> G.nodes[0]['label'] = 'A'
>>> G.nodes[3]['label'] = 'B'
>>> G.nodes(data=True)
NodeDataView({0: {'label': 'A'}, 1: {}, 2: {}, 3: {'label': 'B'}})
>>> G.edges()
EdgeView([(0, 1), (1, 2), (2, 3)])
>>> predicted = node_classification.harmonic_function(G)
>>> predicted
['A', 'A', 'B', 'B']
```

References


3.4.2.2 Local and Global Consistency

Function for computing Local and global consistency algorithm by Zhou et al.

References

networkx.algorithms.node_classification.lgc.local_and_global_consistency

local_and_global_consistency \((G, \alpha=0.99, \text{max_iter}=30, \text{label_name}='label')\)

Node classification by Local and Global Consistency

Parameters

- \(G\) (NetworkX Graph)
- \(\alpha\) (float) – Clamping factor
- \(\text{max_iter}\) (int) – Maximum number of iterations allowed
- \(\text{label_name}\) (string) – Name of target labels to predict

Returns predicted – Array of predicted labels

Return type array, shape = [n_samples]

Raises NetworkXError – If no nodes on \(G\) has \(\text{label_name}\).

Examples

```python
>>> from networkx.algorithms import node_classification
>>> G = nx.path_graph(4)
>>> G.nodes[0]['label'] = 'A'
>>> G.nodes[3]['label'] = 'B'
>>> G.nodes(data=True)
NodeDataView({0: {'label': 'A'}, 1: {}, 2: {}, 3: {'label': 'B'}})
>>> G.edges()
EdgeView([(0, 1), (1, 2), (2, 3)])
>>> predicted = node_classification.local_and_global_consistency(G)
>>> predicted
['A', 'A', 'B', 'B']
```

References


3.43 Operators

Unary operations on graphs

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<td>complement(G)</td>
<td>Returns the graph complement of (G).</td>
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<tr>
<td>reverse(G[, copy])</td>
<td>Returns the reverse directed graph of (G).</td>
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3.43.1 networkx.algorithms.operators.unary.complement

complement \((G)\)

Returns the graph complement of \(G\).

Parameters \(G\) (graph) – A NetworkX graph
Returns GC
Return type  A new graph.

Notes

Note that complement() does not create self-loops and also does not produce parallel edges for MultiGraphs. Graph, node, and edge data are not propagated to the new graph.

3.43.2 networkx.algorithms.operators.unary.reverse

reverse ($G$, $copy=True$)

Returns the reverse directed graph of $G$.

Parameters

- $G$ (directed graph) – A NetworkX directed graph
- $copy$ (bool) – If True, then a new graph is returned. If False, then the graph is reversed in place.

Returns $H$ – The reversed $G$.

Return type  directed graph

Operations on graphs including union, intersection, difference.

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<td>compose ($G$, $H$)</td>
<td>Returns a new graph of $G$ composed with $H$.</td>
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<td>union ($G$, $H$, rename, name)</td>
<td>Return the union of graphs $G$ and $H$.</td>
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<td>disjoint_union ($G$, $H$)</td>
<td>Return the disjoint union of graphs $G$ and $H$.</td>
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<td>intersection ($G$, $H$)</td>
<td>Returns a new graph that contains only the edges that exist in both $G$ and $H$.</td>
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<td>difference ($G$, $H$)</td>
<td>Returns a new graph that contains the edges that exist in $G$ but not in $H$.</td>
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<tr>
<td>symmetric_difference ($G$, $H$)</td>
<td>Returns new graph with edges that exist in either $G$ or $H$ but not both.</td>
</tr>
<tr>
<td>full_join ($G$, $H$, rename)</td>
<td>Returns the full join of graphs $G$ and $H$.</td>
</tr>
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3.43.3 networkx.algorithms.operators.binary.compose

compose ($G$, $H$)

Returns a new graph of $G$ composed with $H$.

Composition is the simple union of the node sets and edge sets. The node sets of $G$ and $H$ do not need to be disjoint.

Parameters  $G$, $H$ (graph) – A NetworkX graph

Returns  $C$

Return type  A new graph with the same type as $G$
Notes

It is recommended that G and H be either both directed or both undirected. Attributes from H take precedent over attributes from G.

For MultiGraphs, the edges are identified by incident nodes AND edge-key. This can cause surprises (i.e., edge \((1, 2)\) may or may not be the same in two graphs) if you use MultiGraph without keeping track of edge keys.

3.43.4 networkx.algorithms.operators.binary.union

union\((G, H,\ rename=(None, None), \ name=None)\)

Return the union of graphs G and H.

Parameters

- \(G, H\) (graph) – A NetworkX graph
- rename (bool, default=(None, None)) – Node names of G and H can be changed by specifying the tuple rename=('G-','H-') (for example). Node “u” in G is then renamed “G-u” and “v” in H is renamed “H-v”.
- name (string) – Specify the name for the union graph

Returns U

Return type A union graph with the same type as G.

Notes

To force a disjoint union with node relabeling, use disjoint_union(G,H) or convert_node_labels_to_integers().

Graph, edge, and node attributes are propagated from G and H to the union graph. If a graph attribute is present in both G and H the value from H is used.

See also:

disjoint_union()

3.43.5 networkx.algorithms.operators.binary.disjoint_union

disjoint_union\((G, H)\)

Return the disjoint union of graphs G and H.

This algorithm forces distinct integer node labels.

Parameters \(G, H\) (graph) – A NetworkX graph

Returns U

Return type A union graph with the same type as G.
A new graph is created, of the same class as G. It is recommended that G and H be either both directed or both undirected.

The nodes of G are relabeled 0 to len(G)-1, and the nodes of H are relabeled len(G) to len(G)+len(H)-1.

Graph, edge, and node attributes are propagated from G and H to the union graph. If a graph attribute is present in both G and H the value from H is used.

3.43.6 networkx.algorithms.operators.binary.intersection

intersection \((G, H)\)

Returns a new graph that contains only the edges that exist in both G and H.

The node sets of H and G must be the same.

Parameters  
\(G, H\) (graph) – A NetworkX graph. G and H must have the same node sets.

Returns  
\(G\) \(H\)

Return type  
A new graph with the same type as G.

Notes

Attributes from the graph, nodes, and edges are not copied to the new graph. If you want a new graph of the intersection of G and H with the attributes (including edge data) from G use remove_nodes_from() as follows:

```python
>>> G=nx.path_graph(3)
>>> H=nx.path_graph(5)
>>> R=G.copy()
>>> R.remove_nodes_from(n for n in G if n not in H)
```

3.43.7 networkx.algorithms.operators.binary.difference

difference \((G, H)\)

Returns a new graph that contains the edges that exist in G but not in H.

The node sets of H and G must be the same.

Parameters  
\(G, H\) (graph) – A NetworkX graph. G and H must have the same node sets.

Returns  
\(D\)

Return type  
A new graph with the same type as G.

Notes

Attributes from the graph, nodes, and edges are not copied to the new graph. If you want a new graph of the difference of G and H with with the attributes (including edge data) from G use remove_nodes_from() as follows:

```python
>>> G = nx.path_graph(3)
>>> H = nx.path_graph(5)
>>> R = G.copy()
>>> R.remove_nodes_from(n for n in G if n in H)
```
3.43.8 networkx.algorithms.operators.binary.symmetric_difference

**symmetric_difference** *(G, H)*  
Returns new graph with edges that exist in either G or H but not both.

The node sets of H and G must be the same.

**Parameters**  
G, H *(graph)* – A NetworkX graph. G and H must have the same node sets.

**Returns**  
D

**Return type** A new graph with the same type as G.

**Notes**
Attributes from the graph, nodes, and edges are not copied to the new graph.

3.43.9 networkx.algorithms.operators.binary.full_join

**full_join** *(G, H, rename=(None, None))*  
Returns the full join of graphs G and H.

Full join is the union of G and H in which all edges between G and H are added. The node sets of G and H must be disjoint, otherwise an exception is raised.

**Parameters**  
• G, H *(graph)* – A NetworkX graph  
• rename *(bool, default=(None, None))* – Node names of G and H can be changed by specifying the tuple rename=('G-','H-') (for example). Node “u” in G is then renamed “G-u” and “v” in H is renamed “H-v”.

**Returns**  
U

**Return type** The full join graph with the same type as G.

**Notes**
It is recommended that G and H be either both directed or both undirected. If G is directed, then edges from G to H are added as well as from H to G. Note that full_join() does not produce parallel edges for MultiGraphs. The full join operation of graphs G and H is the same as getting their complement, performing a disjoint union, and finally getting the complement of the resulting graph. Graph, edge, and node attributes are propagated from G and H to the union graph. If a graph attribute is present in both G and H the value from H is used.

**See also:**  
union(), disjoint_union()  

Operations on many graphs.

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<td>Returns the composition of all graphs.</td>
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<td><code>disjoint_union_all</code></td>
<td>Returns the disjoint union of all graphs.</td>
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<td><code>intersection_all</code></td>
<td>Returns a new graph that contains only the edges that</td>
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<td>exist in all graphs.</td>
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### 3.43.10 `networkx.algorithms.operators.all.compose_all`

**compose_all** *(graphs)*

Returns the composition of all graphs.

Composition is the simple union of the node sets and edge sets. The node sets of the supplied graphs need not be disjoint.

**Parameters**

- **graphs** *(list)* – List of NetworkX graphs

**Returns**

- **C**

**Return type**

A graph with the same type as the first graph in list

**Raises**

- **ValueError** – If `graphs` is an empty list.

**Notes**

It is recommended that the supplied graphs be either all directed or all undirected.

Graph, edge, and node attributes are propagated to the union graph. If a graph attribute is present in multiple graphs, then the value from the last graph in the list with that attribute is used.

### 3.43.11 `networkx.algorithms.operators.all.union_all`

**union_all**(graphs, rename=(None, ))

Returns the union of all graphs.

The graphs must be disjoint, otherwise an exception is raised.

**Parameters**

- **graphs** *(list of graphs)* – List of NetworkX graphs
- **rename** *(bool, default=(None, None))* – Node names of G and H can be changed by specifying the tuple rename=('G-','H-') (for example). Node “u” in G is then renamed “G-u” and “v” in H is renamed “H-v”.

**Returns**

- **U**

**Return type**

A graph with the same type as the first graph in list

**Raises**

- **ValueError** – If `graphs` is an empty list.

**Notes**

To force a disjoint union with node relabeling, use `disjoint_union_all(G,H)` or `convert_node_labels_to_integers()`.

Graph, edge, and node attributes are propagated to the union graph. If a graph attribute is present in multiple graphs, then the value from the last graph in the list with that attribute is used.

**See also:**

- `union()`, `disjoint_union_all()`
3.43.12 networkx.algorithms.operators.all.disjoint_union_all

disjoint_union_all (graphs)
   Returns the disjoint union of all graphs.

   This operation forces distinct integer node labels starting with 0 for the first graph in the list and numbering
   consecutively.

   Parameters  graphs (list) – List of NetworkX graphs
   Returns  U
   Return type  A graph with the same type as the first graph in list
   Raises  ValueError – If graphs is an empty list.

   Notes

   It is recommended that the graphs be either all directed or all undirected.

   Graph, edge, and node attributes are propagated to the union graph. If a graph attribute is present in multiple
   graphs, then the value from the last graph in the list with that attribute is used.

3.43.13 networkx.algorithms.operators.all.intersection_all

intersection_all (graphs)
   Returns a new graph that contains only the edges that exist in all graphs.

   All supplied graphs must have the same node set.

   Parameters  graphs (list) – List of NetworkX graphs
   Returns  R
   Return type  A new graph with the same type as the first graph in list
   Raises  ValueError – If graphs is an empty list.

   Notes

   Attributes from the graph, nodes, and edges are not copied to the new graph.

   Graph products.

   cartesian_product(G, H)        Returns the Cartesian product of G and H.
   lexicographic_product(G, H)   Returns the lexicographic product of G and H.
   rooted_product(G, H, root)   Return the rooted product of graphs G and H rooted at
                                 root in H.
   strong_product(G, H)          Returns the strong product of G and H.
   tensor_product(G, H)          Returns the tensor product of G and H.
   power(G, k)                  Returns the specified power of a graph.

3.43.14 networkx.algorithms.operators.product.cartesian_product

cartesian_product (G, H)
   Returns the Cartesian product of G and H.
The Cartesian product $P$ of the graphs $G$ and $H$ has a node set that is the Cartesian product of the node sets, $\mathcal{V}(P) = \mathcal{V}(G) \times \mathcal{V}(H)$. $P$ has an edge $((u, v), (x, y))$ if and only if either $u$ is equal to $x$ and both $v$ and $y$ are adjacent in $H$ or if $v$ is equal to $y$ and both $u$ and $x$ are adjacent in $G$.

**Parameters** $G, H$ (graphs) – Networkx graphs.

**Returns** $P$ – The Cartesian product of $G$ and $H$. $P$ will be a multi-graph if either $G$ or $H$ is a multi-graph. Will be a directed if $G$ and $H$ are directed, and undirected if $G$ and $H$ are undirected.

**Return type** NetworkX graph

**Raises** *NetworkXError* – If $G$ and $H$ are not both directed or both undirected.

**Notes**

Node attributes in $P$ are two-tuple of the $G$ and $H$ node attributes. Missing attributes are assigned None.

**Examples**

```python
>>> G = nx.Graph()
>>> H = nx.Graph()
>>> G.add_node(0, a1=True)
>>> H.add_node('a', a2='Spam')
>>> P = nx.cartesian_product(G, H)
>>> list(P)
[(0, 'a')]
```

Edge attributes and edge keys (for multigraphs) are also copied to the new product graph.

### 3.43.15 networkx.algorithms.operators.product.lexicographic_product

**lexicographic_product** $(G, H)$

Returns the lexicographical product of $G$ and $H$.

The lexicographical product $P$ of the graphs $G$ and $H$ has a node set that is the Cartesian product of the node sets, $\mathcal{V}(P) = \mathcal{V}(G) \times \mathcal{V}(H)$. $P$ has an edge $((u, v), (x, y))$ if and only if $(u, v)$ is an edge in $G$ or $u == v$ and $(x, y)$ is an edge in $H$.

**Parameters** $G, H$ (graphs) – Networkx graphs.

**Returns** $P$ – The Cartesian product of $G$ and $H$. $P$ will be a multi-graph if either $G$ or $H$ is a multi-graph. Will be a directed if $G$ and $H$ are directed, and undirected if $G$ and $H$ are undirected.

**Return type** NetworkX graph

**Raises** *NetworkXError* – If $G$ and $H$ are not both directed or both undirected.

**Notes**

Node attributes in $P$ are two-tuple of the $G$ and $H$ node attributes. Missing attributes are assigned None.

**Examples**
>>> G = nx.Graph()
>>> H = nx.Graph()
>>> G.add_node(0, a1=True)
>>> H.add_node('a', a2='Spam')
>>> P = nx.lexicographic_product(G, H)
>>> list(P)
[(0, 'a')]

Edge attributes and edge keys (for multigraphs) are also copied to the new product graph

### 3.43.16 networkx.algorithms.operators.product.rooted_product

**rooted_product** *(G, H, root)*

Return the rooted product of graphs G and H rooted at root in H.

A new graph is constructed representing the rooted product of the inputted graphs, G and H, with a root in H. A rooted product duplicates H for each nodes in G with the root of H corresponding to the node in G. Nodes are renamed as the direct product of G and H. The result is a subgraph of the cartesian product.

**Parameters**

- **G, H (graph)** – A NetworkX graph
- **root (node)** – A node in H

**Returns** R

**Return type** The rooted product of G and H with a specified root in H

**Notes**

The nodes of R are the Cartesian Product of the nodes of G and H. The nodes of G and H are not relabeled.

### 3.43.17 networkx.algorithms.operators.product.strong_product

**strong_product** *(G, H)*

Returns the strong product of G and H.

The strong product \( P \) of the graphs \( G \) and \( H \) has a node set that is the Cartesian product of the node sets, \( V(P) = V(G) \times V(H) \). \( P \) has an edge \((u, v), (x, y)\) if and only if \( u == v \) and \((x, y)\) is an edge in \( H \), or \( x == y \) and \((u, v)\) is an edge in \( G \), or \((u, v)\) is an edge in \( G \) and \((x, y)\) is an edge in \( H \).

**Parameters** G, H (graphs) – Networkx graphs.

**Returns** P – The Cartesian product of G and H. P will be a multi-graph if either G or H is a multi-graph. Will be directed if G and H are directed, and undirected if G and H are undirected.

**Return type** NetworkX graph

**Raises** NetworkXError – If G and H are not both directed or both undirected.

**Notes**

Node attributes in P are two-tuple of the G and H node attributes. Missing attributes are assigned None.
Examples

```python
>>> G = nx.Graph()
>>> H = nx.Graph()
>>> G.add_node(0, a1=True)
>>> H.add_node('a', a2='Spam')
>>> P = nx.tensor_product(G, H)
>>> list(P)
[(0, 'a')]
```

Edge attributes and edge keys (for multigraphs) are also copied to the new product graph

3.43.18 networkx.algorithms.operators.product.tensor_product

tensor_product  

Returns the tensor product of G and H.

The tensor product $P$ of the graphs $G$ and $H$ has a node set that is the tensor product of the node sets, $V(P) = V(G) \times V(H)$. $P$ has an edge $((u, v), (x, y))$ if and only if $(u, x)$ is an edge in $G$ and $(v, y)$ is an edge in $H$.

Tensor product is sometimes also referred to as the categorical product, direct product, cardinal product or conjunction.

Parameters  

G, H (graphs) – NetworkX graphs.

Returns  

P – The tensor product of G and H. P will be a multi-graph if either G or H is a multi-graph, will be a directed if G and H are directed, and undirected if G and H are undirected.

Return type  

NetworkX graph

Raises  

NetworkXError – If G and H are not both directed or both undirected.

Notes

Node attributes in P are two-tuple of the G and H node attributes. Missing attributes are assigned None.

Examples

```python
>>> G = nx.Graph()
>>> H = nx.Graph()
>>> G.add_node(0, a1=True)
>>> H.add_node('a', a2='Spam')
>>> P = nx.tensor_product(G, H)
>>> list(P)
[(0, 'a')]
```

Edge attributes and edge keys (for multigraphs) are also copied to the new product graph

3.43.19 networkx.algorithms.operators.product.power

power  

Returns the specified power of a graph.

The $k$, denoted $G^k$, is a graph on the same set of nodes in which two distinct nodes $u$ and $v$ are adjacent in $G^k$ if and only if the shortest path distance between $u$ and $v$ in $G$ is at most $k$. 

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Parameters

- **G (graph)** – A NetworkX simple graph object.
- **k (positive integer)** – The power to which to raise the graph G.

Returns

G to the power k.

Return type

NetworkX simple graph

Raises

- **ValueError** – If the exponent k is not positive.
- **NetworkXNotImplemented** – If G is not a simple graph.

Examples

The number of edges will never decrease when taking successive powers:

```python
>>> G = nx.path_graph(4)
>>> list(nx.power(G, 2).edges)
[(0, 1), (0, 2), (1, 2), (1, 3), (2, 3)]
>>> list(nx.power(G, 3).edges)
[(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3)]
```

The k\textsuperscript{th} power of a cycle graph on *n* nodes is the complete graph on *n* nodes, if k is at least n / 2:

```python
>>> G = nx.cycle_graph(5)
>>> H = nx.complete_graph(5)
>>> nx.is_isomorphic(nx.power(G, 2), H)
True
>>> G = nx.cycle_graph(8)
>>> H = nx.complete_graph(8)
>>> nx.is_isomorphic(nx.power(G, 4), H)
True
```

References

Notes

This definition of “power graph” comes from Exercise 3.1.6 of *Graph Theory* by Bondy and Murty\textsuperscript{1}.

3.44 Planarity

```python
check_planarity(G[, counterexample])
```

Check if a graph is planar and return a counterexample or an embedding.

### 3.44.1 networkx.algorithms.planarity.check_planarity

**check_planarity** *(G, counterexample=False)*

Check if a graph is planar and return a counterexample or an embedding.

A graph is planar if it can be drawn in a plane without any edge intersections.

**Parameters**

- **G** *(NetworkX graph)*
- **counterexample** *(bool)* – A Kuratowski subgraph (to prove non-planarity) is only returned if set to true.

**Returns** *(is_planar, certificate)* – *is_planar* is true if the graph is planar. If the graph is planar, *certificate* is a PlanarEmbedding otherwise it is a Kuratowski subgraph.

**Return type** *(bool, NetworkX graph)* tuple

**Notes**

A (combinatorial) embedding consists of cyclic orderings of the incident edges at each vertex. Given such an embedding there are multiple approaches discussed in literature to drawing the graph (subject to various constraints, e.g. integer coordinates), see e.g. [2].

The planarity check algorithm and extraction of the combinatorial embedding is based on the Left-Right Planarity Test [1].

A counterexample is only generated if the corresponding parameter is set, because the complexity of the counterexample generation is higher.

**References**

- **class PlanarEmbedding**(incoming_graph_data=None, **attr)**
  Represents a planar graph with its planar embedding.

  The planar embedding is given by a **combinatorial embedding**.

  **Neighbor ordering:**

  In comparison to a usual graph structure, the embedding also stores the order of all neighbors for every vertex. The order of the neighbors can be given in clockwise (cw) direction or counterclockwise (ccw) direction. This order is stored as edge attributes in the underlying directed graph. For the edge *(u, v)* the edge attribute ‘cw’ is set to the neighbor of *u* that follows immediately after *v* in clockwise direction.

  In order for a PlanarEmbedding to be valid it must fulfill multiple conditions. It is possible to check if these conditions are fulfilled with the method **check_structure()**. The conditions are:

  - Edges must go in both directions (because the edge attributes differ)
  - Every edge must have a ‘cw’ and ‘ccw’ attribute which corresponds to a correct planar embedding.
  - A node with non zero degree must have a node attribute ‘first_nbr’.

  As long as a PlanarEmbedding is invalid only the following methods should be called:

  - **add_half_edge_ccw()**
  - **add_half_edge_cw()**
  - **connect_components()**
• add_half_edge_first()

Even though the graph is a subclass of nx.DiGraph, it can still be used for algorithms that require undirected graphs, because the method is_directed() is overridden. This is possible, because a valid PlanarGraph must have edges in both directions.

Half edges:

In methods like add_half_edge_ccw the term “half-edge” is used, which is a term that is used in doubly connected edge lists. It is used to emphasize that the edge is only in one direction and there exists another half-edge in the opposite direction. While conventional edges always have two faces (including outer face) next to them, it is possible to assign each half-edge exactly one face. For a half-edge (u, v) that is orientated such that u is below v then the face that belongs to (u, v) is to the right of this half-edge.

Examples

Create an embedding of a star graph (compare nx.star_graph(3)):

```python
>>> G = nx.PlanarEmbedding()
>>> G.add_half_edge_cw(0, 1, None)
>>> G.add_half_edge_cw(0, 2, 1)
>>> G.add_half_edge_cw(0, 3, 2)
>>> G.add_half_edge_cw(1, 0, None)
>>> G.add_half_edge_cw(2, 0, None)
>>> G.add_half_edge_cw(3, 0, None)
```

Alternatively the same embedding can also be defined in counterclockwise orientation. The following results in exactly the same PlanarEmbedding:

```python
>>> G = nx.PlanarEmbedding()
>>> G.add_half_edge_ccw(0, 1, None)
>>> G.add_half_edge_ccw(0, 3, 1)
>>> G.add_half_edge_ccw(0, 2, 3)
>>> G.add_half_edge_ccw(1, 0, None)
>>> G.add_half_edge_ccw(2, 0, None)
>>> G.add_half_edge_ccw(3, 0, None)
```

After creating a graph, it is possible to validate that the PlanarEmbedding object is correct:

```python
>>> G.check_structure()
```

add_half_edge_ccw(start_node, end_node, reference_neighbor)

Adds a half-edge from start_node to end_node.

The half-edge is added counter clockwise next to the existing half-edge (start_node, reference_neighbor).

Parameters

- **start_node (node)** – Start node of inserted edge.
- **end_node (node)** – End node of inserted edge.
- **reference_neighbor (node)** – End node of reference edge.

Raises NetworkXException – If the reference_neighbor does not exist.

See also:

add_half_edge_cw(), connect_components(), add_half_edge_first()
add_half_edge_cw(start_node, end_node, reference_neighbor)
Add a half-edge from start_node to end_node.

The half-edge is added clockwise next to the existing half-edge (start_node, reference_neighbor).

Parameters
• start_node (node) – Start node of inserted edge.
• end_node (node) – End node of inserted edge.
• reference_neighbor (node) – End node of reference edge.

Raises NetworkXException – If the reference_neighbor does not exist.

See also:
add_half_edge_ccw(), connect_components(), add_half_edge_first()

add_half_edge_first(start_node, end_node)
The added half-edge is inserted at the first position in the order.

Parameters
• start_node (node)
• end_node (node)

See also:
add_half_edge_ccw(), add_half_edge_cw(), connect_components()

check_structure()
Runs without exceptions if this object is valid.

Checks that the following properties are fulfilled:
• Edges go in both directions (because the edge attributes differ).
• Every edge has a ‘cw’ and ‘ccw’ attribute which corresponds to a correct planar embedding.
• A node with a degree larger than 0 has a node attribute ‘first_nbr’.

Running this method verifies that the underlying Graph must be planar.

Raises NetworkXException – This exception is raised with a short explanation if the PlanarEmbedding is invalid.

connect_components(v, w)
Adds half-edges for (v, w) and (w, v) at some position.

This method should only be called if v and w are in different components, or it might break the embedding. This especially means that if connect_components(v, w) is called it is not allowed to call connect_components(w, v) afterwards. The neighbor orientations in both directions are all set correctly after the first call.

Parameters
• v (node)
• w (node)

See also:
add_half_edge_ccw(), add_half_edge_cw(), add_half_edge_first()

get_data()
Converts the adjacency structure into a better readable structure.
Returns **embedding** – A dict mapping all nodes to a list of neighbors sorted in clockwise order.

**Return type** dict

**See also:**

*set_data()*

**is_directed()**

A valid PlanarEmbedding is undirected.

All reverse edges are contained, i.e. for every existing half-edge `(v, w)` the half-edge in the opposite direction `(w, v)` is also contained.

**neighbors_cw_order(v)**

Generator for the neighbors of `v` in clockwise order.

**Parameters** `v` *(node)*

**Yields** `node`

**next_face_half_edge(v, w)**

Returns the following half-edge left of a face.

**Parameters**

* `v` *(node)*
* `w` *(node)*

**Returns** half-edge

**Return type** tuple

**set_data(data)**

Inserts edges according to given sorted neighbor list.

The input format is the same as the output format of `get_data()`.

**Parameters** `data` *(dict)* – A dict mapping all nodes to a list of neighbors sorted in clockwise order.

**See also:**

*get_data()*

**traverse_face(v, w, mark_half_edges=None)**

Returns nodes on the face that belong to the half-edge `(v, w)`.

The face that is traversed lies to the right of the half-edge (in an orientation where `v` is below `w`).

Optionally it is possible to pass a set to which all encountered half-edges are added. Before calling this method, this set must not include any half-edges that belong to the face.

**Parameters**

* `v` *(node)* – Start node of half-edge.
* `w` *(node)* – End node of half-edge.
* `mark_half_edges` *(set, optional)* – Set to which all encountered half-edges are added.

**Returns** `face` – A list of nodes that lie on this face.

**Return type** list
3.45 Planar Drawing

combinatorial_embedding_to_pos(embedding, ...)

Assigns every node a (x, y) position based on the given embedding. The algorithm iteratively inserts nodes of the input graph in a certain order and rearranges previously inserted nodes so that the planar drawing stays valid. This is done efficiently by only maintaining relative positions during the node placements and calculating the absolute positions at the end. For more information see 1.

Parameters

- **embedding** *(nx.PlanarEmbedding)* – This defines the order of the edges
- **fully_triangulate** *(bool)* – If set to True the algorithm adds edges to a copy of the input embedding and makes it chordal.

Returns **pos** – Maps each node to a tuple that defines the (x, y) position

Return type **dict**

References

3.46 Reciprocity

Algorithms to calculate reciprocity in a directed graph.

reciprocity(G[, nodes]) Compute the reciprocity in a directed graph.

overall_reciprocity(G) Compute the reciprocity for the whole graph.

3.46.1 networkx.algorithms.reciprocity.reciprocity

reciprocity (G, nodes=None)

Compute the reciprocity in a directed graph.

The reciprocity of a directed graph is defined as the ratio of the number of edges pointing in both directions to the total number of edges in the graph. Formally, \( r = \frac{|(u, v) \in G, (v, u) \in G|}{|G|} \).

The reciprocity of a single node \( u \) is defined similarly, it is the ratio of the number of edges in both directions to the total number of edges attached to node \( u \).

Parameters

- **G** *(graph)* – A networkx directed graph
- **nodes** *(container of nodes, optional (default=whole graph))* – Compute reciprocity for nodes in this container.

Returns **out** – Reciprocity keyed by node label.

---


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Return type dictionary

Notes

The reciprocity is not defined for isolated nodes. In such cases this function will return None.

3.46.2 networkx.algorithms.reciprocity.overall_reciprocity

overall_reciprocity(G)
Compute the reciprocity for the whole graph.
See the doc of reciprocity for the definition.
Parameters G (graph) – A networkx graph

3.47 Rich Club

Functions for computing rich-club coefficients.

rich_club_coefficient(G[, normalized, Q, seed])
Returns the rich-club coefficient of the graph G.

3.47.1 networkx.algorithms.richclub.rich_club_coefficient

rich_club_coefficient (G, normalized=True, Q=100, seed=None)
Returns the rich-club coefficient of the graph G.
For each degree k, the rich-club coefficient is the ratio of the number of actual to the number of potential edges for nodes with degree greater than k:
\[
\phi(k) = \frac{2E_k}{N_k(N_k - 1)}
\]
where \(N_k\) is the number of nodes with degree larger than \(k\), and \(E_k\) is the number of edges among those nodes.

Parameters

- G (NetworkX graph) – Undirected graph with neither parallel edges nor self-loops.
- normalized (bool (optional)) – Normalize using randomized network as in\(^1\)
- Q (float (optional, default=100)) – If normalized is True, perform \(Q \times m\) double-edge swaps, where \(m\) is the number of edges in \(G\), to use as a null-model for normalization.
- seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns rc – A dictionary, keyed by degree, with rich-club coefficient values.

Return type dictionary

Examples

```python
>>> G = nx.Graph([(0, 1), (0, 2), (1, 2), (1, 3), (1, 4), (4, 5)])
>>> rc = nx.rich_club_coefficient(G, normalized=False, seed=42)
>>> rc[0]
0.4
```

Notes

The rich club definition and algorithm are found in\(^1\). This algorithm ignores any edge weights and is not defined for directed graphs or graphs with parallel edges or self loops.

Estimates for appropriate values of ρ are found in\(^2\).

References

3.48 Shortest Paths

Compute the shortest paths and path lengths between nodes in the graph.

These algorithms work with undirected and directed graphs.

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<td><code>shortest_path(G[, source, target, weight, ...])</code></td>
<td>Compute shortest paths in the graph.</td>
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<td>Returns True if G has a path from source to target.</td>
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3.48.1 networkx.algorithms.shortest_paths.generic.shortest_path

`shortest_path(G, source=None, target=None, weight=None, method='dijkstra')`

Compute shortest paths in the graph.

Parameters

- `G` (NetworkX graph)
- `source` (node, optional) – Starting node for path. If not specified, compute shortest paths for each possible starting node.
- `target` (node, optional) – Ending node for path. If not specified, compute shortest paths to all possible nodes.
- `weight` (None or string, optional (default = None)) – If None, every edge has weight/distance/cost 1. If a string, use this edge attribute as the edge weight. Any edge attribute not present defaults to 1.
- `method` (string, optional (default = ‘dijkstra’)) – The algorithm to use to compute the path. Supported options: ‘dijkstra’, ‘bellman-ford’. Other inputs produce a ValueError. If weight is None, unweighted graph methods are used, and this suggestion is ignored.


Returns

path – All returned paths include both the source and target in the path.

If the source and target are both specified, return a single list of nodes in a shortest path from the source to the target.

If only the source is specified, return a dictionary keyed by targets with a list of nodes in a shortest path from the source to one of the targets.

If only the target is specified, return a dictionary keyed by sources with a list of nodes in a shortest path from one of the sources to the target.

If neither the source nor target are specified return a dictionary of dictionaries with path[source][target]=[list of nodes in path].

Return type list or dictionary

Raises

• NodeNotFound – If source is not in G.
• ValueError – If method is not among the supported options.

Examples

```python
g = nx.path_graph(5)
>>> print(nx.shortest_path(g, source=0, target=4))
[0, 1, 2, 3, 4]
>>> p = nx.shortest_path(g, source=0) # target not specified
>>> p[4]
[0, 1, 2, 3, 4]
>>> p = nx.shortest_path(g, target=4) # source not specified
>>> p[0]
[0, 1, 2, 3, 4]
>>> p = nx.shortest_path(g) # source, target not specified
>>> p[0][4]
[0, 1, 2, 3, 4]
```

Notes

There may be more than one shortest path between a source and target. This returns only one of them.

See also:

all_pairs_shortest_path(), all_pairs_dijkstra_path(),
all_pairs_bellman_ford_path(), single_source_shortest_path(),
single_source_dijkstra_path(), single_source_bellman_ford_path()
• **target** (*node*) – Ending node for path.

• **weight** (*None or string, optional (default = None)*) – If None, every edge has weight/distance/cost 1. If a string, use this edge attribute as the edge weight. Any edge attribute not present defaults to 1.

• **method** (*string, optional (default = ‘dijkstra’)*) – The algorithm to use to compute the path lengths. Supported options: ‘dijkstra’, ‘bellman-ford’. Other inputs produce a ValueError. If `weight` is None, unweighted graph methods are used, and this suggestion is ignored.

**Returns** paths – A generator of all paths between source and target.

**Return type** generator of lists

**Raises**

• **ValueError** – If `method` is not among the supported options.

• **NetworkXNoPath** – If `target` cannot be reached from `source`.

**Examples**

```python
>>> G = nx.Graph()
>>> nx.add_path(G, [0, 1, 2])
>>> nx.add_path(G, [0, 10, 2])
>>> print([p for p in nx.all_shortest_paths(G, source=0, target=2)])
[[0, 1, 2], [0, 10, 2]]
```

**Notes**

There may be many shortest paths between the source and target.

**See also:**

`shortest_path()`, `single_source_shortest_path()`, `all_pairs_shortest_path()`

### 3.48.3 networkx.algorithms.shortest_paths.generic.shortest_path_length

**shortest_path_length** (*G, source=None, target=None, weight=None, method=’dijkstra’*)

Compute shortest path lengths in the graph.

**Parameters**

• **G** (*NetworkX graph*)

• **source** (*node, optional*) – Starting node for path. If not specified, compute shortest path lengths using all nodes as source nodes.

• **target** (*node, optional*) – Ending node for path. If not specified, compute shortest path lengths using all nodes as target nodes.

• **weight** (*None or string, optional (default = None)*) – If None, every edge has weight/distance/cost 1. If a string, use this edge attribute as the edge weight. Any edge attribute not present defaults to 1.

• **method** (*string, optional (default = ‘dijkstra’)*) – The algorithm to use to compute the path length. Supported options: ‘dijkstra’, ‘bellman-ford’. Other inputs produce a ValueError. If `weight` is None, unweighted graph methods are used, and this suggestion is ignored.
Returns

- **length** – If the source and target are both specified, return the length of the shortest path from the source to the target.

  If only the source is specified, return a dict keyed by target to the shortest path length from the source to that target.

  If only the target is specified, return a dict keyed by source to the shortest path length from that source to the target.

  If neither the source nor target are specified, return an iterator over (source, dictionary) where dictionary is keyed by target to shortest path length from source to that target.

Return type: int or iterator

Raises

- **NodeNotFound** – If source is not in G.
- **NetworkXNoPath** – If no path exists between source and target.
- **ValueError** – If method is not among the supported options.

Examples

```python
>>> G = nx.path_graph(5)
>>> nx.shortest_path_length(G, source=0, target=4)
4
>>> p = nx.shortest_path_length(G, source=0)  # target not specified
4
>>> p[4]
4
>>> p = nx.shortest_path_length(G, target=4)  # source not specified
4
>>> p[0]
4
>>> p = dict(nx.shortest_path_length(G))  # source, target not specified
>>> p[0][4]
4
```

Notes

The length of the path is always 1 less than the number of nodes involved in the path since the length measures the number of edges followed.

For digraphs this returns the shortest directed path length. To find path lengths in the reverse direction use G.reverse(copy=False) first to flip the edge orientation.

See also:

- `all_pairs_shortest_path_length()`, `all_pairs_dijkstra_path_length()`, `all_pairs_bellman_ford_path_length()`, `single_source_shortest_path_length()`, `single_source_dijkstra_path_length()`, `single_source_bellman_ford_path_length()`

3.48.4 networkx.algorithms.shortest_paths.generic.average_shortest_path_length

**average_shortest_path_length** *(G, weight=None, method=None)*

Returns the average shortest path length.
The average shortest path length is

\[ a = \sum_{s, t \in V} \frac{d(s, t)}{n(n-1)} \]

where \( V \) is the set of nodes in \( G \), \( d(s, t) \) is the shortest path from \( s \) to \( t \), and \( n \) is the number of nodes in \( G \).

Parameters

- \( G \) (NetworkX graph)
- weight (None or string, optional (default = None)) – If None, every edge has weight/distance/cost 1. If a string, use this edge attribute as the edge weight. Any edge attribute not present defaults to 1.
- method (string, optional (default = ‘unweighted’ or ‘dijkstra’)) – The algorithm to use to compute the path lengths. Supported options are ‘unweighted’, ‘dijkstra’, ‘bellman-ford’, ‘floyd-warshall’ and ‘floyd-warshall-numpy’. Other method values produce a ValueError. The default method is ‘unweighted’ if weight is None, otherwise the default method is ‘dijkstra’.

Raises

- NetworkXPointlessConcept – If \( G \) is the null graph (that is, the graph on zero nodes).
- NetworkXError – If \( G \) is not connected (or not weakly connected, in the case of a directed graph).
- ValueError – If method is not among the supported options.

Examples

```python
>>> G = nx.path_graph(5)
>>> nx.average_shortest_path_length(G)
2.0
```

For disconnected graphs, you can compute the average shortest path length for each component

```python
>>> G = nx.Graph([(1, 2), (3, 4)])
>>> for C in (G.subgraph(c).copy() for c in connected_components(G)):
...    print(nx.average_shortest_path_length(C))
1.0
1.0
```

3.48.5 networkx.algorithms.shortest_paths.generic.has_path

has_path \((G, source, target)\)

Returns True if \( G \) has a path from \( source \) to \( target \).

Parameters

- \( G \) (NetworkX graph)
- source (node) – Starting node for path
- target (node) – Ending node for path
3.48.6 Advanced Interface

Shortest path algorithms for unweighted graphs.

<table>
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<th>Description</th>
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<td>Compute shortest path between source and all other nodes reachable from source.</td>
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**networkx.algorithms.shortest_paths.unweighted.single_source_shortest_path**

**`single_source_shortest_path(G, source[, cutoff])`**

Compute shortest path between source and all other nodes reachable from source.

**Parameters**

- **G** (*NetworkX graph*)
- **source** (*node label*) – Starting node for path
- **cutoff** (*integer, optional*) – Depth to stop the search. Only paths of length <= cutoff are returned.

**Returns**

- **lengths** – Dictionary, keyed by target, of shortest paths.

**Return type**

- dictionary

**Examples**

```python
>>> G = nx.path_graph(5)
>>> path = nx.single_source_shortest_path(G, 0)
>>> path[4]
[0, 1, 2, 3, 4]
```

**Notes**

The shortest path is not necessarily unique. So there can be multiple paths between the source and each target node, all of which have the same ‘shortest’ length. For each target node, this function returns only one of those paths.

**See also:**

- `shortest_path()`
networkx.algorithms.shortest_paths.unweighted.single_source_shortest_path_length

`single_source_shortest_path_length(G, source, cutoff=None)`

Compute the shortest path lengths from source to all reachable nodes.

**Parameters**

- `G` (*NetworkX graph*)
- `source` (*node*) – Starting node for path
- `cutoff` (*integer, optional*) – Depth to stop the search. Only paths of length <= cutoff are returned.

**Returns**

- `lengths` – Dict keyed by node to shortest path length to source.
- `Return type` *dict*

**Examples**

```python
>>> G = nx.path_graph(5)
>>> length = nx.single_source_shortest_path_length(G, 0)
>>> length[4]
4
>>> for node in length:
...     print('{:}: {}'.format(node, length[node]))
0: 0
1: 1
2: 2
3: 3
4: 4
```

**See also:**

- `shortest_path_length()`

networkx.algorithms.shortest_paths.unweighted.single_target_shortest_path

`single_target_shortest_path(G, target, cutoff=None)`

Compute shortest path to target from all nodes that reach target.

**Parameters**

- `G` (*NetworkX graph*)
- `target` (*node label*) – Target node for path
- `cutoff` (*integer, optional*) – Depth to stop the search. Only paths of length <= cutoff are returned.

**Returns**

- `lengths` – Dictionary, keyed by target, of shortest paths.
- `Return type` *dictionary*

**Examples**
NetworkX Reference, Release 2.5rc1.dev20191214022633

```python
>>> G = nx.path_graph(5, create_using=nx.DiGraph())
>>> path = nx.single_target_shortest_path(G, 4)
>>> path[0]
[0, 1, 2, 3, 4]
```

### Notes

The shortest path is not necessarily unique. So there can be multiple paths between the source and each target node, all of which have the same ‘shortest’ length. For each target node, this function returns only one of those paths.

See also:

shortest_path(), single_source_shortest_path()

---

networkx.algorithms.shortest_paths.unweighted.single_target_shortest_path_length

**single_target_shortest_path_length** *(G, target, cutoff=None)*

Compute the shortest path lengths to target from all reachable nodes.

#### Parameters

- **G** *(NetworkX graph)*
- **target** *(node)* – Target node for path
- **cutoff** *(integer, optional)* – Depth to stop the search. Only paths of length <= cutoff are returned.

#### Returns

- **lengths** – (source, shortest path length) iterator

#### Return type

iterator

#### Examples

```python
>>> G = nx.path_graph(5, create_using=nx.DiGraph())
>>> length = dict(nx.single_target_shortest_path_length(G, 4))
>>> length[0]
4
>>> for node in range(5):
...     print('{}: {}'.format(node, length[node]))
0: 4
1: 3
2: 2
3: 1
4: 0
```

See also:

single_source_shortest_path_length(), shortest_path_length()

---

networkx.algorithms.shortest_paths.unweighted.bidirectional_shortest_path

**bidirectional_shortest_path** *(G, source, target)*

Returns a list of nodes in a shortest path between source and target.

---

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Parameters

- **G** (*NetworkX graph*)
- **source** (*node label*) – starting node for path
- **target** (*node label*) – ending node for path

Returns **path** – List of nodes in a path from source to target.

Return type **list**

Raises **NetworkXNoPath** – If no path exists between source and target.

See also: `shortest_path()`

Notes

This algorithm is used by `shortest_path(G, source, target)`.


**all_pairs_shortest_path** (*G, cutoff=None*)

Compute shortest paths between all nodes.

Parameters

- **G** (*NetworkX graph*)
- **cutoff** (*integer, optional*) – Depth at which to stop the search. Only paths of length at most `cutoff` are returned.

Returns **lengths** – Dictionary, keyed by source and target, of shortest paths.

Return type **dictionary**

Examples

```python
>>> G = nx.path_graph(5)
>>> path = dict(nx.all_pairs_shortest_path(G))
>>> print(path[0][4])
[0, 1, 2, 3, 4]
```

See also: `floyd_warshall()`

[networkx.algorithms.shortest_paths.unweighted.all_pairs_shortest_path_length](https://networkx.github.io/documentation/stable/reference/algorithms/generated/networkx.algorithms.shortest_paths.unweighted.all_pairs_shortest_path_length.html)

**all_pairs_shortest_path_length** (*G, cutoff=None*)

Computes the shortest path lengths between all nodes in `G`.

Parameters

- **G** (*NetworkX graph*)
- **cutoff** (*integer, optional*) – Depth at which to stop the search. Only paths of length at most `cutoff` are returned.
Returns lengths – (source, dictionary) iterator with dictionary keyed by target and shortest path length as the key value.

Return type iterator

Notes

The iterator returned only has reachable node pairs.

Examples

```python
>>> G = nx.path_graph(5)
>>> length = dict(nx.all_pairs_shortest_path_length(G))
>>> for node in [0, 1, 2, 3, 4]:
...    print('1 - {}: {}'.format(node, length[1][node]))
1 - 0: 1
1 - 1: 0
1 - 2: 1
1 - 3: 2
1 - 4: 3
```

networkx.algorithms.shortest_paths.unweighted.predecessor

predecessor (G, source, target=None, cutoff=None, return_seen=None)

Returns dict of predecessors for the path from source to all nodes in G

Parameters

- G (NetworkX graph)
- source (node label) – Starting node for path
- target (node label, optional) – Ending node for path. If provided only predecessors between source and target are returned
- cutoff (integer, optional) – Depth to stop the search. Only paths of length <= cutoff are returned.

Returns pred – Dictionary, keyed by node, of predecessors in the shortest path.

Return type dictionary

Examples

```python
>>> G = nx.path_graph(4)
>>> list(G)
[0, 1, 2, 3]
>>> nx.predecessor(G, 0)
[0: [], 1: [0], 2: [1], 3: [2]]
```
Shortest path algorithms for weighed graphs.

<table>
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<tr>
<th>Function Name</th>
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<td><code>dijkstra_path(G, source, target[, weight])</code></td>
<td>Returns the shortest weighted path from source to target in G.</td>
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<td><code>dijkstra_path_length(G, source, target[, weight])</code></td>
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<td>Find shortest weighted paths and lengths from a given set of source nodes.</td>
</tr>
<tr>
<td><code>multi_source_dijkstra_path(G, sources[, target, ...])</code></td>
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<td><code>multi_source_dijkstra_path_length(G, sources)</code></td>
<td>Find shortest weighted path lengths in G from a given set of source nodes.</td>
</tr>
<tr>
<td><code>all_pairs_dijkstra(G[, cutoff, weight])</code></td>
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<td><code>all_pairs_dijkstra_path_length(G[, cutoff, ...])</code></td>
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<td>Returns the shortest path from source to target in a weighted graph G.</td>
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<td><code>bellman_ford_path_length(G, source, target)</code></td>
<td>Returns the shortest path length from source to target in a weighted graph.</td>
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<tr>
<td><code>single_source_bellman_ford(G, source[, ...])</code></td>
<td>Compute shortest paths and lengths in a weighted graph G.</td>
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<td><code>single_source_bellman_ford_path(G, source[, ...])</code></td>
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<td><code>single_source_bellman_ford_path_length(G, source)</code></td>
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<td><code>all_pairs_bellman_ford_path(G[, weight])</code></td>
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<tr>
<td><code>all_pairs_bellman_ford_path_length(G[, weight])</code></td>
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</tr>
<tr>
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<td><code>goldberg_radzik(G, source[, weight])</code></td>
<td>Compute shortest path lengths and predecessors on shortest paths in weighted graphs.</td>
</tr>
<tr>
<td><code>johnson(G[, weight])</code></td>
<td>Uses Johnson’s Algorithm to compute shortest paths.</td>
</tr>
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</table>
networkx.algorithms.shortest_paths.weighted.dijkstra_predecessor_and_distance

dijkstra_predecessor_and_distance (G, source, cutoff=None, weight='weight')

Compute weighted shortest path length and predecessors.

Uses Dijkstra’s Method to obtain the shortest weighted paths and return dictionaries of predecessors for each node and distance for each node from the source.

Parameters

- **G** *(NetworkX graph)*
- **source** *(node label)* – Starting node for path
- **cutoff** *(integer or float, optional)* – Depth to stop the search. Only return paths with length <= cutoff.
- **weight** *(string or function)* – If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining u to v will be G. edges[u, v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

  If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

Returns **pred**, **distance** – Returns two dictionaries representing a list of predecessors of a node and the distance to each node. Warning: If target is specified, the dicts are incomplete as they only contain information for the nodes along a path to target.

Return type **dictionaries**

Raises **NodeNotFound** – If source is not in G.

Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

The list of predecessors contains more than one element only when there are more than one shortest paths to the key node.

Examples

```python
>>> import networkx as nx
>>> G = nx.path_graph(5, create_using = nx.DiGraph())
>>> pred, dist = nx.dijkstra_predecessor_and_distance(G, 0)
>>> sorted(pred.items())
[(0, []), (1, [0]), (2, [1]), (3, [2]), (4, [3])]
>>> sorted(dist.items())
[(0, 0), (1, 1), (2, 2), (3, 3), (4, 4)]

>>> pred, dist = nx.dijkstra_predecessor_and_distance(G, 0, 1)
>>> sorted(pred.items())
[(0, []), (1, [0])]
>>> sorted(dist.items())
[(0, 0), (1, 1)]
```
networkx.algorithms.shortest_paths.weighted.dijkstra_path

dijkstra_path(G, source, target, weight='weight')

Returns the shortest weighted path from source to target in G.

Uses Dijkstra’s Method to compute the shortest weighted path between two nodes in a graph.

Parameters

• G (NetworkX graph)
• source (node) – Starting node
• target (node) – Ending node
• weight (string or function) – If this is a string, then edge weights will be accessed via
  the edge attribute with this key (that is, the weight of the edge joining u to v will be G.
  edges[u, v][weight]). If no such edge attribute exists, the weight of the edge is
  assumed to be one.
  If this is a function, the weight of an edge is the value returned by the function. The func-
  tion must accept exactly three positional arguments: the two endpoints of an edge and the
  dictionary of edge attributes for that edge. The function must return a number.

Returns path – List of nodes in a shortest path.

Return type list

Raises

• NodeNotFound – If source is not in G.
• NetworkXNoPath – If no path exists between source and target.

Examples

```python
>>> G=nx.path_graph(5)
>>> print(nx.dijkstra_path(G,0,4))
[0, 1, 2, 3, 4]
```

Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

The weight function can be used to hide edges by returning None. So weight = lambda u, v, d: 1
if d['color']=='red' else None will find the shortest red path.

The weight function can be used to include node weights.

```python
>>> def func(u, v, d):
...   node_u_wt = G.nodes[u].get('node_weight', 1)
...   node_v_wt = G.nodes[v].get('node_weight', 1)
...   edge_wt = d.get('weight', 1)
...   return node_u_wt/2 + node_v_wt/2 + edge_wt
```

In this example we take the average of start and end node weights of an edge and add it to the weight of the
edge.

The function single_source_dijkstra() computes both path and length-of-path if you need both, use that.
See also:

`bidirectional_dijkstra()`, `bellman_ford_path()`, `single_source_dijkstra()`

diastar_path_length \((G, \text{source}, \text{target}, \text{weight}='weight')\)

Returns the shortest weighted path length in \(G\) from source to target.

Uses Dijkstra’s Method to compute the shortest weighted path length between two nodes in a graph.

**Parameters**

- **\(G\)** *(NetworkX graph)*
- **source** *(node label)* – starting node for path
- **target** *(node label)* – ending node for path
- **weight** *(string or function)* – If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining \(u\) to \(v\) will be \(G.\)edges[\(u\), \(v\)][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one. If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

**Returns** **length** – Shortest path length.

**Return type** number

**Raises**

- **NodeNotFound** – If source is not in \(G\).
- **NetworkXNoPath** – If no path exists between source and target.

**Examples**

```python
>>> G=nx.path_graph(5)
>>> print(nx.dijkstra_path_length(G,0,4))
4
```

**Notes**

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

The weight function can be used to hide edges by returning None. So `weight = lambda u, v, d: 1 if d['color']="red" else None` will find the shortest red path.

The function `single_source_dijkstra()` computes both path and length-of-path if you need both, use that.

**See also:**

`bidirectional_dijkstra()`, `bellman_ford_path_length()`, `single_source_dijkstra()`
single_source_dijkstra (G, source=None, target=None, cutoff=None, weight='weight')

Find shortest weighted paths and lengths from a source node.

Compute the shortest path length between source and all other reachable nodes for a weighted graph.

Uses Dijkstra’s algorithm to compute shortest paths and lengths between a source and all other reachable nodes in a weighted graph.

Parameters

- G (NetworkX graph)
- source (node label) – Starting node for path
- target (node label, optional) – Ending node for path
- cutoff (integer or float, optional) – Depth to stop the search. Only return paths with length <= cutoff.
- weight (string or function) – If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining u to v will be G.edges[u, v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

Returns distance, path – If target is None, paths and lengths to all nodes are computed. The return value is a tuple of two dictionaries keyed by target nodes. The first dictionary stores distance to each target node. The second stores the path to each target node. If target is not None, returns a tuple (distance, path), where distance is the distance from source to target and path is a list representing the path from source to target.

Return type pair of dictionaries, or numeric and list.

Raises  NodeNotFound – If source is not in G.

Examples

```python
>>> G = nx.path_graph(5)
>>> length, path = nx.single_source_dijkstra(G, 0)
>>> print(length[4])
4
>>> for node in [0, 1, 2, 3, 4]:
...    print('{}: {}'.format(node, length[node]))
0: 0
1: 1
2: 2
3: 3
4: 4
>>> path[4]  # [0, 1, 2, 3, 4]
[0, 1, 2, 3, 4]
>>> length, path = nx.single_source_dijkstra(G, 0, 1)
>>> length
1
>>> path
[0, 1]
```
Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

The weight function can be used to hide edges by returning None. So weight = lambda u, v, d: 1 if d['color']=='red' else None will find the shortest red path.

Based on the Python cookbook recipe (119466) at http://aspn.activestate.com/ASPN/Cookbook/Python/Recipe/119466

This algorithm is not guaranteed to work if edge weights are negative or are floating point numbers (overflows and roundoff errors can cause problems).

See also:

single_source_dijkstra_path(), single_source_dijkstra_path_length(), single_source_bellman_ford()

networkx.algorithms.shortest_paths.weighted.single_source_dijkstra_path

double_source_dijkstra_path (G, source, cutoff=None, weight='weight')

Find shortest weighted paths in G from a source node.

Parameters

• G (NetworkX graph)

• source (node) – Starting node for path.

• cutoff (integer or float, optional) – Depth to stop the search. Only return paths with length <= cutoff.

• weight (string or function) – If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining u to v will be G.edges[u, v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

   If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

Returns paths – Dictionary of shortest path lengths keyed by target.

Return type dictionary

Raises NodeNotFound – If source is not in G.

Examples

```python
>>> G=nx.path_graph(5)
>>> path=nx.single_source_dijkstra_path(G,0)
>>> path[4]
[0, 1, 2, 3, 4]
```
Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.
The weight function can be used to hide edges by returning None. So weight = lambda u, v, d: 1
if d['color']=="red" else None will find the shortest red path.

See also:

single_source_dijkstra(), single_source_bellman_ford()

networkx.algorithms.shortest_paths.weighted.single_source_dijkstra_path_length

single_source_dijkstra_path_length (G, source, cutoff=None, weight='weight')
Find shortest weighted path lengths in G from a source node.

Parameters

• G (NetworkX graph)
• source (node label) – Starting node for path
• cutoff (integer or float, optional) – Depth to stop the search. Only return paths with length <= cutoff.
• weight (string or function) – If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining u to v will be G. edges[u, v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

Returns length – Dict keyed by node to shortest path length from source.

Return type dict

Raises NodeNotFound – If source is not in G.

Examples

```python
>>> G = nx.path_graph(5)
>>> length = nx.single_source_dijkstra_path_length(G, 0)
>>> length[4]
4
>>> for node in [0, 1, 2, 3, 4]:
...     print('{}: {}'.format(node, length[node]))
0: 0
1: 1
2: 2
3: 3
4: 4
```
Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.
The weight function can be used to hide edges by returning None. So weight = lambda u, v, d: 1
if d['color'] == "red" else None will find the shortest red path.
See also:

single_source_dijkstra(), single_source_bellman_ford_path_length()

networkx.algorithms.shortest_paths.weighted.multi_source_dijkstra

multi_source_dijkstra (G, sources, target=None, cutoff=None, weight='weight')
Find shortest weighted paths and lengths from a given set of source nodes.
Uses Dijkstra's algorithm to compute the shortest paths and lengths between one of the source nodes and the
given target, or all other reachable nodes if not specified, for a weighted graph.

Parameters

• G (NetworkX graph)
• sources (non-empty set of nodes) – Starting nodes for paths. If this is just a set containing a
single node, then all paths computed by this function will start from that node. If there are
two or more nodes in the set, the computed paths may begin from any one of the start nodes.
• target (node label, optional) – Ending node for path
• cutoff (integer label, optional) – Depth to stop the search. Only return paths with length
<= cutoff.
• weight (string or function) – If this is a string, then edge weights will be accessed via
the edge attribute with this key (that is, the weight of the edge joining u to v will be G.
edges[u, v][weight]). If no such edge attribute exists, the weight of the edge is
assumed to be one.
If this is a function, the weight of an edge is the value returned by the function. The func-
tion must accept exactly three positional arguments: the two endpoints of an edge and the
dictionary of edge attributes for that edge. The function must return a number.

Returns distance, path – If target is None, returns a tuple of two dictionaries keyed by node. The
first dictionary stores distance from one of the source nodes. The second stores the path from
one of the sources to that node. If target is not None, returns a tuple of (distance, path) where
distance is the distance from source to target and path is a list representing the path from source
to target.

Return type pair of dictionaries, or numeric and list

Examples

```python
>>> G = nx.path_graph(5)
>>> length, path = nx.multi_source_dijkstra(G, {0, 4})
>>> for node in [0, 1, 2, 3, 4]:
...     print('{}: {}'.format(node, length[node]))
0: 0
1: 1
2: 2
```
(continues on next page)
3: 1
4: 0
>>> path[1]
[0, 1]
>>> path[3]
[4, 3]

>>> length, path = nx.multi_source_dijkstra(G, {0, 4}, 1)
>>> length
1
>>> path
[0, 1]

Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

The weight function can be used to hide edges by returning None. So weight = lambda u, v, d: 1 if d['color']="red" else None will find the shortest red path.

Based on the Python cookbook recipe (119466) at http://aspn.activestate.com/ASPN/Cookbook/Python/Recipe/119466

This algorithm is not guaranteed to work if edge weights are negative or are floating point numbers (overflows and roundoff errors can cause problems).

Raises

• ValueError – If sources is empty.
• NodeNotFound – If any of sources is not in G.

See also:

multi_source_dijkstra_path(), multi_source_dijkstra_path_length()
If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

**Returns** paths – Dictionary of shortest paths keyed by target.

**Return type** dictionary

**Examples**

```python
>>> G = nx.path_graph(5)
>>> path = nx.multi_source_dijkstra_path(G, {0, 4})
>>> path[1]
[0, 1]
>>> path[3]
[4, 3]
```

**Notes**

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

The weight function can be used to hide edges by returning None. So weight = lambda u, v, d: 1 if d['color']=="red" else None will find the shortest red path.

**Raises**

- **ValueError** – If sources is empty.
- **NodeNotFound** – If any of sources is not in G.

**See also:**

`multi_source_dijkstra()`, `multi_source_bellman_ford()`

**networkx.algorithms.shortest_paths.weighted.multi_source_dijkstra_path_length**

Find shortest weighted path lengths in G from a given set of source nodes.

Compute the shortest path length between any of the source nodes and all other reachable nodes for a weighted graph.

**Parameters**

- **G** *(NetworkX graph)*
- **sources** *(non-empty set of nodes)* – Starting nodes for paths. If this is just a set containing a single node, then all paths computed by this function will start from that node. If there are two or more nodes in the set, the computed paths may begin from any one of the start nodes.
- **cutoff** *(integer or float, optional)* – Depth to stop the search. Only return paths with length <= cutoff.
- **weight** *(string or function)* – If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining u to v will be G.edges[u, v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.
If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

**Returns** length  – Dict keyed by node to shortest path length to nearest source.

**Return type**  dict

### Examples

```python
>>> G = nx.path_graph(5)
>>> length = nx.multi_source_dijkstra_path_length(G, {0, 4})
>>> for node in [0, 1, 2, 3, 4]:
...     print('{}: {}'.format(node, length[node]))
0: 0
1: 1
2: 2
3: 1
4: 0
```

### Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

The weight function can be used to hide edges by returning None. So `weight = lambda u, v, d: 1 if d['color']=='red' else None` will find the shortest red path.

**Raises**

- **ValueError** – If `sources` is empty.
- **NodeNotFound** – If any of `sources` is not in `G`.

**See also:**

`multi_source_dijkstra()`

### networkx.algorithms.shortest_paths.weighted.all_pairs_dijkstra

**all_pairs_dijkstra** *(G, cutoff=None, weight='weight')*

Find shortest weighted paths and lengths between all nodes.

**Parameters**

- **G** *(NetworkX graph)*
- **cutoff** *(integer or float, optional)* – Depth to stop the search. Only return paths with length <= `cutoff`.
- **weight** *(string or function)* – If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining u to v will be `G.edge[u][v][weight]`). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.
Yields (node, (distance, path)) ((node obj, (dict, dict)) – Each source node has two associated
dicts. The first holds distance keyed by target and the second holds paths keyed by target.
(See single_source_dijkstra for the source/target node terminology.) If desired you can apply
dict() to this function to create a dict keyed by source node to the two dicts.

Examples

```python
>>> G = nx.path_graph(5)
>>> len_path = dict(nx.all_pairs_dijkstra(G))
>>> print(len_path[3][0][1])
2
>>> for node in [0, 1, 2, 3, 4]:
...     print('3 - {}: {}'.format(node, len_path[3][0][node]))
3 - 0: 3
3 - 1: 2
3 - 2: 1
3 - 3: 0
3 - 4: 1
>>> len_path[3][1][1]
[3, 2, 1]
>>> for n, (dist, path) in nx.all_pairs_dijkstra(G):
...     print(path[1])
[0, 1]
[1]
[2, 1]
[3, 2, 1]
[4, 3, 2, 1]
```

Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.
The yielded dicts only have keys for reachable nodes.

networkx.algorithms.shortest_paths.weighted.all_pairs_dijkstra_path

all_pairs_dijkstra_path (G, cutoff=None, weight='weight')
Compute shortest paths between all nodes in a weighted graph.

Parameters

- G (NetworkX graph)
- cutoff (integer or float, optional) – Depth to stop the search. Only return paths with length <= cutoff.
- weight (string or function) – If this is a string, then edge weights will be accessed via
  the edge attribute with this key (that is, the weight of the edge joining u to v will be G.
  edges[u, v][weight]). If no such edge attribute exists, the weight of the edge is
  assumed to be one.
  If this is a function, the weight of an edge is the value returned by the function. The func-
  tion must accept exactly three positional arguments: the two endpoints of an edge and the
  dictionary of edge attributes for that edge. The function must return a number.

Returns distance – Dictionary, keyed by source and target, of shortest paths.
Return type dictionary

Examples

```python
>>> G = nx.path_graph(5)
>>> path = dict(nx.all_pairs_dijkstra_path(G))
>>> print(path[0][4])
[0, 1, 2, 3, 4]
```

Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

See also:

floyd_warshall(), all_pairs_bellman_ford_path()

networkx.algorithms.shortest_paths.weighted.all_pairs_dijkstra_path_length

all_pairs_dijkstra_path_length(G, cutoff=None, weight='weight')

Compute shortest path lengths between all nodes in a weighted graph.

Parameters

- **G** (*NetworkX graph*)
- **cutoff (integer or float, optional)** – Depth to stop the search. Only return paths with length <= cutoff.
- **weight (string or function)** – If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining u to v will be G.edges[u, v][weight]). If no such edge attribute exists, the weight of the edge is assumed to be one. If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

Returns **distance** – (source, dictionary) iterator with dictionary keyed by target and shortest path length as the key value.

Return type iterator

Examples

```python
>>> G = nx.path_graph(5)
>>> length = dict(nx.all_pairs_dijkstra_path_length(G))
>>> for node in [0, 1, 2, 3, 4]:
...     print ('1 - {}: {}'.format (node, length[1][node]))
1 - 0: 1
1 - 1: 0
1 - 2: 1
1 - 3: 2
1 - 4: 3
>>> length[3][2]
```

(continues on next page)
Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed. The dictionary returned only has keys for reachable node pairs.

networkx.algorithms.shortest_paths.weighted.bidirectional_dijkstra

bidirectional_dijkstra \((G, \text{source}, \text{target}, \text{weight}='weight')\)

Dijkstra’s algorithm for shortest paths using bidirectional search.

Parameters

- \(G\) (NetworkX graph)
- \(\text{source}\) (node) – Starting node.
- \(\text{target}\) (node) – Ending node.
- \(\text{weight}\) (string or function) – If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining \(u\) to \(v\) will be \(G.\)edges\([u, v]\)[weight]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

Returns \(\text{length, path}\) – length is the distance from source to target. path is a list of nodes on a path from source to target.

Return type number and list

Raises

- \(\text{NodeNotFound}\) – If either \(\text{source}\) or \(\text{target}\) is not in \(G\).
- \(\text{NetworkXNoPath}\) – If no path exists between source and target.

Examples

```python
>>> G = nx.path_graph(5)
>>> length, path = nx.bidirectional_dijkstra(G, 0, 4)
>>> print(length)
4
>>> print(path)
[0, 1, 2, 3, 4]
```
Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

In practice bidirectional Dijkstra is much more than twice as fast as ordinary Dijkstra.

Ordinary Dijkstra expands nodes in a sphere-like manner from the source. The radius of this sphere will eventually be the length of the shortest path. Bidirectional Dijkstra will expand nodes from both the source and the target, making two spheres of half this radius. Volume of the first sphere is $\pi r^2$ while the others are $2\pi r^2/2^2$, making up half the volume.

This algorithm is not guaranteed to work if edge weights are negative or are floating point numbers (overflows and roundoff errors can cause problems).

See also:
shortest_path(), shortest_path_length()

networkx.algorithms.shortest_paths.weighted.bellman_ford_path

bellman_ford_path(G, source, target, weight='weight')

Returns the shortest path from source to target in a weighted graph G.

Parameters
- G (NetworkX graph)
- source (node) – Starting node
- target (node) – Ending node
- weight (string, optional (default='weight')) – Edge data key corresponding to the edge weight

Returns path – List of nodes in a shortest path.

Return type list

Raises
- NodeNotFound – If source is not in G.
- NetworkXNoPath – If no path exists between source and target.

Examples

```python
>>> G=nx.path_graph(5)
>>> print(nx.bellman_ford_path(G, 0, 4))
[0, 1, 2, 3, 4]
```

Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

See also:
dijkstra_path(), bellman_ford_path_length()
networkx.algorithms.shortest_paths.weighted.bellman_ford_path_length

bellman_ford_path_length \((G, \text{source}, \text{target}, \text{weight}=\text{weight})\)
Returns the shortest path length from source to target in a weighted graph.

Parameters

- \(G\) (NetworkX graph)
- source (node label) – starting node for path
- target (node label) – ending node for path
- weight (string, optional (default=\text{weight})) – Edge data key corresponding to the edge weight

Returns length – Shortest path length.

Return type number

Raises

- NodeNotFound – If source is not in \(G\).
- NetworkXNoPath – If no path exists between source and target.

Examples

```python
>>> G=nx.path_graph(5)
>>> print(nx.bellman_ford_path_length(G,0,4))
4
```

Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

See also:
dijkstra_path_length(), bellman_ford_path()

networkx.algorithms.shortest_paths.weighted.single_source_bellman_ford

single_source_bellman_ford \((G, \text{source}, \text{target}=\text{None}, \text{weight}=\text{weight})\)
Compute shortest paths and lengths in a weighted graph \(G\).

Uses Bellman-Ford algorithm for shortest paths.

Parameters

- \(G\) (NetworkX graph)
- source (node label) – Starting node for path
- target (node label, optional) – Ending node for path

Returns distance, path – If target is None, returns a tuple of two dictionaries keyed by node. The first dictionary stores distance from one of the source nodes. The second stores the path from one of the sources to that node. If target is not None, returns a tuple of (distance, path) where distance is the distance from source to target and path is a list representing the path from source to target.

3.48. Shortest Paths
Return type  pair of dictionaries, or numeric and list

Raises  

NodeNotFound – If source is not in G.

Examples

```python
g = nx.path_graph(5)
length, path = nx.single_source_bellman_ford(g, 0)
print(length[4])
>>> for node in [0, 1, 2, 3, 4]:
...    print('{}: {}'.format(node, length[node]))
0: 0
1: 1
2: 2
3: 3
4: 4
>>> path[4]
[0, 1, 2, 3, 4]
>>> length, path = nx.single_source_bellman_ford(g, 0, 1)
>>> length
1
>>> path
[0, 1]
```

Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

See also:

single_source_dijkstra(), single_source_bellman_ford_path(),
single_source_bellman_ford_path_length()

networkx.algorithms.shortest_paths.weighted.single_source_bellman_ford_path

single_source_bellman_ford_path (G, source, weight='weight')

Compute shortest path between source and all other reachable nodes for a weighted graph.

Parameters

- G (NetworkX graph)
- source (node) – Starting node for path.
- weight (string, optional (default='weight')) – Edge data key corresponding to the edge weight

Returns paths – Dictionary of shortest path lengths keyed by target.

Return type  dictionary

Raises  

NodeNotFound – If source is not in G.
Examples

```python
>>> G=nx.path_graph(5)
>>> path=nx.single_source_bellman_ford_path(G,0)
>>> path[4]
[0, 1, 2, 3, 4]
```

Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

See also:

- `single_source_dijkstra()`, `single_source_bellman_ford()`

networkx.algorithms.shortest_paths.weighted.single_source_bellman_ford_path_length

`single_source_bellman_ford_path_length(G, source, weight='weight')`

Compute the shortest path length between source and all other reachable nodes for a weighted graph.

Parameters

- `G` *(NetworkX graph)*
- `source` *(node label)* – Starting node for path
- `weight` *(string, optional (default='weight'))* – Edge data key corresponding to the edge weight.

Returns `length` – (target, shortest path length) iterator

Return type iterator

Raises `NodeNotFound` – If `source` is not in `G`.

Examples

```python
>>> G = nx.path_graph(5)
>>> length = dict(nx.single_source_bellman_ford_path_length(G, 0))
>>> length[4]
4
>>> for node in [0, 1, 2, 3, 4]:
...     print('{}: {}'.format(node, length[node]))
0: 0
1: 1
2: 2
3: 3
4: 4
```

Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

See also:

- `single_source_dijkstra()`, `single_source_bellman_ford()`
networkx.algorithms.shortest_paths.weighted.all_pairs_bellman_ford_path

all_pairs_bellman_ford_path\( (G, \text{weight}='\text{weight}') \)

Compute shortest paths between all nodes in a weighted graph.

**Parameters**

- **G** *(NetworkX graph)*
- **weight** *(string, optional (default='weight'))* – Edge data key corresponding to the edge weight

**Returns**

- **distance** – Dictionary, keyed by source and target, of shortest paths.

**Return type**

dictionary

**Examples**

```python
>>> G = nx.path_graph(5)
>>> path = dict(nx.all_pairs_bellman_ford_path(G))
>>> print(path[0][4])
[0, 1, 2, 3, 4]
```

**Notes**

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

**See also:**
floyd_warshall(), all_pairs_dijkstra_path()

networkx.algorithms.shortest_paths.weighted.all_pairs_bellman_ford_path_length

all_pairs_bellman_ford_path_length\( (G, \text{weight}='\text{weight}') \)

Compute shortest path lengths between all nodes in a weighted graph.

**Parameters**

- **G** *(NetworkX graph)*
- **weight** *(string, optional (default='weight'))* – Edge data key corresponding to the edge weight

**Returns**

- **distance** – (source, dictionary) iterator with dictionary keyed by target and shortest path length as the key value.

**Return type**

iterator

**Examples**

```python
>>> G = nx.path_graph(5)
>>> length = dict(nx.all_pairs_bellman_ford_path_length(G))
>>> for node in [0, 1, 2, 3, 4]:
...     print('1 - {}: {}'.format(node, length[1][node]))
1 - 0: 1
1 - 1: 0
```

(continues on next page)
1 - 2: 1
1 - 3: 2
1 - 4: 3

>>> length[3][2]
1
>>> length[2][2]
0

Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.
The dictionary returned only has keys for reachable node pairs.

networkx.algorithms.shortest_paths.weighted.bellman_ford_predecessor_and_distance

bellman_ford_predecessor_and_distance \((G, \text{source}, \text{target}=None, \text{weight}='weight')\)
Compute shortest path lengths and predecessors on shortest paths in weighted graphs.
The algorithm has a running time of \(O(mn)\) where \(n\) is the number of nodes and \(m\) is the number of edges. It is slower than Dijkstra but can handle negative edge weights.

Parameters

• \(G\) (NetworkX graph) – The algorithm works for all types of graphs, including directed graphs and multigraphs.
• \(\text{source}\) (node label) – Starting node for path
• \(\text{weight}\) (string or function) – If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining \(u\) to \(v\) will be \(G.\) edges[\(u, v\)][weight\]). If no such edge attribute exists, the weight of the edge is assumed to be one.

If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

Returns \(\text{pred}, \text{dist}\) – Returns two dictionaries keyed by node to predecessor in the path and to the distance from the source respectively.

Return type  dictionaries

Raises

• \(\text{NodeNotFound}\) – If \(\text{source}\) is not in \(G\).
• \(\text{NetworkXUnbounded}\) – If the (di)graph contains a negative cost (di)cycle, the algorithm raises an exception to indicate the presence of the negative cost (di)cycle. Note: any negative weight edge in an undirected graph is a negative cost cycle.

Examples
```python
>>> import networkx as nx
>>> G = nx.path_graph(5, create_using = nx.DiGraph())
>>> pred, dist = nx.bellman_ford_predecessor_and_distance(G, 0)
>>> sorted(pred.items())
[(0, []), (1, [0]), (2, [1]), (3, [2]), (4, [3])]
>>> sorted(dist.items())
[(0, 0), (1, 1), (2, 2), (3, 3), (4, 4)]

```
Examples

```python
>>> import networkx as nx
>>> G = nx.cycle_graph(5, create_using = nx.DiGraph())
>>> print(nx.negative_edge_cycle(G))
False
>>> G[1][2]['weight'] = -7
>>> print(nx.negative_edge_cycle(G))
True
```

Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

This algorithm uses bellman_ford_predecessor_and_distance() but finds negative cycles on any component by first adding a new node connected to every node, and starting bellman_ford_predecessor_and_distance on that node. It then removes that extra node.

`networkx.algorithms.shortest_paths.weighted.goldberg_radzik`

goldberg_radzik(G, source, weight='weight')

Compute shortest path lengths and predecessors on shortest paths in weighted graphs.

The algorithm has a running time of \(O(mn)\) where \(n\) is the number of nodes and \(m\) is the number of edges. It is slower than Dijkstra but can handle negative edge weights.

**Parameters**

- **G** (*NetworkX graph*) – The algorithm works for all types of graphs, including directed graphs and multigraphs.
- **source** (*node label*) – Starting node for path
- **weight** (*string or function*) – If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining \(u\) to \(v\) will be \(G\).edges[\(u, v\)][weight]\)). If no such edge attribute exists, the weight of the edge is assumed to be one.
  
  If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

**Returns** **pred, dist** – Returns two dictionaries keyed by node to predecessor in the path and to the distance from the source respectively.

**Return type** dictionaries

**Raises**

- **NodeNotFound** – If **source** is not in **G**.
- **NetworkXUnbounded** – If the (di)graph contains a negative cost (di)cycle, the algorithm raises an exception to indicate the presence of the negative cost (di)cycle. Note: any negative weight edge in an undirected graph is a negative cost cycle.
### Examples

```python
>>> import networkx as nx

>>> G = nx.path_graph(5, create_using = nx.DiGraph())
>>> pred, dist = nx.goldberg_radzik(G, 0)
>>> sorted(pred.items())
[(0, None), (1, 0), (2, 1), (3, 2), (4, 3)]

>>> sorted(dist.items())
[(0, 0), (1, 1), (2, 2), (3, 3), (4, 4)]

>>> G = nx.cycle_graph(5, create_using = nx.DiGraph())

>>> G[1][2]['weight'] = -7

>>> nx.goldberg_radzik(G, 0)
Traceback (most recent call last):
...
networkx.exception.NetworkXUnbounded: Negative cost cycle detected.
```

### Notes

Edge weight attributes must be numerical. Distances are calculated as sums of weighted edges traversed.

The dictionaries returned only have keys for nodes reachable from the source.

In the case where the (di)graph is not connected, if a component not containing the source contains a negative cost (di)cycle, it will not be detected.

### networkx.algorithms.shortest_paths.weighted.johnson

**johnson** *(G, weight='weight')*

Uses Johnson’s Algorithm to compute shortest paths.

Johnson’s Algorithm finds a shortest path between each pair of nodes in a weighted graph even if negative weights are present.

**Parameters**

- **G** *(NetworkX graph)*
  - **weight** *(string or function)* – If this is a string, then edge weights will be accessed via the edge attribute with this key (that is, the weight of the edge joining u to v will be `G.edges[u, v][weight]`). If no such edge attribute exists, the weight of the edge is assumed to be one.

  If this is a function, the weight of an edge is the value returned by the function. The function must accept exactly three positional arguments: the two endpoints of an edge and the dictionary of edge attributes for that edge. The function must return a number.

**Returns**

- **distance** – Dictionary, keyed by source and target, of shortest paths.

**Return type**

- **dictionary**

**Raises**

- **NetworkXError** – If given graph is not weighted.

### Examples
```python
>>> import networkx as nx
>>> graph = nx.DiGraph()
>>> graph.add_weighted_edges_from([('0', '3', 3), ('0', '1', -5),
... ('0', '2', 2), ('1', '2', 4), ('2', '3', 1)])
>>> paths = nx.johnson(graph, weight='weight')
>>> paths['0']['2']
['0', '1', '2']
```

Notes

Johnson’s algorithm is suitable even for graphs with negative weights. It works by using the Bellman–Ford algorithm to compute a transformation of the input graph that removes all negative weights, allowing Dijkstra’s algorithm to be used on the transformed graph.

The time complexity of this algorithm is $O(n^2 \log n + nm)$, where $n$ is the number of nodes and $m$ the number of edges in the graph. For dense graphs, this may be faster than the Floyd–Warshall algorithm.

See also:

- floyd_warshall_predecessor_and_distance()
- floyd_warshall_numpy()
- all_pairs_shortest_path()
- all_pairs_shortest_path_length()
- all_pairs_dijkstra_path()
- bellman_ford_predecessor_and_distance()
- all_pairs_bellman_ford_path()
- all_pairs_bellman_ford_path_length()

### 3.48.7 Dense Graphs

Floyd-Warshall algorithm for shortest paths.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
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<tr>
<td>floyd_warshall(G[, weight])</td>
<td>Find all-pairs shortest path lengths using Floyd’s algorithm.</td>
</tr>
<tr>
<td>floyd_warshall_predecessor_and_distance([G[, weight]])</td>
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<td>Floyd_warshall_numpy(G[, nodelist, weight])</td>
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<td>reconstruct_path(source, target, predecessors)</td>
<td>Reconstruct a path from source to target using the predecessors dict as returned by floyd_warshall_predecessor_and_distance</td>
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</table>

networkx.algorithms.shortest_paths.dense.floyd_warshall

floyd_warshall (G, weight='weight')

Find all-pairs shortest path lengths using Floyd’s algorithm.

Parameters

- G (NetworkX graph)
- weight (string, optional (default= ‘weight’)) – Edge data key corresponding to the edge weight.

Returns distance – A dictionary, keyed by source and target, of shortest paths distances between nodes.

Return type dict
Notes

Floyd’s algorithm is appropriate for finding shortest paths in dense graphs or graphs with negative weights when Dijkstra’s algorithm fails. This algorithm can still fail if there are negative cycles. It has running time $O(n^3)$ with running space of $O(n^2)$.

See also:

`floyd_warshall_predecessor_and_distance()`, `floyd_warshall_numpy()`,
`all_pairs_shortest_path()`, `all_pairs_shortest_path_length()`

networkx.algorithms.shortest_paths.dense.floyd_warshall_predecessor_and_distance

`floyd_warshall_predecessor_and_distance(G, weight='weight')`
Find all-pairs shortest path lengths using Floyd’s algorithm.

Parameters

- `G` (NetworkX graph)
- `weight` (string, optional (default= `weight`) – Edge data key corresponding to the edge weight.

Returns `predecessor`, `distance` – Dictionaries, keyed by source and target, of predecessors and distances in the shortest path.

Return type dictionaries

Examples

```python
>>> G = nx.DiGraph()
>>> G.add_weighted_edges_from([('s', 'u', 10), ('s', 'x', 5),
...     ('u', 'v', 1), ('u', 'x', 2), ('v', 'y', 1), ('x', 'u', 3),
...     ('x', 'v', 5), ('x', 'y', 2), ('y', 's', 7), ('y', 'v', 6)])
>>> predecessors, _ = nx.floyd_warshall_predecessor_and_distance(G)
>>> print(nx.reconstruct_path('s', 'v', predecessors))
['s', 'x', 'u', 'v']
```

Notes

Floyd’s algorithm is appropriate for finding shortest paths in dense graphs or graphs with negative weights when Dijkstra’s algorithm fails. This algorithm can still fail if there are negative cycles. It has running time $O(n^3)$ with running space of $O(n^2)$.

See also:

`floyd_warshall()`, `floyd_warshall_numpy()`,
`all_pairs_shortest_path()`, `all_pairs_shortest_path_length()`

networkx.algorithms.shortest_paths.dense.floyd_warshall_numpy

`floyd_warshall_numpy(G, nodelist=None, weight='weight')`
Find all-pairs shortest path lengths using Floyd’s algorithm.

Parameters
• \texttt{G} (NetworkX graph)
• \texttt{nodelist} (list, optional) – The rows and columns are ordered by the nodes in nodelist. If nodelist is None then the ordering is produced by \texttt{G.nodes()}. 
• \texttt{weight} (string, optional (default= ’weight’) – Edge data key corresponding to the edge weight.

Returns distance – A matrix of shortest path distances between nodes. If there is no path between to nodes the corresponding matrix entry will be \texttt{Inf}.

Return type NumPy matrix

Notes

Floyd’s algorithm is appropriate for finding shortest paths in dense graphs or graphs with negative weights when Dijkstra’s algorithm fails. This algorithm can still fail if there are negative cycles. It has running time \(O(n^3)\) with running space of \(O(n^2)\).

\texttt{networkx.algorithms.shortest_paths.dense.reconstruct_path}

\texttt{reconstruct_path(source, target, predecessors)}
Reconstruct a path from source to target using the predecessors dict as returned by \texttt{floyd_warshall_predecessor_and_distance}

Parameters

• \texttt{source} (node) – Starting node for path
• \texttt{target} (node) – Ending node for path
• \texttt{predecessors} (dictionary) – Dictionary, keyed by source and target, of predecessors in the shortest path, as returned by \texttt{floyd_warshall_predecessor_and_distance}

Returns

\texttt{path} – A list of nodes containing the shortest path from source to target

If source and target are the same, an empty list is returned

Return type list

Notes

This function is meant to give more applicability to the \texttt{floyd_warshall_predecessor_and_distance} function

See also:
\texttt{floyd_warshall_predecessor_and_distance()}

### 3.48.8 A* Algorithm

Shortest paths and path lengths using the A* (“A star”) algorithm.

\texttt{astar_path(G, source, target[, heuristic, ...])} Returns a list of nodes in a shortest path between source and target using the A* (“A-star”) algorithm.

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<td><code>astar_path_length</code></td>
<td>Returns the length of the shortest path between source and target using the A* (“A-star”) algorithm.</td>
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</table>

`networkx.algorithms.shortest_paths.astar.astar_path`

`astar_path` \((G, \text{source}, \text{target}, \text{heuristic}=\text{None}, \text{weight}='\text{weight}')\)

Returns a list of nodes in a shortest path between source and target using the A* (“A-star”) algorithm.

There may be more than one shortest path. This returns only one.

**Parameters**

- `G` (*NetworkX graph*)
- `source` (*node*) – Starting node for path
- `target` (*node*) – Ending node for path
- `heuristic` (*function*) – A function to evaluate the estimate of the distance from the a node to the target. The function takes two nodes arguments and must return a number.
- `weight` (*string, optional (default='weight')*) – Edge data key corresponding to the edge weight.

**Raises** `NetworkXNoPath` – If no path exists between source and target.

**Examples**

```python
>>> G = nx.path_graph(5)
>>> print(nx.astar_path(G, 0, 4))
[0, 1, 2, 3, 4]
>>> G = nx.grid_graph(dim=[3, 3])  # nodes are two-tuples (x,y)
>>> nx.set_edge_attributes(G, {e: e[1][0]*2 for e in G.edges()}, 'cost')
>>> def dist(a, b):
...     (x1, y1) = a
...     (x2, y2) = b
...     return ((x1 - x2) ** 2 + (y1 - y2) ** 2) ** 0.5
>>> print(nx.astar_path(G, (0, 0), (2, 2), heuristic=dist, weight='cost'))
[(0, 0), (0, 1), (0, 2), (1, 2), (2, 2)]
```

**See also:**

`shortest_path()`, `dijkstra_path()`

`networkx.algorithms.shortest_paths.astar.astar_path_length`

`astar_path_length` \((G, \text{source}, \text{target}, \text{heuristic}=\text{None}, \text{weight}='\text{weight}')\)

Returns the length of the shortest path between source and target using the A* (“A-star”) algorithm.

**Parameters**

- `G` (*NetworkX graph*)
- `source` (*node*) – Starting node for path
- `target` (*node*) – Ending node for path
• **heuristic** *(function)* – A function to evaluate the estimate of the distance from the a node to the target. The function takes two nodes arguments and must return a number.

**Raises** *NetworkXNoPath* – If no path exists between source and target.

See also:  
astar_path()

### 3.49 Similarity Measures

Functions measuring similarity using graph edit distance.

The graph edit distance is the number of edge/node changes needed to make two graphs isomorphic.

The default algorithm/implementation is sub-optimal for some graphs. The problem of finding the exact Graph Edit Distance (GED) is NP-hard so it is often slow. If the simple interface `graph_edit_distance` takes too long for your graph, try `optimize_graph_edit_distance` and/or `optimize_edit_paths`.

At the same time, I encourage capable people to investigate alternative GED algorithms, in order to improve the choices available.

<table>
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<tr>
<th>Function</th>
<th>Description</th>
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<tr>
<td><code>graph_edit_distance(G1, G2[, node_match, ...])</code></td>
<td>Returns GED (graph edit distance) between graphs G1 and G2.</td>
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<tr>
<td><code>optimal_edit_paths(G1, G2[, node_match, ...])</code></td>
<td>Returns all minimum-cost edit paths transforming G1 to G2.</td>
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<td><code>optimize_graph_edit_distance(G1, G2[, ...])</code></td>
<td>Returns consecutive approximations of GED (graph edit distance) between G1 and G2.</td>
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<tr>
<td><code>optimize_edit_paths(G1, G2[, node_match, ...])</code></td>
<td>GED (graph edit distance) calculation: advanced interface.</td>
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<tr>
<td><code>simrank_similarity(G[, source, target, ...])</code></td>
<td>Returns the SimRank similarity of nodes in the graph G.</td>
</tr>
<tr>
<td><code>simrank_similarity_numpy(G[, source, ...])</code></td>
<td>Calculate SimRank of nodes in G using matrices with numpy.</td>
</tr>
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</table>

#### 3.49.1 networkx.algorithms.similarity.graph_edit_distance

**graph_edit_distance** *(G1, G2, node_match=None, edge_match=None, node_subst_cost=None, node_del_cost=None, node_ins_cost=None, edge_subst_cost=None, edge_del_cost=None, edge_ins_cost=None, upper_bound=None)*

Returns GED (graph edit distance) between graphs G1 and G2.

Graph edit distance is a graph similarity measure analogous to Levenshtein distance for strings. It is defined as minimum cost of edit path (sequence of node and edge edit operations) transforming graph G1 to graph isomorphic to G2.

**Parameters**

- **G1, G2** *(graphs)* – The two graphs G1 and G2 must be of the same type.

- **node_match** *(callable)* – A function that returns True if node n1 in G1 and n2 in G2 should be considered equal during matching.

  The function will be called like

  `node_match(G1.nodes[n1], G2.nodes[n2])`.

  That is, the function will receive the node attribute dictionaries for n1 and n2 as inputs.
Ignored if node_subst_cost is specified. If neither node_match nor node_subst_cost are specified then node attributes are not considered.

• edge_match (callable) – A function that returns True if the edge attribute dictionaries for the pair of nodes (u1, v1) in G1 and (u2, v2) in G2 should be considered equal during matching.

The function will be called like

    edge_match(G1[u1][v1], G2[u2][v2]).

That is, the function will receive the edge attribute dictionaries of the edges under consideration.

Ignored if edge_subst_cost is specified. If neither edge_match nor edge_subst_cost are specified then edge attributes are not considered.

• node_subst_cost, node_del_cost, node_ins_cost (callable) – Functions that return the costs of node substitution, node deletion, and node insertion, respectively.

The functions will be called like

    node_subst_cost(G1.nodes[n1], G2.nodes[n2]), node_del_cost(G1.nodes[n1]), node_ins_cost(G2.nodes[n2]).

That is, the functions will receive the node attribute dictionaries as inputs. The functions are expected to return positive numeric values.

Function node_subst_cost overrides node_match if specified. If neither node_match nor node_subst_cost are specified then default node substitution cost of 0 is used (node attributes are not considered during matching).

If node_del_cost is not specified then default node deletion cost of 1 is used. If node_ins_cost is not specified then default node insertion cost of 1 is used.

• edge_subst_cost, edge_del_cost, edge_ins_cost (callable) – Functions that return the costs of edge substitution, edge deletion, and edge insertion, respectively.

The functions will be called like

    edge_subst_cost(G1[u1][v1], G2[u2][v2]), edge_del_cost(G1[u1][v1]), edge_ins_cost(G2[u2][v2]).

That is, the functions will receive the edge attribute dictionaries as inputs. The functions are expected to return positive numeric values.

Function edge_subst_cost overrides edge_match if specified. If neither edge_match nor edge_subst_cost are specified then default edge substitution cost of 0 is used (edge attributes are not considered during matching).

If edge_del_cost is not specified then default edge deletion cost of 1 is used. If edge_ins_cost is not specified then default edge insertion cost of 1 is used.

• upper_bound (numeric) – Maximum edit distance to consider. Return None if no edit distance under or equal to upper_bound exists.

Examples

```python
>>> G1 = nx.cycle_graph(6)
>>> G2 = nx.wheel_graph(7)
>>> nx.graph_edit_distance(G1, G2)
7.0
```
See also:

optimal_edit_paths(), optimize_graph_edit_distance(), is_isomorphic()

References

3.49.2 networkx.algorithms.similarity.optimal_edit_paths

optimal_edit_paths(G1, G2, node_match=None, edge_match=None, node_subst_cost=None, node_del_cost=None, node_ins_cost=None, edge_subst_cost=None, edge_del_cost=None, edge_ins_cost=None, upper_bound=None)

Returns all minimum-cost edit paths transforming G1 to G2.

Graph edit path is a sequence of node and edge edit operations transforming graph G1 to graph isomorphic to G2. Edit operations include substitutions, deletions, and insertions.

Parameters

- G1, G2 (graphs) – The two graphs G1 and G2 must be of the same type.
- node_match (callable) – A function that returns True if node n1 in G1 and n2 in G2 should be considered equal during matching.
  
  The function will be called like
  
  node_match(G1.nodes[n1], G2.nodes[n2]).
  
  That is, the function will receive the node attribute dictionaries for n1 and n2 as inputs.
  
  Ignored if node_subst_cost is specified. If neither node_match nor node_subst_cost are specified then node attributes are not considered.
- edge_match (callable) – A function that returns True if the edge attribute dictionaries for the pair of nodes (u1, v1) in G1 and (u2, v2) in G2 should be considered equal during matching.
  
  The function will be called like
  
  edge_match(G1[u1][v1], G2[u2][v2]).
  
  That is, the function will receive the edge attribute dictionaries of the edges under consideration.
  
  Ignored if edge_subst_cost is specified. If neither edge_match nor edge_subst_cost are specified then edge attributes are not considered.
- node_subst_cost, node_del_cost, node_ins_cost (callable) – Functions that return the costs of node substitution, node deletion, and node insertion, respectively.
  
  The functions will be called like
  
  node_subst_cost(G1.nodes[n1], G2.nodes[n2]), node_del_cost(G1.nodes[n1]), node_ins_cost(G2.nodes[n2]).
  
  That is, the functions will receive the node attribute dictionaries as inputs. The functions are expected to return positive numeric values.
  
  Function node_subst_cost overrides node_match if specified. If neither node_match nor node_subst_cost are specified then default node substitution cost of 0 is used (node attributes are not considered during matching).
  
  If node_del_cost is not specified then default node deletion cost of 1 is used. If node_ins_cost is not specified then default node insertion cost of 1 is used.
• **edge_subst_cost, edge_del_cost, edge_ins_cost** *(callable)* – Functions that return the costs of edge substitution, edge deletion, and edge insertion, respectively.

The functions will be called like

\[
\text{edge_subst_cost}(G1[u1][v1], G2[u2][v2]), \quad \text{edge_del_cost}(G1[u1][v1]), \quad \text{edge_ins_cost}(G2[u2][v2]).
\]

That is, the functions will receive the edge attribute dictionaries as inputs. The functions are expected to return positive numeric values.

Function edge_subst_cost overrides edge_match if specified. If neither edge_match nor edge_subst_cost are specified then default edge substitution cost of 0 is used (edge attributes are not considered during matching).

If edge_del_cost is not specified then default edge deletion cost of 1 is used. If edge_ins_cost is not specified then default edge insertion cost of 1 is used.

• **upper_bound** *(numeric)* – Maximum edit distance to consider.

Returns

• **edit_paths** *(list of tuples (node_edit_path, edge_edit_path))* – node_edit_path : list of tuples (u, v) edge_edit_path : list of tuples ((u1, v1), (u2, v2))

• **cost** *(numeric)* – Optimal edit path cost (graph edit distance).

**Examples**

```python
>>> G1 = nx.cycle_graph(4)
>>> G2 = nx.wheel_graph(5)
>>> paths, cost = nx.optimal_edit_paths(G1, G2)
>>> len(paths)
40
>>> cost
5.0
```

See also:

*graph_edit_distance(), optimize_edit_paths()*

**References**

3.49.3 networkx.algorithms.similarity.optimize_graph_edit_distance

**optimize_graph_edit_distance** *(G1, G2, node_match=None, edge_match=None, node_subst_cost=None, node_del_cost=None, node_ins_cost=None, edge_subst_cost=None, edge_del_cost=None, edge_ins_cost=None, upper_bound=None)*

Returns consecutive approximations of GED (graph edit distance) between graphs G1 and G2.

Graph edit distance is a graph similarity measure analogous to Levenshtein distance for strings. It is defined as minimum cost of edit path (sequence of node and edge edit operations) transforming graph G1 to graph isomorphic to G2.

**Parameters**

• **G1, G2** *(graphs)* – The two graphs G1 and G2 must be of the same type.
• **node_match** *(callable)* – A function that returns True if node n1 in G1 and n2 in G2 should be considered equal during matching.

The function will be called like

    node_match(G1.nodes[n1], G2.nodes[n2]).

That is, the function will receive the node attribute dictionaries for n1 and n2 as inputs.

Ignored if node_subst_cost is specified. If neither node_match nor node_subst_cost are specified then node attributes are not considered.

• **edge_match** *(callable)* – A function that returns True if the edge attribute dictionaries for the pair of nodes (u1, v1) in G1 and (u2, v2) in G2 should be considered equal during matching.

The function will be called like

    edge_match(G1[u1][v1], G2[u2][v2]).

That is, the function will receive the edge attribute dictionaries of the edges under consideration.

Ignored if edge_subst_cost is specified. If neither edge_match nor edge_subst_cost are specified then edge attributes are not considered.

• **node_subst_cost, node_del_cost, node_ins_cost** *(callable)* – Functions that return the costs of node substitution, node deletion, and node insertion, respectively.

The functions will be called like

    node_subst_cost(G1.nodes[n1], G2.nodes[n2]),
    node_del_cost(G1.nodes[n1]),
    node_ins_cost(G2.nodes[n2]).

That is, the functions will receive the node attribute dictionaries as inputs. The functions are expected to return positive numeric values.

Function node_subst_cost overrides node_match if specified. If neither node_match nor node_subst_cost are specified then default node substitution cost of 0 is used (node attributes are not considered during matching).

If node_del_cost is not specified then default node deletion cost of 1 is used. If node_ins_cost is not specified then default node insertion cost of 1 is used.

• **edge_subst_cost, edge_del_cost, edge_ins_cost** *(callable)* – Functions that return the costs of edge substitution, edge deletion, and edge insertion, respectively.

The functions will be called like

    edge_subst_cost(G1[u1][v1], G2[u2][v2]),
    edge_del_cost(G1[u1][v1]),
    edge_ins_cost(G2[u2][v2]).

That is, the functions will receive the edge attribute dictionaries as inputs. The functions are expected to return positive numeric values.

Function edge_subst_cost overrides edge_match if specified. If neither edge_match nor edge_subst_cost are specified then default edge substitution cost of 0 is used (edge attributes are not considered during matching).

If edge_del_cost is not specified then default edge deletion cost of 1 is used. If edge_ins_cost is not specified then default edge insertion cost of 1 is used.

• **upper_bound** *(numeric)* – Maximum edit distance to consider.

Returns
Return type  Generator of consecutive approximations of graph edit distance.

Examples

```python
>>> G1 = nx.cycle_graph(6)
>>> G2 = nx.wheel_graph(7)
>>> for v in nx.optimize_graph_edit_distance(G1, G2):
...    minv = v
>>> minv
7.0
```

See also:

`graph_edit_distance()`, `optimize_edit_paths()`

References

3.49.4 networkx.algorithms.similarity.optimize_edit_paths

`optimize_edit_paths(G1, G2, node_match=None, edge_match=None, node_subst_cost=None, node_del_cost=None, node_ins_cost=None, edge_subst_cost=None, edge_del_cost=None, edge_ins_cost=None, upper_bound=None, strictly_decreasing=True)`

GED (graph edit distance) calculation: advanced interface.

Graph edit path is a sequence of node and edge edit operations transforming graph G1 to graph isomorphic to G2. Edit operations include substitutions, deletions, and insertions.

Graph edit distance is defined as minimum cost of edit path.

Parameters

- **G1, G2 (graphs)** – The two graphs G1 and G2 must be of the same type.
- **node_match (callable)** – A function that returns True if node n1 in G1 and n2 in G2 should be considered equal during matching.
  
  The function will be called like
  
  `node_match(G1.nodes[n1], G2.nodes[n2]).`
  
  That is, the function will receive the node attribute dictionaries for n1 and n2 as inputs.

  Ignored if node_subst_cost is specified. If neither node_match nor node_subst_cost are specified then node attributes are not considered.

- **edge_match (callable)** – A function that returns True if the edge attribute dictionaries for the pair of nodes (u1, v1) in G1 and (u2, v2) in G2 should be considered equal during matching.
  
  The function will be called like
  
  `edge_match(G1[u1][v1], G2[u2][v2]).`
  
  That is, the function will receive the edge attribute dictionaries of the edges under consideration.

  Ignored if edge_subst_cost is specified. If neither edge_match nor edge_subst_cost are specified then edge attributes are not considered.
• **node_subst_cost**, **node_del_cost**, **node_ins_cost** *(callable)* – Functions that return the costs of node substitution, node deletion, and node insertion, respectively.

  The functions will be called like

  ```python
  node_subst_cost(G1.nodes[n1], G2.nodes[n2]), node_del_cost(G1.nodes[n1]), node_ins_cost(G2.nodes[n2]).
  ```

  That is, the functions will receive the node attribute dictionaries as inputs. The functions are expected to return positive numeric values.

  Function `node_subst_cost` overrides `node_match` if specified. If neither `node_match` nor `node_subst_cost` are specified then default node substitution cost of 0 is used (node attributes are not considered during matching).

  If `node_del_cost` is not specified then default node deletion cost of 1 is used. If `node_ins_cost` is not specified then default node insertion cost of 1 is used.

• **edge_subst_cost**, **edge_del_cost**, **edge_ins_cost** *(callable)* – Functions that return the costs of edge substitution, edge deletion, and edge insertion, respectively.

  The functions will be called like

  ```python
  edge_subst_cost(G1[u1][v1], G2[u2][v2]), edge_del_cost(G1[u1][v1]), edge_ins_cost(G2[u2][v2]).
  ```

  That is, the functions will receive the edge attribute dictionaries as inputs. The functions are expected to return positive numeric values.

  Function `edge_subst_cost` overrides `edge_match` if specified. If neither `edge_match` nor `edge_subst_cost` are specified then default edge substitution cost of 0 is used (edge attributes are not considered during matching).

  If `edge_del_cost` is not specified then default edge deletion cost of 1 is used. If `edge_ins_cost` is not specified then default edge insertion cost of 1 is used.

• **upper_bound** *(numeric)* – Maximum edit distance to consider.

• **strictly_decreasing** *(bool)* – If True, return consecutive approximations of strictly decreasing cost. Otherwise, return all edit paths of cost less than or equal to the previous minimum cost.

  **Returns**

  `node_edit_path` : list of tuples (u, v)  
  `edge_edit_path` : list of tuples ((u1, v1), (u2, v2))  
  `cost` : numeric

  **Return type** Generator of tuples `(node_edit_path, edge_edit_path, cost)`

  **See also:**

  `graph_edit_distance()`, `optimize_graph_edit_distance()`, `optimal_edit_paths()`

**References**

3.49.5 networkx.algorithms.similarity.simrank_similarity

**simrank_similarity** *(G, source=None, target=None, importance_factor=0.9, max_iterations=100, tolerance=0.0001)*

  Returns the SimRank similarity of nodes in the graph G.

  SimRank is a similarity metric that says “two objects are considered to be similar if they are referenced by similar objects.”

The pseudo-code definition from the paper is:

```python
def simrank(G, u, v):
    in_neighbors_u = G.predecessors(u)
    in_neighbors_v = G.predecessors(v)
    scale = C / (len(in_neighbors_u) * len(in_neighbors_v))
    return scale * sum(simrank(G, w, x)
                         for w, x in product(in_neighbors_u,
                                             in_neighbors_v))
```

where $G$ is the graph, $u$ is the source, $v$ is the target, and $C$ is a float decay or importance factor between 0 and 1.

The SimRank algorithm for determining node similarity is defined in Equation 2.

**Parameters**

- $G$ (*NetworkX graph*) – A NetworkX graph
- $source$ (*node*) – If this is specified, the returned dictionary maps each node $v$ in the graph to the similarity between $source$ and $v$.
- $target$ (*node*) – If both $source$ and $target$ are specified, the similarity value between $source$ and $target$ is returned. If $target$ is specified but $source$ is not, this argument is ignored.
- $importance\_factor$ (*float*) – The relative importance of indirect neighbors with respect to direct neighbors.
- $max\_iterations$ (*integer*) – Maximum number of iterations.
- $tolerance$ (*float*) – Error tolerance used to check convergence. When an iteration of the algorithm finds that no similarity value changes more than this amount, the algorithm halts.

**Returns**

- $similarity$ – If $source$ and $target$ are both None, this returns a dictionary of dictionaries, where keys are node pairs and value are similarity of the pair of nodes.
  
  If $source$ is not None but $target$ is, this returns a dictionary mapping node to the similarity of $source$ and that node.
  
  If neither $source$ nor $target$ is None, this returns the similarity value for the given pair of nodes.

**Return type** dictionary or float

**Examples**

If the nodes of the graph are numbered from zero to $n - 1$, where $n$ is the number of nodes in the graph, you can create a SimRank matrix from the return value of this function where the node numbers are the row and column indices of the matrix:

```python
>>> import networkx as nx
>>> from numpy import array
>>> G = nx.cycle_graph(4)
>>> sim = nx.simrank_similarity(G)
>>> lol = [[sim[u][v] for v in sorted(sim[u])] for u in sorted(sim)]
>>> sim_array = array(lol)
```

---

3.49.6 `networkx.algorithms.similarity.simrank_similarity_numpy`

`simrank_similarity_numpy`\( (G, \text{source}=\text{None}, \text{target}=\text{None}, \text{importance\_factor}=0.9, \text{max\_iterations}=100, \text{tolerance}=0.0001) \)

Calculate SimRank of nodes in \( G \) using matrices with `numpy`.

The SimRank algorithm for determining node similarity is defined in\(^1\).

**Parameters**

- **\( G \) (NetworkX graph)** – A NetworkX graph
- **\text{source} (node)** – If this is specified, the returned dictionary maps each node \( v \) in the graph to the similarity between \text{source} and \( v \).
- **\text{target} (node)** – If both \text{source} and \text{target} are specified, the similarity value between \text{source} and \text{target} is returned. If \text{target} is specified but \text{source} is not, this argument is ignored.
- **\text{importance\_factor} (float)** – The relative importance of indirect neighbors with respect to direct neighbors.
- **\text{max\_iterations} (integer)** – Maximum number of iterations.
- **\text{tolerance} (float)** – Error tolerance used to check convergence. When an iteration of the algorithm finds that no similarity value changes more than this amount, the algorithm halts.

**Returns**

**\text{similarity}** – If \text{source} and \text{target} are both \text{None}, this returns a dictionary of dictionaries, where keys are node pairs and value are similarity of the pair of nodes.

If \text{source} is not \text{None} but \text{target} is, this returns a dictionary mapping node to the similarity of \text{source} and that node.

If neither \text{source} nor \text{target} is \text{None}, this returns the similarity value for the given pair of nodes.

**Return type** dictionary or float

**Examples**

```python
>>> import networkx as nx
>>> from numpy import array
>>> G = nx.cycle_graph(4)
>>> sim = nx.simrank_similarity_numpy(G)
```

**References**

3.50 Simple Paths

all_simple_paths(G, source, target[, cutoff])  Generate all simple paths in the graph G from source to target.

is_simple_path(G, nodes)  Returns True if and only if the given nodes form a simple path in G.

shortest_simple_paths(G, source, target[, ...])  Generate all simple paths in the graph G from source to target.

3.50.1 networkx.algorithms.simple_paths.all_simple_paths

all_simple_paths (G, source, target, cutoff=0)
Generate all simple paths in the graph G from source to target.

A simple path is a path with no repeated nodes.

Parameters

- G (NetworkX graph)
- source (node) – Starting node for path
- target (nodes) – Single node or iterable of nodes at which to end path
- cutoff (integer, optional) – Depth to stop the search. Only paths of length <= cutoff are returned.

Returns path_generator – A generator that produces lists of simple paths. If there are no paths between the source and target within the given cutoff the generator produces no output.

Return type generator

Examples

This iterator generates lists of nodes:

```python
>>> G = nx.complete_graph(4)
>>> for path in nx.all_simple_paths(G, source=0, target=3):
...    print(path)
...
[0, 1, 2, 3]
[0, 1, 3]
[0, 2, 1, 3]
[0, 2, 3]
[0, 3]
```

You can generate only those paths that are shorter than a certain length by using the cutoff keyword argument:

```python
>>> paths = nx.all_simple_paths(G, source=0, target=3, cutoff=2)
>>> print(list(paths))
[[0, 1, 3], [0, 2, 3], [0, 3]]
```

To get each path as the corresponding list of edges, you can use the networkx.utils.pairwise() helper function:

```python
>>> paths = nx.all_simple_paths(G, source=0, target=3)
>>> for path in map(nx.utils.pairwise, paths):
...    print(list(path))
...    [(0, 1), (1, 2), (2, 3)]
```

(continues on next page)
Pass an iterable of nodes as target to generate all paths ending in any of several nodes:

```python
>>> G = nx.complete_graph(4)
>>> for path in nx.all_simple_paths(G, source=0, target=[3, 2]):
...     print(path)
...  [0, 1, 2]
  [0, 1, 2, 3]
  [0, 1, 3]
  [0, 1, 3, 2]
  [0, 2]
  [0, 2, 1, 3]
  [0, 2, 3]
  [0, 3]
  [0, 3, 1, 2]
  [0, 3, 2]
```

Iterate over each path from the root nodes to the leaf nodes in a directed acyclic graph using a functional programming approach:

```python
>>> from itertools import chain, product, starmap, partial
>>> chaini = chain.from_iterable
>>> G = nx.DiGraph([(0, 1), (1, 2), (0, 3), (3, 2)])
>>> roots = (v for v, d in G.in_degree if d == 0)
>>> leaves = (v for v, d in G.out_degree if d == 0)
>>> all_paths = partial(nx.all_simple_paths, G)
>>> list(chaini(starmap(all_paths, product(roots, leaves))))
[[0, 1, 2], [0, 3, 2]]
```

The same list computed using an iterative approach:

```python
>>> G = nx.DiGraph([(0, 1), (1, 2), (0, 3), (3, 2)])
>>> roots = (v for v, d in G.in_degree if d == 0)
>>> leaves = (v for v, d in G.out_degree if d == 0)
>>> all_paths = []
>>> for root in roots:
...     for leaf in leaves:
...         paths = nx.all_simple_paths(G, root, leaf)
...         all_paths.extend(paths)
>>> all_paths
[[0, 1, 2], [0, 3, 2]]
```

Iterate over each path from the root nodes to the leaf nodes in a directed acyclic graph passing all leaves together to avoid unnecessary compute:
```python
>>> G = nx.DiGraph([(0, 1), (2, 1), (1, 3), (1, 4)])
>>> roots = (v for v, d in G.in_degree() if d == 0)
>>> leaves = [v for v, d in G.out_degree() if d == 0]
>>> all_paths = []
>>> for root in roots:
...    paths = nx.all_simple_paths(G, root, leaves)
...    all_paths.extend(paths)
>>> all_paths
[[0, 1, 3], [0, 1, 4], [2, 1, 3], [2, 1, 4]]
```

Notes

This algorithm uses a modified depth-first search to generate the paths. A single path can be found in $O(V + E)$ time but the number of simple paths in a graph can be very large, e.g. $O(n!)$ in the complete graph of order $n$.

References

See also:

all_shortest_paths(), shortest_path()

3.50.2 networkx.algorithms.simple_paths.is_simple_path

\texttt{is\_simple\_path}(G, \texttt{nodes})

Returns True if and only if the given nodes form a simple path in $G$.

A simple path in a graph is a nonempty sequence of nodes in which no node appears more than once in the sequence, and each adjacent pair of nodes in the sequence is adjacent in the graph.

Parameters \texttt{nodes} (list) -- A list of one or more nodes in the graph $G$.

Returns Whether the given list of nodes represents a simple path in $G$.

Return type \texttt{bool}

Notes

A list of zero nodes is not a path and a list of one node is a path. Here’s an explanation why.

This function operates on node paths. One could also consider edge paths. There is a bijection between node paths and edge paths.

The length of a path is the number of edges in the path, so a list of nodes of length $n$ corresponds to a path of length $n - 1$. Thus the smallest edge path would be a list of zero edges, the empty path. This corresponds to a list of one node.

To convert between a node path and an edge path, you can use code like the following:

```python
>>> from networkx.utils import pairwise
>>> nodes = [0, 1, 2, 3]
>>> edges = list(pairwise(nodes))
>>> edges
[(0, 1), (1, 2), (2, 3)]
```

**Examples**

```python
>>> nodes = [edges[0][0]] + [v for u, v in edges]
>>> nodes
[0, 1, 2, 3]
```

3.50.3 `networkx.algorithms.simple_paths.shortest_simple_paths`

**shortest_simple_paths** *(G, source, target, weight=None)*

Generate all simple paths in the graph G from source to target, starting from shortest ones. A simple path is a path with no repeated nodes.

If a weighted shortest path search is to be used, no negative weights are allowed.

**Parameters**

- **G** (*NetworkX graph*)
- **source** (*node*) – Starting node for path
- **target** (*node*) – Ending node for path
- **weight** (*string*) – Name of the edge attribute to be used as a weight. If None all edges are considered to have unit weight. Default value None.

**Returns** **path_generator** – A generator that produces lists of simple paths, in order from shortest to longest.

**Return type** generator

**Raises**

- **NetworkXNoPath** – If no path exists between source and target.
- **NetworkXError** – If source or target nodes are not in the input graph.
- **NetworkXNotImplemented** – If the input graph is a Multi[Di]Graph.

**Examples**

```python
>>> G = nx.cycle_graph(7)
>>> paths = list(nx.shortest_simple_paths(G, 0, 3))
>>> print(paths)
[[0, 1, 2, 3], [0, 6, 5, 4, 3]]
```

You can use this function to efficiently compute the k shortest/best paths between two nodes.
>>> from itertools import islice
>>> def k_shortest_paths(G, source, target, k, weight=None):
...     return list(islice(nx.shortest_simple_paths(G, source, target,
                        weight=weight), k))
>>> for path in k_shortest_paths(G, 0, 3, 2):
...     print(path)
[0, 1, 2, 3]
[0, 6, 5, 4, 3]

Notes

This procedure is based on algorithm by Jin Y. Yen\(^1\). Finding the first \( K \) paths requires \( O(KN^3) \) operations.

See also:

all_shortest_paths(), shortest_path(), all_simple_paths()

References

3.51 Small-world

Functions for estimating the small-world-ness of graphs.

A small world network is characterized by a small average shortest path length, and a large clustering coefficient.

Small-worldness is commonly measured with the coefficient sigma or omega.

Both coefficients compare the average clustering coefficient and shortest path length of a given graph against the same quantities for an equivalent random or lattice graph.

For more information, see the Wikipedia article on small-world network\(^1\).

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
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<tbody>
<tr>
<td>random_reference(G, niter, connectivity, seed)</td>
<td>Compute a random graph by swapping edges of a given graph.</td>
</tr>
<tr>
<td>lattice_reference(G, niter, D, ...)</td>
<td>Latticize the given graph by swapping edges.</td>
</tr>
<tr>
<td>sigma(G, niter, nrand, seed)</td>
<td>Returns the small-world coefficient (sigma) of the given graph.</td>
</tr>
<tr>
<td>omega(G, niter, nrand, seed)</td>
<td>Returns the small-world coefficient (omega) of a graph</td>
</tr>
</tbody>
</table>

3.51.1 networkx.algorithms.smallworld.random_reference

random_reference (G, niter=1, connectivity=True, seed=None)

Compute a random graph by swapping edges of a given graph.

Parameters

- **G** *(graph)* – An undirected graph with 4 or more nodes.
- **niter** *(integer (optional, default=1)) – An edge is rewired approximately \( n_{iter} \) times.

---


\(^1\) Small-world network: https://en.wikipedia.org/wiki/Small-world_network
• **connectivity** (boolean (optional, default=True)) – When True, ensure connectivity for the randomized graph.

• **seed** (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

**Returns** G – The randomized graph.

**Return type** graph

**Notes**

The implementation is adapted from the algorithm by Maslov and Sneppen (2002)\(^1\).

**References**

**3.51.2 networkx.algorithms.smallworld.lattice_reference**

**lattice_reference** \((G, niter=1, D=None, connectivity=True, seed=None)\)

Latticize the given graph by swapping edges.

**Parameters**

- **G** (graph) – An undirected graph with 4 or more nodes.
- **niter** (integer (optional, default=1)) – An edge is rewired approximatively niter times.
- **D** (numpy.array (optional, default=None)) – Distance to the diagonal matrix.
- **connectivity** (boolean (optional, default=True)) – Ensure connectivity for the latticized graph when set to True.
- **seed** (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

**Returns** G – The latticized graph.

**Return type** graph

**Notes**

The implementation is adapted from the algorithm by Sporns et al.\(^1\), which is inspired from the original work by Maslov and Sneppen(2002)\(^2\).

**References**

**3.51.3 networkx.algorithms.smallworld.sigma**

**sigma** \((G, niter=100, nrand=10, seed=None)\)

Returns the small-world coefficient (sigma) of the given graph.

The small-world coefficient is defined as: \[ \text{sigma} = \frac{C}{C_r} / \frac{L}{L_r} \] where \(C\) and \(L\) are respectively the average clustering coefficient and average shortest path length of \(G\). \(C_r\) and \(L_r\) are respectively the average clustering coefficient and average shortest path length of an equivalent random graph.


A graph is commonly classified as small-world if $\sigma > 1$.

**Parameters**

- **$G$ (NetworkX graph)** – An undirected graph.
- **niter** (*integer (optional, default=100)*) – Approximate number of rewiring per edge to compute the equivalent random graph.
- **nrand** (*integer (optional, default=10)) – Number of random graphs generated to compute the average clustering coefficient ($C_r$) and average shortest path length ($L_r$).
- **seed** (*integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

**Returns** $\sigma$ – The small-world coefficient of $G$.

**Return type** float

**Notes**

The implementation is adapted from Humphries et al.\(^1\).

**References**

3.51.4 **networkx.algorithms.smallworld.omega**

omega ($G, niter=100, nrand=10, seed=None$)

Returns the small-world coefficient (omega) of a graph

The small-world coefficient of a graph $G$ is:

$$\omega = L_r/L - C/C_l$$

where $C$ and $L$ are respectively the average clustering coefficient and average shortest path length of $G$. $L_r$ is the average shortest path length of an equivalent random graph and $C_l$ is the average clustering coefficient of an equivalent lattice graph.

The small-world coefficient (omega) ranges between -1 and 1. Values close to 0 means the $G$ features small-world characteristics. Values close to -1 means $G$ has a lattice shape whereas values close to 1 means $G$ is a random graph.

**Parameters**

- **$G$ (NetworkX graph)** – An undirected graph.
- **niter** (*integer (optional, default=100)) – Approximate number of rewiring per edge to compute the equivalent random graph.
- **nrand** (*integer (optional, default=10)) – Number of random graphs generated to compute the average clustering coefficient ($C_r$) and average shortest path length ($L_r$).
- **seed** (*integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

**Returns** $\omega$ – The small-world coefficient (omega)

---


Return type  float

Notes
The implementation is adapted from the algorithm by Telesford et al.\textsuperscript{1}.

References

3.52 s metric

\texttt{s\_metric}(G[, normalized])  Returns the s-metric of graph.

3.52.1 networkx.algorithms.smetric.s_metric

\texttt{s\_metric}(G, normalized=True)  Returns the s-metric of graph.

The s-metric is defined as the sum of the products \text{deg}(u)\text{deg}(v) for every edge \((u,v)\) in G. If norm is provided construct the s-max graph and compute it’s s_metric, and return the normalized s value.

Parameters

- \texttt{G} (graph) – The graph used to compute the s-metric.
- \texttt{normalized} (bool (optional)) – Normalize the value.

Returns  s – The s-metric of the graph.

Return type  float

References

3.53 Sparsifiers

Functions for computing sparsifiers of graphs.

\texttt{spanner}(G, stretch[, weight, seed])  Returns a spanner of the given graph with the given stretch.

3.53.1 networkx.algorithms.sparsifiers.spanner

\texttt{spanner} (G, stretch, weight=None, seed=None)  Returns a spanner of the given graph with the given stretch.

A spanner of a graph G = (V, E) with stretch t is a subgraph H = (V, E_S) such that \text{E_S} is a subset of E and the distance between any pair of nodes in H is at most t times the distance between the nodes in G.

Parameters

- \texttt{G} (\texttt{NetworkX} graph) – An undirected simple graph.

• **stretch** (*float*) – The stretch of the spanner.

• **weight** (*object*) – The edge attribute to use as distance.

• **seed** (*integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

**Returns** A spanner of the given graph with the given stretch.

**Return type** NetworkX graph

**Raises** ValueError – If a stretch less than 1 is given.

**Notes**

This function implements the spanner algorithm by Baswana and Sen, see [1].

This algorithm is a randomized las vegas algorithm: The expected running time is $O(km)$ where $k = (\text{stretch} + 1) / 2$ and $m$ is the number of edges in $G$. The returned graph is always a spanner of the given graph with the specified stretch. For weighted graphs the number of edges in the spanner is $O(k * n^a(1 + 1 / k))$ where $k$ is defined as above and $n$ is the number of nodes in $G$. For unweighted graphs the number of edges is $O(n^a(1 + 1 / k) + kn)$.

**References**


### 3.54 Structural holes

Functions for computing measures of structural holes.

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<th>Description</th>
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<td><code>constraint(G[, nodes, weight])</code></td>
<td>Returns the constraint on all nodes in the graph $G$.</td>
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<tr>
<td><code>effective_size(G[, nodes, weight])</code></td>
<td>Returns the effective size of all nodes in the graph $G$.</td>
</tr>
<tr>
<td><code>local_constraint(G, u, v[, weight])</code></td>
<td>Returns the local constraint on the node $u$ with respect to the node $v$ in the graph $G$.</td>
</tr>
</tbody>
</table>

### 3.54.1 networkx.algorithms.structuralholes.constraint

**constraint** (*G, nodes=None, weight=None*)

Returns the constraint on all nodes in the graph $G$.

The constraint is a measure of the extent to which a node $v$ is invested in those nodes that are themselves invested in the neighbors of $v$. Formally, the constraint on $v$, denoted $c(v)$, is defined by

$$ c(v) = \sum_{w \in N(v)\setminus\{v\}} \ell(v, w) $$

where $N(v)$ is the subset of the neighbors of $v$ that are either predecessors or successors of $v$ and $\ell(v, w)$ is the local constraint on $v$ with respect to $w$.

**Parameters**

• **G** *(NetworkX graph)* – The graph containing `v`. This can be either directed or undirected.

• **nodes** *(container, optional)* – Container of nodes in the graph `G` to compute the constraint. If None, the constraint of every node is computed.

• **weight** *(None or string, optional)* – If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

**Returns** Dictionary with nodes as keys and the constraint on the node as values.

**Return type** `dict`

**See also:** `local_constraint()`

### References

**3.54.2 networkx.algorithms.structuralholes.effective_size**

**effective_size** *(G, nodes=None, weight=None)*

Returns the effective size of all nodes in the graph `G`.

The **effective size** of a node’s ego network is based on the concept of redundancy. A person’s ego network has redundancy to the extent that her contacts are connected to each other as well. The nonredundant part of a person’s relationships it’s the effective size of her ego network. Formally, the effective size of a node `u`, denoted `e(u)`, is defined by

\[
e(u) = \sum_{v \in N(u) \setminus \{u\}} \left(1 - \sum_{w \in N(v)} p_{uw} m_{vw}\right)
\]

where `N(u)` is the set of neighbors of `u` and `p_{uw}` is the normalized mutual weight of the (directed or undirected) edges joining `u` and `v`, for each vertex `u` and `v`. And `m_{vw}` is the mutual weight of `v` and `w` divided by `v` highest mutual weight with any of its neighbors. The **mutual weight** of `u` and `v` is the sum of the weights of edges joining them (edge weights are assumed to be one if the graph is unweighted).

For the case of unweighted and undirected graphs, Borgatti proposed a simplified formula to compute effective size:

\[
e(u) = n - \frac{2t}{n}
\]

where `t` is the number of ties in the ego network (not including ties to ego) and `n` is the number of nodes (excluding ego).

**Parameters**

• **G** *(NetworkX graph)* – The graph containing `v`. Directed graphs are treated like undirected graphs when computing neighbors of `v`.

• **nodes** *(container, optional)* – Container of nodes in the graph `G` to compute the effective size. If None, the effective size of every node is computed.

• **weight** *(None or string, optional)* – If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

**Returns** Dictionary with nodes as keys and the constraint on the node as values.

**Return type** `dict`
Burt also defined the related concept of **efficiency** of a node’s ego network, which is its effective size divided by the degree of that node\(^1\). So you can easily compute efficiency:

```python
>>> G = nx.DiGraph()
>>> G.add_edges_from(((0, 1), (0, 2), (1, 0), (2, 1)))
>>> esize = nx.effective_size(G)
>>> efficiency = {n: v / G.degree(n) for n, v in esize.items()}
```

**See also:**

`constraint()`

### References

#### 3.54.3 networkx.algorithms. structuralholes. local_constraint

**local_constraint** \((G, u, v, \text{weight}=\text{None})\)

Returns the local constraint on the node \(u\) with respect to the node \(v\) in the graph \(G\).

Formally, the **local constraint on \(u\) with respect to \(v\)**, denoted \(\ell(v)\), is defined by

\[
\ell(u, v) = \left( p_{uv} + \sum_{w \in N(v)} p_{uw} p_{wv} \right)^2,
\]

where \(N(v)\) is the set of neighbors of \(v\) and \(p_{uw}\) is the normalized mutual weight of the (directed or undirected) edges joining \(u\) and \(v\), for each vertex \(u\) and \(v\). The **mutual weight** of \(u\) and \(v\) is the sum of the weights of edges joining them (edge weights are assumed to be one if the graph is unweighted).

**Parameters**

- \(G\) (*NetworkX graph*) — The graph containing \(u\) and \(v\). This can be either directed or undirected.
- \(u\) (*node*) — A node in the graph \(G\).
- \(v\) (*node*) — A node in the graph \(G\).
- \(\text{weight} (\text{None or string, optional})\) — If None, all edge weights are considered equal. Otherwise holds the name of the edge attribute used as weight.

**Returns** The constraint of the node \(v\) in the graph \(G\).

**Return type** `float`

**See also:**

`constraint()`

### References

#### 3.55 Swap

Swap edges in a graph.

---

**double_edge_swap** *(G, nswap=1, max_tries=100, seed=None)*

Swap two edges in the graph while keeping the node degrees fixed.

A double-edge swap removes two randomly chosen edges \( u-v \) and \( x-y \) and creates the new edges \( u-x \) and \( v-y \):

\[
\begin{array}{cc}
\text{u--v} & \text{u v} \\
\text{becomes} & | | \\
\text{x--y} & \text{x y}
\end{array}
\]

If either the edge \( u-x \) or \( v-y \) already exist no swap is performed and another attempt is made to find a suitable edge pair.

**Parameters**

- **G** *(graph)* – An undirected graph
- **nswap** *(integer (optional, default=1))* – Number of double-edge swaps to perform
- **max_tries** *(integer (optional))* – Maximum number of attempts to swap edges
- **seed** *(integer, random_state, or None (default))* – Indicator of random number generation state. See *Randomness*.

**Returns**

- **G** – The graph after double edge swaps.

**Return type**

- **graph**

**Notes**

Does not enforce any connectivity constraints.

The graph \( G \) is modified in place.

**3.5.2 networkx.algorithms.swap.connected_double_edge_swap**

**connected_double_edge_swap** *(G, nswap=1, _window_threshold=3, seed=None)*

Attempts the specified number of double-edge swaps in the graph \( G \).

A double-edge swap removes two randomly chosen edges \((u, v)\) and \((x, y)\) and creates the new edges \((u, x)\) and \((v, y)\):

\[
\begin{array}{cc}
\text{u--v} & \text{u v} \\
\text{becomes} & | | \\
\text{x--y} & \text{x y}
\end{array}
\]

If either \((u, x)\) or \((v, y)\) already exist, then no swap is performed so the actual number of swapped edges is always at most \( n\text{swap} \).

**Parameters**
• G (graph) – An undirected graph
• nswap (integer (optional, default=1)) – Number of double-edge swaps to perform
• _window_threshold (integer) – The window size below which connectedness of the graph will be checked after each swap.

The “window” in this function is a dynamically updated integer that represents the number of swap attempts to make before checking if the graph remains connected. It is an optimization used to decrease the running time of the algorithm in exchange for increased complexity of implementation.

If the window size is below this threshold, then the algorithm checks after each swap if the graph remains connected by checking if there is a path joining the two nodes whose edge was just removed. If the window size is above this threshold, then the algorithm performs do all the swaps in the window and only then check if the graph is still connected.

• seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns The number of successful swaps
Return type int

 Raises NetworkXError – If the input graph is not connected, or if the graph has fewer than four nodes.

Notes

The initial graph G must be connected, and the resulting graph is connected. The graph G is modified in place.

References

3.56 Tournament

Functions concerning tournament graphs.

A tournament graph is a complete oriented graph. In other words, it is a directed graph in which there is exactly one directed edge joining each pair of distinct nodes. For each function in this module that accepts a graph as input, you must provide a tournament graph. The responsibility is on the caller to ensure that the graph is a tournament graph.

To access the functions in this module, you must access them through the networkx.algorithms.tournament module:

```python
>>> import networkx as nx
>>> from networkx.algorithms import tournament

>>> G = nx.DiGraph([(0, 1), (1, 2), (2, 0)])
>>> tournament.is_tournament(G)
True
```

<table>
<thead>
<tr>
<th>hamiltonian_path(G)</th>
<th>Returns a Hamiltonian path in the given tournament graph.</th>
</tr>
</thead>
<tbody>
<tr>
<td>is_reachable(G, s, t)</td>
<td>Decides whether there is a path from s to t in the tournament.</td>
</tr>
</tbody>
</table>

Continued on next page
3.56.1 networkx.algorithms.tournament.hamiltonian_path

hamiltonian_path(G)
Returns a Hamiltonian path in the given tournament graph.

Each tournament has a Hamiltonian path. If furthermore, the tournament is strongly connected, then the returned
Hamiltonian path is a Hamiltonian cycle (by joining the endpoints of the path).

Parameters G (NetworkX graph) – A directed graph representing a tournament.

Returns Whether the given graph is a tournament graph.

Return type bool

Notes
This is a recursive implementation with an asymptotic running time of \(O(n^2)\), ignoring multiplicative polylog-
arithmetic factors, where \(n\) is the number of nodes in the graph.

3.56.2 networkx.algorithms.tournament.is_reachable

is_reachable(G, s, t)
Decides whether there is a path from \(s\) to \(t\) in the tournament.

This function is more theoretically efficient than the reachability checks than the shortest path algorithms in
networkx.algorithms.shortest_paths.

The given graph must be a tournament, otherwise this function’s behavior is undefined.

Parameters
- G (NetworkX graph) – A directed graph representing a tournament.
- s (node) – A node in the graph.
- t (node) – A node in the graph.

Returns Whether there is a path from \(s\) to \(t\) in \(G\).

Return type bool

Notes
Although this function is more theoretically efficient than the generic shortest path functions, a speedup requires
the use of parallelism. Though it may in the future, the current implementation does not use parallelism, thus
you may not see much of a speedup.

This algorithm comes from [1].
### References

#### 3.56.3 networkx.algorithms.tournament.is_strongly_connected

**is_strongly_connected**(G)

Decides whether the given tournament is strongly connected.

This function is more theoretically efficient than the `is_strongly_connected()` function.

The given graph must be a tournament, otherwise this function’s behavior is undefined.

**Parameters**

- `G` (*NetworkX* graph) – A directed graph representing a tournament.

**Returns**

Whether the tournament is strongly connected.

**Return type**

`bool`

**Notes**

Although this function is more theoretically efficient than the generic strong connectivity function, a speedup requires the use of parallelism. Though it may in the future, the current implementation does not use parallelism, thus you may not see much of a speedup.

This algorithm comes from [1].

### References

#### 3.56.4 networkx.algorithms.tournament.is_tournament

**is_tournament**(G)

Returns True if and only if G is a tournament.

A tournament is a directed graph, with neither self-loops nor multi-edges, in which there is exactly one directed edge joining each pair of distinct nodes.

**Parameters**

- `G` (*NetworkX* graph) – A directed graph representing a tournament.

**Returns**

Whether the given graph is a tournament graph.

**Return type**

`bool`

**Notes**

Some definitions require a self-loop on each node, but that is not the convention used here.

#### 3.56.5 networkx.algorithms.tournament.random_tournament

**random_tournament**(n, seed=None)

Returns a random tournament graph on n nodes.

**Parameters**

- `n` (*int*) – The number of nodes in the returned graph.
- `seed` (*integer, random_state, or None (default)*) – Indicator of random number generation state. See Randomness.

**Returns**

Whether the given graph is a tournament graph.
Return type: bool

Notes

This algorithm adds, for each pair of distinct nodes, an edge with uniformly random orientation. In other words, \( \binom{n}{2} \) flips of an unbiased coin decide the orientations of the edges in the graph.

3.56.6 networkx.algorithms.tournament.score_sequence

score_sequence \((G)\)

Returns the score sequence for the given tournament graph.

The score sequence is the sorted list of the out-degrees of the nodes of the graph.

Parameters

\- G (NetworkX graph) – A directed graph representing a tournament.

Returns

A sorted list of the out-degrees of the nodes of \( G \).

Return type: list

3.57 Traversal

3.57.1 Depth First Search

Basic algorithms for depth-first searching the nodes of a graph.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
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<tbody>
<tr>
<td>dfs_edges ((G[, \text{source}, \text{depth}\text{_limit}]))</td>
<td>Iterate over edges in a depth-first-search (DFS).</td>
</tr>
<tr>
<td>dfs_tree ((G[, \text{source}, \text{depth}\text{_limit}]))</td>
<td>Returns oriented tree constructed from a depth-first-search from source.</td>
</tr>
<tr>
<td>dfs_predecessors ((G[, \text{source}, \text{depth}\text{_limit}]))</td>
<td>Returns dictionary of predecessors in depth-first-search from source.</td>
</tr>
<tr>
<td>dfs_successors ((G[, \text{source}, \text{depth}\text{_limit}]))</td>
<td>Returns dictionary of successors in depth-first-search from source.</td>
</tr>
<tr>
<td>dfs_preorder_nodes ((G[, \text{source}, \text{depth}\text{_limit}]))</td>
<td>Generate nodes in a depth-first-search pre-ordering starting at source.</td>
</tr>
<tr>
<td>dfs_postorder_nodes ((G[, \text{source}, \text{depth}\text{_limit}]))</td>
<td>Generate nodes in a depth-first-search post-ordering starting at source.</td>
</tr>
<tr>
<td>dfs_labeled_edges ((G[, \text{source}, \text{depth}\text{_limit}]))</td>
<td>Iterate over edges in a depth-first-search (DFS) labeled by type.</td>
</tr>
</tbody>
</table>

networkx.algorithms.traversal.depth_first_search.dfs_edges

dfs_edges \((G, \text{source}=\text{None}, \text{depth}\text{\_limit}=\text{None})\)

Iterate over edges in a depth-first-search (DFS).

Perform a depth-first-search over the nodes of \( G \) and yield the edges in order. This may not generate all edges in \( G \) (see edge_dfs).

Parameters

\- G (NetworkX graph)
• **source** (*node, optional*) – Specify starting node for depth-first search and return edges in the component reachable from source.

• **depth_limit** (*int, optional (default=len(G))*) – Specify the maximum search depth.

**Returns** edges – A generator of edges in the depth-first-search.

**Return type** generator

### Examples

```python
>>> G = nx.path_graph(5)
>>> list(nx.dfs_edges(G, source=0))
[(0, 1), (1, 2), (2, 3), (3, 4)]
>>> list(nx.dfs_edges(G, source=0, depth_limit=2))
[(0, 1), (1, 2)]
```

### Notes

If a source is not specified then a source is chosen arbitrarily and repeatedly until all components in the graph are searched.

The implementation of this function is adapted from David Eppstein’s depth-first search function in PADS, with modifications to allow depth limits based on the Wikipedia article “Depth-limited search”.

**See also:**

dfs_preorder_nodes(), dfs_postorder_nodes(), dfs_labeled_edges(), edge_dfs(), bfs_edges()

#### networkx.algorithms.traversal.depth_first_search.dfs_tree

**dfs_tree** (*G, source=None, depth_limit=None*)

Returns oriented tree constructed from a depth-first-search from source.

**Parameters**

- **G** (*NetworkX graph*)
- **source** (*node, optional*) – Specify starting node for depth-first search.
- **depth_limit** (*int, optional (default=len(G))*) – Specify the maximum search depth.

**Returns** **T** – An oriented tree

**Return type** NetworkX DiGraph

### Examples

```python
>>> G = nx.path_graph(5)
>>> T = nx.dfs_tree(G, source=0, depth_limit=2)
>>> list(T.edges())
[(0, 1), (1, 2)]
>>> T = nx.dfs_tree(G, source=0)
>>> list(T.edges())
[(0, 1), (1, 2), (2, 3), (3, 4)]
```
See also:

dfs_preorder_nodes(), dfs_postorder_nodes(), dfs_labeled_edges(), edge_dfs(),
bfs_tree()

networkx.algorithms.traversal.depth_first_search.dfs_predecessors
dfs_predecessors \((G, source=None, depth_limit=None)\)

Returns dictionary of predecessors in depth-first-search from source.

Parameters

- \(G\) (NetworkX graph)
- \(source\) (node, optional) – Specify starting node for depth-first search.
- \(depth\_limit\) (int, optional (default=len(G))) – Specify the maximum search depth.

Returns \(pred\) – A dictionary with nodes as keys and predecessor nodes as values.

Return type dict

Examples

```python
>>> G = nx.path_graph(4)
>>> nx.dfs_predecessors(G, source=0)
{1: 0, 2: 1, 3: 2}
>>> nx.dfs_predecessors(G, source=0, depth_limit=2)
{1: 0, 2: 1}
```

Notes

If a source is not specified then a source is chosen arbitrarily and repeatedly until all components in the graph are searched.

The implementation of this function is adapted from David Eppstein’s depth-first search function in PADS, with modifications to allow depth limits based on the Wikipedia article “Depth-limited search”.

See also:

dfs_preorder_nodes(), dfs_postorder_nodes(), dfs_labeled_edges(), edge_dfs(),
bfs_tree()

networkx.algorithms.traversal.depth_first_search.dfs_successors
dfs_successors \((G, source=None, depth_limit=None)\)

Returns dictionary of successors in depth-first-search from source.

Parameters

- \(G\) (NetworkX graph)
- \(source\) (node, optional) – Specify starting node for depth-first search.
- \(depth\_limit\) (int, optional (default=len(G))) – Specify the maximum search depth.

Returns \(succ\) – A dictionary with nodes as keys and list of successor nodes as values.

Return type dict
Examples

```python
>>> G = nx.path_graph(5)
>>> nx.dfs_successors(G, source=0)
{0: [1], 1: [2], 2: [3], 3: [4])
>>> nx.dfs_successors(G, source=0, depth_limit=2)
{0: [1], 1: [2])
```

Notes

If a source is not specified then a source is chosen arbitrarily and repeatedly until all components in the graph are searched.

The implementation of this function is adapted from David Eppstein’s depth-first search function in PADS, with modifications to allow depth limits based on the Wikipedia article “Depth-limited search”.

See also:
dfs_preorder_nodes(), dfs_postorder_nodes(), dfs_labeled_edges(), edge_dfs(), bfs_tree()

dfs_preorder_nodes

Generate nodes in a depth-first-search pre-ordering starting at source.

Parameters

- `G` *(NetworkX graph)*
- `source` *(node, optional)* – Specify starting node for depth-first search and return nodes in the component reachable from source.
- `depth_limit` *(int, optional (default=len(G)))* – Specify the maximum search depth.

Returns `nodes` – A generator of nodes in a depth-first-search pre-ordering.

Return type generator

Examples

```python
>>> G = nx.path_graph(5)
>>> list(nx.dfs_preorder_nodes(G, source=0))
[0, 1, 2, 3, 4]
>>> list(nx.dfs_preorder_nodes(G, source=0, depth_limit=2))
[0, 1, 2]
```

Notes

If a source is not specified then a source is chosen arbitrarily and repeatedly until all components in the graph are searched.

The implementation of this function is adapted from David Eppstein’s depth-first search function in PADS, with modifications to allow depth limits based on the Wikipedia article “Depth-limited search”.

See also:
dfs_edges(), dfs_postorder_nodes(), dfs_labeled_edges(), bfs_edges()

networkx.algorithms.traversal.depth_first_search.dfs_postorder_nodes

dfs_postorder_nodes (G, source=None, depth_limit=None)
Generate nodes in a depth-first-search post-ordering starting at source.

Parameters

• G (NetworkX graph)
• source (node, optional) – Specify starting node for depth-first search.
• depth_limit (int, optional (default=len(G))) – Specify the maximum search depth.

Returns nodes – A generator of nodes in a depth-first-search post-ordering.

Return type generator

Examples

>>> G = nx.path_graph(5)
>>> list(nx.dfs_postorder_nodes(G, source=0))
[4, 3, 2, 1, 0]
>>> list(nx.dfs_postorder_nodes(G, source=0, depth_limit=2))
[1, 0]

Notes

If a source is not specified then a source is chosen arbitrarily and repeatedly until all components in the graph are searched.

The implementation of this function is adapted from David Eppstein’s depth-first search function in PADS, with modifications to allow depth limits based on the Wikipedia article “Depth-limited search”.

See also:

dfs_edges(), dfs_preorder_nodes(), dfs_labeled_edges(), edge_dfs(), bfs_tree()

networkx.algorithms.traversal.depth_first_search.dfs_labeled_edges

dfs_labeled_edges (G, source=None, depth_limit=None)
Iterate over edges in a depth-first-search (DFS) labeled by type.

Parameters

• G (NetworkX graph)
• source (node, optional) – Specify starting node for depth-first search and return edges in the component reachable from source.
• depth_limit (int, optional (default=len(G))) – Specify the maximum search depth.

Returns edges – A generator of triples of the form (u, v, d), where (u, v) is the edge being explored in the depth-first search and d is one of the strings ‘forward’, ‘nontree’, or ‘reverse’. A ‘forward’ edge is one in which u has been visited but v has not. A ‘nontree’ edge is one in which both u and v have been visited but the edge is not in the DFS tree. A ‘reverse’ edge is on in which both u and v have been visited and the edge is in the DFS tree.
Return type  generator

Examples
The labels reveal the complete transcript of the depth-first search algorithm in more detail than, for example, `dfs_edges()`:

```python
>>> from pprint import pprint
>>> G = nx.DiGraph([(0, 1), (1, 2), (2, 1)])
>>> pprint(list(nx.dfs_labeled_edges(G, source=0)))
[(0, 0, 'forward'),
 (0, 1, 'forward'),
 (1, 2, 'forward'),
 (2, 1, 'nontree'),
 (1, 2, 'reverse'),
 (0, 1, 'reverse'),
 (0, 0, 'reverse')]```

Notes
If a source is not specified then a source is chosen arbitrarily and repeatedly until all components in the graph are searched.

The implementation of this function is adapted from David Eppstein’s depth-first search function in PADS, with modifications to allow depth limits based on the Wikipedia article “Depth-limited search”.

See also:
`dfs_edges()`, `dfs_preorder_nodes()`, `dfs_postorder_nodes()`

3.57.2 Breadth First Search
Basic algorithms for breadth-first searching the nodes of a graph.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>bfs_edges(G, source[, reverse, depth_limit])</code></td>
<td>Iterate over edges in a breadth-first-search starting at source.</td>
</tr>
<tr>
<td><code>bfs_tree(G, source[, reverse, depth_limit])</code></td>
<td>Returns an oriented tree constructed from of a breadth-first-search starting at source.</td>
</tr>
<tr>
<td><code>bfs_predecessors(G, source[, depth_limit])</code></td>
<td>Returns an iterator of predecessors in breadth-first-search from source.</td>
</tr>
<tr>
<td><code>bfs_successors(G, source[, depth_limit])</code></td>
<td>Returns an iterator of successors in breadth-first-search from source.</td>
</tr>
</tbody>
</table>

networkx.algorithms.traversal.breadth_first_search.bfs_edges

`bfs_edges (G, source[, reverse=False, depth_limit=None])`
Iterate over edges in a breadth-first-search starting at source.

Parameters
- **G** (NetworkX graph)
• **source** (*node*) – Specify starting node for breadth-first search; this function iterates over only those edges in the component reachable from this node.

• **reverse** (*bool*, optional) – If True traverse a directed graph in the reverse direction

• **depth_limit** (*int*, optional(*default=len(G)*)) – Specify the maximum search depth

**Returns** edges – A generator of edges in the breadth-first-search.

**Return type** generator

**Examples**

To get the edges in a breadth-first search:

```python
>>> G = nx.path_graph(3)
>>> list(nx.bfs_edges(G, 0))
[(0, 1), (1, 2)]
>>> list(nx.bfs_edges(G, source=0, depth_limit=1))
[(0, 1)]
```

To get the nodes in a breadth-first search order:

```python
>>> G = nx.path_graph(3)
>>> root = 2
>>> edges = nx.bfs_edges(G, root)
>>> nodes = [root] + [v for u, v in edges]
>>> nodes
[2, 1, 0]
```

**Notes**

The naming of this function is very similar to bfs_edges. The difference is that ‘edge_bfs’ yields edges even if they extend back to an already explored node while ‘bfs_edges’ yields the edges of the tree that results from a breadth-first-search (BFS) so no edges are reported if they extend to already explored nodes. That means ‘edge_bfs’ reports all edges while ‘bfs_edges’ only report those traversed by a node-based BFS. Yet another description is that ‘bfs_edges’ reports the edges traversed during BFS while ‘edge_bfs’ reports all edges in the order they are explored.


**See also:**

bfs_tree(), dfs_edges(), edge_bfs()

**networkx.algorithms.traversal.breadth_first_search.bfs_tree**

bfs_tree (*G*, *source*, reverse=False, depth_limit=None)

Returns an oriented tree constructed from of a breadth-first-search starting at source.

**Parameters**

• **G** (*NetworkX graph*)

• **source** (*node*) – Specify starting node for breadth-first search

• **reverse** (*bool*, optional) – If True traverse a directed graph in the reverse direction
• **depth_limit** *(int, optional=default=len(G))* – Specify the maximum search depth

**Returns** T – An oriented tree

**Return type** NetworkX DiGraph

### Examples

```python
>>> G = nx.path_graph(3)
>>> print(list(nx.bfs_tree(G, 1).edges()))
[(1, 0), (1, 2)]
>>> H = nx.Graph()
>>> nx.add_path(H, [0, 1, 2, 3, 4, 5, 6])
>>> nx.add_path(H, [2, 7, 8, 9, 10])
>>> print(sorted(list(nx.bfs_tree(H, source=3, depth_limit=3).edges())))
[(1, 0), (2, 1), (2, 7), (3, 2), (3, 4), (4, 5), (5, 6), (7, 8)]
```

### Notes


**See also:**

```
dfs_tree(), bfs_edges(), edge_bfs()
```

### networkx.algorithms.traversal.breadth_first_search.bfs_predecessors

**bfs_predecessors** *(G, source, depth_limit=None)*

Returns an iterator of predecessors in breadth-first-search from source.

**Parameters**

- **G** *(NetworkX graph)*
- **source** *(node)* – Specify starting node for breadth-first search
- **depth_limit** *(int, optional=default=len(G))* – Specify the maximum search depth

**Returns** pred – *(node, predecessors)* iterator where predecessors is the list of predecessors of the node.

**Return type** iterator

### Examples

```python
>>> G = nx.path_graph(3)
>>> print(dict(nx.bfs_predecessors(G, 0)))
{1: 0, 2: 1}
>>> H = nx.Graph()
>>> H.add_edges_from([(0, 1), (0, 2), (1, 3), (1, 4), (2, 5), (2, 6)])
>>> print(dict(nx.bfs_predecessors(H, 0)))
{1: 0, 2: 0, 3: 1, 4: 1, 5: 2, 6: 2}
>>> M = nx.Graph()
>>> nx.add_path(M, [0, 1, 2, 3, 4, 5, 6])
>>> nx.add_path(M, [2, 7, 8, 9, 10])
```
>>> print(sorted(nx.bfs_predecessors(M, source=1, depth_limit=3)))
[(0, 1), (2, 1), (3, 2), (4, 3), (7, 2), (8, 7)]

Notes


See also:
bfs_tree(), bfs_edges(), edge_bfs()

networkx.algorithms.traversal.breadth_first_search.bfs_successors

bfs_successors (G, source, depth_limit=None)
Returns an iterator of successors in breadth-first-search from source.

Parameters

- **G** (NetworkX graph)
- **source** (node) – Specify starting node for breadth-first search
- **depth_limit** (int, optional(default=len(G))) – Specify the maximum search depth

Returns **succ** – (node, successors) iterator where successors is the list of successors of the node.

Return type iterator

Examples

```python
>>> G = nx.path_graph(3)
>>> print(dict(nx.bfs_successors(G,0)))
{0: [1], 1: [2]}
>>> H = nx.Graph()
>>> H.add_edges_from([(0, 1), (0, 2), (1, 3), (1, 4), (2, 5), (2, 6)])
>>> print(dict(nx.bfs_successors(H, 0)))
{0: [1, 2], 1: [3, 4], 2: [5, 6]}
>>> G = nx.Graph()
>>> nx.add_path(G, [0, 1, 2, 3, 4, 5, 6])
>>> nx.add_path(G, [2, 7, 8, 9, 10])
>>> print(dict(nx.bfs_successors(G, source=1, depth_limit=3)))
{1: [0, 2], 2: [3, 7], 3: [4], 7: [8]}
```

Notes


See also:
bfs_tree(), bfs_edges(), edge_bfs()
3.57.3 Beam search

Basic algorithms for breadth-first searching the nodes of a graph.

*bfs_beam_edges*(G, source, value[, width])  
Iterates over edges in a beam search.

**networkx.algorithms.traversal.beamsearch.bfs_beam_edges**

*bfs_beam_edges* (*G, source, value, width=None*)

Iterates over edges in a beam search.

The beam search is a generalized breadth-first search in which only the “best” *w* neighbors of the current node are enqueued, where *w* is the beam width and “best” is an application-specific heuristic. In general, a beam search with a small beam width might not visit each node in the graph.

**Parameters**

- *G* (*NetworkX graph*)
- *source* (*node*) – Starting node for the breadth-first search; this function iterates over only those edges in the component reachable from this node.
- *value* (*function*) – A function that takes a node of the graph as input and returns a real number indicating how “good” it is. A higher value means it is more likely to be visited sooner during the search. When visiting a new node, only the *width* neighbors with the highest *value* are enqueued (in decreasing order of *value*).
- *width* (*int* (*default* = *None*)*) – The beam width for the search. This is the number of neighbors (ordered by *value*) to enqueue when visiting each new node.

**Yields** *edge* – Edges in the beam search starting from *source*, given as a pair of nodes.

**Examples**

To give nodes with, for example, a higher centrality precedence during the search, set the *value* function to return the centrality value of the node:

```python
>>> G = nx.karate_club_graph()
>>> centrality = nx.eigenvector_centrality(G)
>>> source = 0
>>> width = 5
>>> for u, v in nx.bfs_beam_edges(G, source, centrality.get, width):
...    print((u, v))
```

3.57.4 Depth First Search on Edges

**Depth First Search on Edges**

Algorithms for a depth-first traversal of edges in a graph.

*edge_dfs* (*G[, source, orientation]*)

A directed, depth-first-search of edges in *G*, beginning at *source*. 

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networkx.algorithms.traversal.edgedfs.edge_dfs

`edge_dfs(G, source=None, orientation=None)`

A directed, depth-first-search of edges in `G`, beginning at `source`.

Yield the edges of `G` in a depth-first-search order continuing until all edges are generated.

**Parameters**

- `G (graph)` – A directed/undirected graph/multigraph.
- `source (node, list of nodes)` – The node from which the traversal begins. If None, then a source is chosen arbitrarily and repeatedly until all edges from each node in the graph are searched.
- `orientation (None | 'original' | 'reverse' | 'ignore' (default: None))` – For directed graphs and directed multigraphs, edge traversals need not respect the original orientation of the edges. When set to ‘reverse’ every edge is traversed in the reverse direction. When set to ‘ignore’, every edge is treated as undirected. When set to ‘original’, every edge is treated as directed. In all three cases, the yielded edge tuples add a last entry to indicate the direction in which that edge was traversed. If orientation is None, the yielded edge has no direction indicated. The direction is respected, but not reported.

**Yields** `edge (directed edge)` – A directed edge indicating the path taken by the depth-first traversal. For graphs, `edge` is of the form `(u, v)` where `u` and `v` are the tail and head of the edge as determined by the traversal. For multigraphs, `edge` is of the form `(u, v, key)`, where `key` is the key of the edge. When the graph is directed, then `u` and `v` are always in the order of the actual directed edge. If orientation is not None then the edge tuple is extended to include the direction of traversal (‘forward’ or ‘reverse’) on that edge.

**Examples**

```python
g = nx.Graph([(0, 1), (1, 0), (1, 0), (2, 1), (3, 1)])
list(nx.edge_dfs(g, sources=(0,)))
```

```python
[(0, 1), (1, 2), (1, 3)]
```

```python
g = nx.DiGraph([(0, 1), (1, 0)])
list(nx.edge_dfs(g, sources=(0,)))
```

```python
[(0, 1), (1, 0), (2, 1), (3, 1)]
```

```python
g = nx.MultiGraph([(0, 1), (1, 0), (1, 0), (2, 1), (3, 1)])
list(nx.edge_dfs(g, sources=(0,)))
```

```python
[(0, 1, 0), (1, 0, 1), (0, 1, 2), (1, 2, 0), (1, 3, 0)]
```

```python
g = nx.MultiDiGraph([(0, 1), (1, 0), (1, 0), (2, 1), (3, 1)])
list(nx.edge_dfs(g, sources=(0,)))
```

```python
[(0, 1, 0), (1, 0, 1), (1, 0, 0), (2, 1, 0), (3, 1, 0)]
```

```python
g = nx.DiGraph([(0, 1), (1, 0)])
list(nx.edge_dfs(g, sources=(0,), orientation='ignore'))
```

```python
[(0, 1, 'forward'), (1, 0, 'forward'), (2, 1, 'reverse'), (3, 1, 'reverse')]
```

```python
g = nx.MultiDiGraph([(0, 1), (1, 0), (1, 0), (2, 1), (3, 1)])
list(nx.edge_dfs(g, sources=(0,), orientation='ignore'))
```

```python
[(0, 1, 'forward'), (1, 0, 0, 'forward'), (1, 0, 1, 'reverse'), (2, 1, 0, 'reverse'), (3, 1, 0, 'reverse')]
```
Notes

The goal of this function is to visit edges. It differs from the more familiar depth-first traversal of nodes, as provided by `networkx.algorithms.traversal.depth_first_search.dfs_edges()`, in that it does not stop once every node has been visited. In a directed graph with edges [(0, 1), (1, 2), (2, 1)], the edge (2, 1) would not be visited if not for the functionality provided by this function.

See also:

dfs_edges()

3.57.5 Breadth First Search on Edges

Breadth First Search on Edges

Algorithms for a breadth-first traversal of edges in a graph.

\[edge_bfs(G[, source, orientation])\]

A directed, breadth-first-search of edges in \(G\), beginning at \(source\).

\[edge_bfs\]

\(edge_bfs\) \((G, source=None, orientation=None)\)

A directed, breadth-first-search of edges in \(G\), beginning at \(source\).

Yield the edges of \(G\) in a breadth-first-search order continuing until all edges are generated.

Parameters

- \(G\) (graph) – A directed/undirected graph/multigraph.
- \(source\) (node, list of nodes) – The node from which the traversal begins. If None, then a source is chosen arbitrarily and repeatedly until all edges from each node in the graph are searched.
- \(orientation\) (None | 'original' | 'reverse' | 'ignore' (default: None)) – For directed graphs and directed multigraphs, edge traversals need not respect the original orientation of the edges. When set to ‘reverse’ every edge is traversed in the reverse direction. When set to ‘ignore’, every edge is treated as undirected. When set to ‘original’, every edge is treated as directed. In all three cases, the yielded edge tuples add a last entry to indicate the direction in which that edge was traversed. If orientation is None, the yielded edge has no direction indicated. The direction is respected, but not reported.

Yields \(edge\) (directed edge) – A directed edge indicating the path taken by the breadth-first-search. For graphs, \(edge\) is of the form \((u, v)\) where \(u\) and \(v\) are the tail and head of the edge as determined by the traversal. For multigraphs, \(edge\) is of the form \((u, v, key)\), where \(key\) is the key of the edge. When the graph is directed, then \(u\) and \(v\) are always in the order of the actual directed edge. If orientation is not None then the edge tuple is extended to include the direction of traversal (‘forward’ or ‘reverse’) on that edge.

Examples
```python
>>> import networkx as nx
>>> nodes = [0, 1, 2, 3]
>>> edges = [(0, 1), (1, 0), (1, 0), (2, 0), (2, 1), (3, 1)]

>>> list(nx.edge_bfs(nx.Graph(edges), nodes))
[(0, 1), (0, 2), (1, 2), (1, 3)]

>>> list(nx.edge_bfs(nx.DiGraph(edges), nodes))
[(0, 1), (1, 0), (2, 0), (2, 1), (3, 1)]

>>> list(nx.edge_bfs(nx.MultiGraph(edges), nodes))
[(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (1, 2, 0), (1, 3, 0)]

>>> list(nx.edge_bfs(nx.MultiDiGraph(edges), nodes))
[(0, 1, 0), (1, 0, 0), (1, 0, 1), (2, 0, 0), (2, 1, 0), (3, 1, 0)]

>>> list(nx.edge_bfs(nx.DiGraph(edges), nodes, orientation='ignore'))
[(0, 1, 'forward'), (1, 0, 'reverse'), (2, 0, 'reverse'), (2, 1, 'reverse'), (3, 1, 'reverse')]

>>> list(nx.edge_bfs(nx.MultiDiGraph(edges), nodes, orientation='ignore'))
[(0, 1, 'forward'), (1, 0, 0, 'reverse'), (1, 0, 1, 'reverse'), (2, 0, 0, 'reverse'), (2, 1, 0, 'reverse'), (3, 1, 0, 'reverse')]
```

### Notes

The goal of this function is to visit edges. It differs from the more familiar breadth-first-search of nodes, as provided by `networkx.algorithms.traversal.breadth_first_search.bfs_edges()`, in that it does not stop once every node has been visited. In a directed graph with edges 
{(0, 1), (1, 2), (2, 1)}, the edge (2, 1) would not be visited if not for the functionality provided by this function.

The naming of this function is very similar to `bfs_edges`. The difference is that ‘edge_bfs’ yields edges even if they extend back to an already explored node while ‘bfs_edges’ yields the edges of the tree that results from a breadth-first-search (BFS) so no edges are reported if they extend to already explored nodes. That means ‘edge_bfs’ reports all edges while ‘bfs_edges’ only report those traversed by a node-based BFS. Yet another description is that ‘bfs_edges’ reports the edges traversed during BFS while ‘edge_bfs’ reports all edges in the order they are explored.

#### See also:

`bfs_edges(), bfs_tree(), edge_dfs()`

### 3.58 Tree

#### 3.58.1 Recognition

#### Recognition Tests

A forest is an acyclic, undirected graph, and a tree is a connected forest. Depending on the subfield, there are various conventions for generalizing these definitions to directed graphs.
In one convention, directed variants of forest and tree are defined in an identical manner, except that the direction of the edges is ignored. In effect, each directed edge is treated as a single undirected edge. Then, additional restrictions are imposed to define *branchings* and *arborescences*.

In another convention, directed variants of forest and tree correspond to the previous convention’s branchings and arborescences, respectively. Then two new terms, *polyforest* and *polytree*, are defined to correspond to the other convention’s forest and tree.

Summarizing:

<table>
<thead>
<tr>
<th>Convention A</th>
<th>Convention B</th>
</tr>
</thead>
<tbody>
<tr>
<td>forest</td>
<td>polyforest</td>
</tr>
<tr>
<td>tree</td>
<td>polytree</td>
</tr>
<tr>
<td>branching</td>
<td>forest</td>
</tr>
<tr>
<td>arborescence</td>
<td>tree</td>
</tr>
</tbody>
</table>

Each convention has its reasons. The first convention emphasizes definitional similarity in that directed forests and trees are only concerned with acyclicity and do not have an in-degree constraint, just as their undirected counterparts do not. The second convention emphasizes functional similarity in the sense that the directed analog of a spanning tree is a spanning arborescence. That is, take any spanning tree and choose one node as the root. Then every edge is assigned a direction such there is a directed path from the root to every other node. The result is a spanning arborescence.

NetworkX follows convention “A”. Explicitly, these are:

**undirected forest** An undirected graph with no undirected cycles.

**undirected tree** A connected, undirected forest.

**directed forest** A directed graph with no undirected cycles. Equivalently, the underlying graph structure (which ignores edge orientations) is an undirected forest. In convention B, this is known as a polyforest.

**directed tree** A weakly connected, directed forest. Equivalently, the underlying graph structure (which ignores edge orientations) is an undirected tree. In convention B, this is known as a polytree.

**branching** A directed forest with each node having, at most, one parent. So the maximum in-degree is equal to 1. In convention B, this is known as a forest.

**arborescence** A directed tree with each node having, at most, one parent. So the maximum in-degree is equal to 1. In convention B, this is known as a tree.

For trees and arborescences, the adjective “spanning” may be added to designate that the graph, when considered as a forest/branching, consists of a single tree/arborescence that includes all nodes in the graph. It is true, by definition, that every tree/arborescence is spanning with respect to the nodes that define the tree/arborescence and so, it might seem redundant to introduce the notion of “spanning”. However, the nodes may represent a subset of nodes from a larger graph, and it is in this context that the term “spanning” becomes a useful notion.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>is_tree(G)</td>
<td>Returns True if G is a tree.</td>
</tr>
<tr>
<td>is_forest(G)</td>
<td>Returns True if G is a forest.</td>
</tr>
<tr>
<td>is_arborescence(G)</td>
<td>Returns True if G is an arborescence.</td>
</tr>
<tr>
<td>is_branching(G)</td>
<td>Returns True if G is a branching.</td>
</tr>
</tbody>
</table>

**networkx.algorithms.tree.recognition.is_tree**

**is_tree(G)**

Returns True if G is a tree.
A tree is a connected graph with no undirected cycles.

For directed graphs, \( G \) is a tree if the underlying graph is a tree. The underlying graph is obtained by treating each directed edge as a single undirected edge in a multigraph.

Parameters
- \( G (\text{graph}) \) – The graph to test.

Returns
- \( b \) – A boolean that is True if \( G \) is a tree.

Return type
- bool

Notes
In another convention, a directed tree is known as a polytree and then tree corresponds to an arborescence.

See also:
- \textit{is_arborescence()}

\texttt{networkx.algorithms.tree.recognition.is_forest}

\texttt{is_forest}(\texttt{G})

Returns True if \( G \) is a forest.

A forest is a graph with no undirected cycles.

For directed graphs, \( G \) is a forest if the underlying graph is a forest. The underlying graph is obtained by treating each directed edge as a single undirected edge in a multigraph.

Parameters
- \( G (\text{graph}) \) – The graph to test.

Returns
- \( b \) – A boolean that is True if \( G \) is a forest.

Return type
- bool

Notes
In another convention, a directed forest is known as a polyforest and then forest corresponds to a branching.

See also:
- \textit{is_branching()}

\texttt{networkx.algorithms.tree.recognition.is_arborescence}

\texttt{is_arborescence}(\texttt{G})

Returns True if \( G \) is an arborescence.

An arborescence is a directed tree with maximum in-degree equal to 1.

Parameters
- \( G (\text{graph}) \) – The graph to test.

Returns
- \( b \) – A boolean that is True if \( G \) is an arborescence.

Return type
- bool
Notes

In another convention, an arborescence is known as a tree.

See also:

is_tree()

networkx.algorithms.tree.recognition.is_branching

is_branching(G)

Returns True if G is a branching.

A branching is a directed forest with maximum in-degree equal to 1.

Parameters

G (directed graph) – The directed graph to test.

Returns

b – A boolean that is True if G is a branching.

Return type

bool

Notes

In another convention, a branching is also known as a forest.

See also:

is_forest()

3.58.2 Branchings and Spanning Arborescences

Algorithms for finding optimum branchings and spanning arborescences.

This implementation is based on:


<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>branching_weight(G[, attr, default])</td>
<td>Returns the total weight of a branching.</td>
</tr>
<tr>
<td>greedy_branching(G[, attr, default, kind, seed])</td>
<td>Returns a branching obtained through a greedy algorithm.</td>
</tr>
<tr>
<td>maximum_branching(G[, attr, default, ...])</td>
<td>Returns a maximum branching from G.</td>
</tr>
<tr>
<td>minimum_branching(G[, attr, default, ...])</td>
<td>Returns a minimum branching from G.</td>
</tr>
<tr>
<td>maximum_spanning_arborescence(G[, attr, ...])</td>
<td>Returns a maximum spanning arborescence from G.</td>
</tr>
<tr>
<td>minimum_spanning_arborescence(G[, attr, ...])</td>
<td>Returns a minimum spanning arborescence from G.</td>
</tr>
<tr>
<td>Edmonds(G[, seed])</td>
<td>Edmonds algorithm for finding optimal branchings and spanning arborescences.</td>
</tr>
</tbody>
</table>

networkx.algorithms.tree.branchings.branching_weight

branching_weight (G, attr='weight', default=1)

Returns the total weight of a branching.
networkx.algorithms.tree.branchings.greedy_branching

greedy_branching (G, attr='weight', default=1, kind='max', seed=None)
Returns a branching obtained through a greedy algorithm.

This algorithm is wrong, and cannot give a proper optimal branching. However, we include it for pedagogical
reasons, as it can be helpful to see what its outputs are.

The output is a branching, and possibly, a spanning arborescence. However, it is not guaranteed to be optimal
in either case.

Parameters

• G (DiGraph) – The directed graph to scan.
• attr (str) – The attribute to use as weights. If None, then each edge will be treated equally
  with a weight of 1.
• default (float) – When attr is not None, then if an edge does not have that attribute,
  default specifies what value it should take.
• kind (str) – The type of optimum to search for: ‘min’ or ‘max’ greedy branching.
• seed (integer, random_state, or None (default)) – Indicator of random number generation
  state. See Randomness.

Returns B – The greedily obtained branching.

Return type directed graph

networkx.algorithms.tree.branchings.maximum_branching

maximum_branching (G, attr='weight', default=1, preserve_attrs=False)
Returns a maximum branching from G.

Parameters

• G ((multi)digraph-like) – The graph to be searched.
• attr (str) – The edge attribute used to in determining optimality.
• default (float) – The value of the edge attribute used if an edge does not have the attribute attr.
• preserve_attrs (bool) – If True, preserve the other attributes of the original graph (that are not passed to attr)

Returns B – A maximum branching.

Return type (multi)digraph-like

networkx.algorithms.tree.branchings.minimum_branching

minimum_branching (G, attr='weight', default=1, preserve_attrs=False)
Returns a minimum branching from G.

Parameters

• G ((multi)digraph-like) – The graph to be searched.
• attr (str) – The edge attribute used to in determining optimality.
networkx.algorithms.tree.branchings.maximum_spanning_arborescence

maximum_spanning_arborescence (G, attr='weight', default=1, preserve_attrs=False)

Returns a maximum spanning arborescence from G.

Parameters

• G ((multi)digraph-like) – The graph to be searched.
• attr (str) – The edge attribute used to in determining optimality.
• default (float) – The value of the edge attribute used if an edge does not have the attribute attr.
• preserve_attrs (bool) – If True, preserve the other attributes of the original graph (that are not passed to attr)

Returns B – A maximum spanning arborescence.

Return type (multi)digraph-like

Raises NetworkXException – If the graph does not contain a maximum spanning arborescence.

networkx.algorithms.tree.branchings.minimum_spanning_arborescence

minimum_spanning_arborescence (G, attr='weight', default=1, preserve_attrs=False)

Returns a minimum spanning arborescence from G.

Parameters

• G ((multi)digraph-like) – The graph to be searched.
• attr (str) – The edge attribute used to in determining optimality.
• default (float) – The value of the edge attribute used if an edge does not have the attribute attr.
• preserve_attrs (bool) – If True, preserve the other attributes of the original graph (that are not passed to attr)

Returns B – A minimum spanning arborescence.

Return type (multi)digraph-like

Raises NetworkXException – If the graph does not contain a minimum spanning arborescence.

networkx.algorithms.tree.branchings.Edmonds

class Edmonds (G, seed=None)

Edmonds algorithm for finding optimal branchings and spanning arborescences.
Methods

- __init__(G[, seed]) Initialize self.
- find_optimum([attr, default, kind, style, ...]) Returns a branching from G.

3.58.3 Encoding and decoding

Functions for encoding and decoding trees.

Since a tree is a highly restricted form of graph, it can be represented concisely in several ways. This module includes functions for encoding and decoding trees in the form of nested tuples and Prüfer sequences. The former requires a rooted tree, whereas the latter can be applied to unrooted trees. Furthermore, there is a bijection from Prüfer sequences to labeled trees.

- from_nested_tuple(sequence[, ...]) Returns the rooted tree corresponding to the given nested tuple.
- to_nested_tuple(T, root[, canonical_form]) Returns a nested tuple representation of the given tree.
- from_prufer_sequence(sequence) Returns the tree corresponding to the given Prüfer sequence.
- to_prufer_sequence(T) Returns the Prüfer sequence of the given tree.

networkx.algorithms.tree.coding.from_nested_tuple

**from_nested_tuple** (sequence, sensible_relabeling=False)

Returns the rooted tree corresponding to the given nested tuple.

The nested tuple representation of a tree is defined recursively. The tree with one node and no edges is represented by the empty tuple, (). A tree with \( k \) subtrees is represented by a tuple of length \( k \) in which each element is the nested tuple representation of a subtree.

**Parameters**

- sequence (tuple) – A nested tuple representing a rooted tree.
- sensible_relabeling (bool) – Whether to relabel the nodes of the tree so that nodes are labeled in increasing order according to their breadth-first search order from the root node.

**Returns** The tree corresponding to the given nested tuple, whose root node is node 0. If sensible_labeling is True, nodes will be labeled in breadth-first search order starting from the root node.

**Return type** NetworkX graph

**Notes**

This function is **not** the inverse of to_nested_tuple(); the only guarantee is that the rooted trees are isomorphic.

**See also:**

to_nested_tuple(), from_prufer_sequence()
Examples

Sensible relabeling ensures that the nodes are labeled from the root starting at 0:

```python
>>> balanced = (((), ()), ((), ()))
>>> T = nx.from_nested_tuple(balanced, sensible_relabeling=True)
>>> edges = [(0, 1), (0, 2), (1, 3), (1, 4), (2, 5), (2, 6)]
>>> all((u, v) in T.edges() or (v, u) in T.edges() for (u, v) in edges)  
True
```

networkx.algorithms.tree.coding.to_nested_tuple

**to_nested_tuple** *(T, root, canonical_form=False)*

Returns a nested tuple representation of the given tree.

The nested tuple representation of a tree is defined recursively. The tree with one node and no edges is represented by the empty tuple, (). A tree with \( k \) subtrees is represented by a tuple of length \( k \) in which each element is the nested tuple representation of a subtree.

**Parameters**

- **T** *(NetworkX graph)*  – An undirected graph object representing a tree.
- **root** *(node)*  – The node in \( T \) to interpret as the root of the tree.
- **canonical_form** *(bool)*  – If True, each tuple is sorted so that the function returns a canonical form for rooted trees. This means “lighter” subtrees will appear as nested tuples before “heavier” subtrees. In this way, each isomorphic rooted tree has the same nested tuple representation.

**Returns**  A nested tuple representation of the tree.

**Return type**  tuple

**Notes**

This function is *not* the inverse of *from_nested_tuple()*; the only guarantee is that the rooted trees are isomorphic.

**See also:**

*from_nested_tuple()* , *to_pruefer_sequence()*

**Examples**

The tree need not be a balanced binary tree:

```python
>>> T = nx.Graph()
>>> T.add_edges_from([[0, 1], [0, 2], [0, 3]])
>>> T.add_edges_from([[1, 4], [1, 5]])
>>> T.add_edges_from([[3, 6], [3, 7]])
>>> root = 0
>>> nx.to_nested_tuple(T, root)
(((), ()), (), ((), ()))
```

Continuing the above example, if **canonical_form** is True, the nested tuples will be sorted:
>>> nx.to_nested_tuple(T, root, canonical_form=True)
(((), (((), ())), (((), ()))), (((), ())))

Even the path graph can be interpreted as a tree:

>>> T = nx.path_graph(4)
>>> root = 0
>>> nx.to_nested_tuple(T, root)
((((),),),)

networkx.algorithms.tree.coding.from_prufer_sequence

from_prufer_sequence(sequence)
Returns the tree corresponding to the given Prüfer sequence.

A Prüfer sequence is a list of \( n - 2 \) numbers between 0 and \( n - 1 \), inclusive. The tree corresponding to a given Prüfer sequence can be recovered by repeatedly joining a node in the sequence with a node with the smallest potential degree according to the sequence.

Parameters sequence (list) – A Prüfer sequence, which is a list of \( n - 2 \) integers between zero and \( n - 1 \), inclusive.

Returns The tree corresponding to the given Prüfer sequence.

Return type NetworkX graph

Notes

There is a bijection from labeled trees to Prüfer sequences. This function is the inverse of the from_prufer_sequence() function.

Sometimes Prüfer sequences use nodes labeled from 1 to \( n \) instead of from 0 to \( n - 1 \). This function requires nodes to be labeled in the latter form. You can use networkx.relabel_nodes() to relabel the nodes of your tree to the appropriate format.

This implementation is from\(^1\) and has a running time of \( O(n) \).

References

See also:

from_nested_tuple(), to_prufer_sequence()

Examples

There is a bijection between Prüfer sequences and labeled trees, so this function is the inverse of the to_prufer_sequence() function:

```python
>> edges = [(0, 3), (1, 3), (2, 3), (3, 4), (4, 5)]
>>> tree = nx.Graph(edges)
>>> sequence = nx.to_prufer_sequence(tree)
>>> sequence
```

(continues on next page)

networkx.algorithms.tree.coding.to_prufer_sequence

to_prufer_sequence (T)
Returns the Prüfer sequence of the given tree.

A Prüfer sequence is a list of n - 2 numbers between 0 and n - 1, inclusive. The tree corresponding to a given Prüfer sequence can be recovered by repeatedly joining a node in the sequence with a node with the smallest potential degree according to the sequence.

Parameters T (NetworkX graph) – An undirected graph object representing a tree.

Returns The Prüfer sequence of the given tree.

Return type list

Raises
• NetworkXPointlessConcept – If the number of nodes in T is less than two.
• NotATree – If T is not a tree.
• KeyError – If the set of nodes in T is not {0, ..., n - 1}.

Notes
There is a bijection from labeled trees to Prüfer sequences. This function is the inverse of the from_prufer_sequence() function.

Sometimes Prüfer sequences use nodes labeled from 1 to n instead of from 0 to n - 1. This function requires nodes to be labeled in the latter form. You can use relabel_nodes() to relabel the nodes of your tree to the appropriate format.

This implementation is from\(^1\) and has a running time of \(O(n)\).

See also:
to_nested_tuple(), from_prufer_sequence()

References

Examples

There is a bijection between Prüfer sequences and labeled trees, so this function is the inverse of the from_prufer_sequence() function:

```python
>>> edges = [(0, 3), (1, 3), (2, 3), (3, 4), (4, 5)]
>>> tree = nx.Graph(edges)
>>> sequence = nx.to_prufer_sequence(tree)
>>> sequence
[3, 3, 3, 4]
```
3.58.4 Operations

Operations on trees.

<table>
<thead>
<tr>
<th><code>join</code>(rooted_trees[, label_attribute])</th>
<th>Returns a new rooted tree with a root node joined with the roots of each of the given rooted trees.</th>
</tr>
</thead>
</table>

networkx.algorithms.tree.operations.join

`join(rooted_trees, label_attribute=None)`

Returns a new rooted tree with a root node joined with the roots of each of the given rooted trees.

**Parameters**

- **rooted_trees** *(list)* – A list of pairs in which each left element is a NetworkX graph object representing a tree and each right element is the root node of that tree. The nodes of these trees will be relabeled to integers.

- **label_attribute** *(str)* – If provided, the old node labels will be stored in the new tree under this node attribute. If not provided, the node attribute '_old' will store the original label of the node in the rooted trees given in the input.

**Returns**

The rooted tree whose subtrees are the given rooted trees. The new root node is labeled 0. Each non-root node has an attribute, as described under the keyword argument label_attribute, that indicates the label of the original node in the input tree.

**Return type** NetworkX graph

**Notes**

Graph, edge, and node attributes are propagated from the given rooted trees to the created tree. If there are any overlapping graph attributes, those from later trees will overwrite those from earlier trees in the tuple of positional arguments.

**Examples**

Join two full balanced binary trees of height $h$ to get a full balanced binary tree of depth $h + 1$:

```python
>>> h = 4
>>> left = nx.balanced_tree(2, h)
>>> right = nx.balanced_tree(2, h)
>>> joined_tree = nx.join([(left, 0), (right, 0)])
>>> nx.is_isomorphic(joined_tree, nx.balanced_tree(2, h + 1))
True
```
3.58.5 Spanning Trees

Algorithms for calculating min/max spanning trees/forests.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>minimum_spanning_tree(G[, weight,...])</code></td>
<td>Returns a minimum spanning tree or forest on an undirected graph G.</td>
</tr>
<tr>
<td><code>maximum_spanning_tree(G[, weight,...])</code></td>
<td>Returns a maximum spanning tree or forest on an undirected graph G.</td>
</tr>
<tr>
<td><code>minimum_spanning_edges(G[, algorithm,...])</code></td>
<td>Generate edges in a minimum spanning forest of an undirected weighted graph.</td>
</tr>
<tr>
<td><code>maximum_spanning_edges(G[, algorithm,...])</code></td>
<td>Generate edges in a maximum spanning forest of an undirected weighted graph.</td>
</tr>
</tbody>
</table>

**networkx.algorithms.tree.mst.minimum_spanning_tree**

`minimum_spanning_tree(G, weight='weight', algorithm='kruskal', ignore_nan=False)`

Returns a minimum spanning tree or forest on an undirected graph G.

**Parameters**

- `G` (*undirected graph*) – An undirected graph. If G is connected, then the algorithm finds a spanning tree. Otherwise, a spanning forest is found.
- `weight` (*str*) – Data key to use for edge weights.
- `algorithm` (*string*) – The algorithm to use when finding a minimum spanning tree. Valid choices are ‘kruskal’, ‘prim’, or ‘boruvka’. The default is ‘kruskal’.
- `ignore_nan` (*bool (default: False)*) – If a NaN is found as an edge weight normally an exception is raised. If ignore_nan is True then that edge is ignored instead.

**Returns**

G – A minimum spanning tree or forest.

**Return type** NetworkX Graph

**Examples**

```python
>>> G = nx.cycle_graph(4)
>>> G.add_edge(0, 3, weight=2)
>>> T = nx.minimum_spanning_tree(G)
>>> sorted(T.edges(data=True))
[(0, 1, {}), (1, 2, {}), (2, 3, {})]
```

**Notes**

For Borůvka’s algorithm, each edge must have a weight attribute, and each edge weight must be distinct.

For the other algorithms, if the graph edges do not have a weight attribute a default weight of 1 will be used.

There may be more than one tree with the same minimum or maximum weight. See `networkx.tree.recognition` for more detailed definitions.

Isolated nodes with self-loops are in the tree as edgeless isolated nodes.
maximum_spanning_tree \( (G, \text{weight}=\text{weight}', \text{algorithm}=\text{kruskal}', \text{ignore.nan}=\text{False}) \)

Returns a maximum spanning tree or forest on an undirected graph \( G \).

Parameters

- \( G \) (undirected graph) – An undirected graph. If \( G \) is connected, then the algorithm finds a spanning tree. Otherwise, a spanning forest is found.
- \( \text{weight} \) (str) – Data key to use for edge weights.
- \( \text{algorithm} \) (string) – The algorithm to use when finding a maximum spanning tree. Valid choices are ‘kruskal’, ‘prim’, or ‘boruvka’. The default is ‘kruskal’.
- \( \text{ignore.nan} \) (bool (default: False)) – If a NaN is found as an edge weight normally an exception is raised. If \( \text{ignore.nan} \) is True then that edge is ignored instead.

Returns \( G \) – A maximum spanning tree or forest.

Return type NetworkX Graph

Examples

```python
>>> G = nx.cycle_graph(4)
>>> G.add_edge(0, 3, weight=2)
>>> T = nx.maximum_spanning_tree(G)
>>> sorted(T.edges(data=True))
[(0, 1, {}), (0, 3, {'weight': 2}), (1, 2, {})]
```

Notes

For Borůvka’s algorithm, each edge must have a weight attribute, and each edge weight must be distinct.

For the other algorithms, if the graph edges do not have a weight attribute a default weight of 1 will be used.

There may be more than one tree with the same minimum or maximum weight. See networkx.tree.recognition for more detailed definitions.

Isolated nodes with self-loops are in the tree as edgeless isolated nodes.

minimum_spanning_edges \( (G, \text{algorithm}=\text{kruskal}', \text{weight}=\text{weight}', \text{keys}=\text{True}, \text{data}=\text{True}, \text{ignore.nan}=\text{False}) \)

Generate edges in a minimum spanning forest of an undirected weighted graph.

A minimum spanning tree is a subgraph of the graph (a tree) with the minimum sum of edge weights. A spanning forest is a union of the spanning trees for each connected component of the graph.

Parameters

- \( G \) (undirected Graph) – An undirected graph. If \( G \) is connected, then the algorithm finds a spanning tree. Otherwise, a spanning forest is found.
- \( \text{algorithm} \) (string) – The algorithm to use when finding a minimum spanning tree. Valid choices are ‘kruskal’, ‘prim’, or ‘boruvka’. The default is ‘kruskal’.
- \( \text{weight} \) (string) – Edge data key to use for weight (default ‘weight’).
• **keys** *(bool)* – Whether to yield edge key in multigraphs in addition to the edge. If \( G \) is not a multigraph, this is ignored.

• **data** *(bool, optional)* – If True yield the edge data along with the edge.

• **ignore_nan** *(bool (default: False))* – If a NaN is found as an edge weight normally an exception is raised. If **ignore_nan** is True then that edge is ignored instead.

**Returns**

**edges** – An iterator over edges in a maximum spanning tree of \( G \). Edges connecting nodes \( u \) and \( v \) are represented as tuples: \((u, v, k, d)\) or \((u, v, k)\) or \((u, v, d)\) or \((u, v)\) or \((u, v, k, d)\) or \((u, v, k)\) or \((u, v, d)\) or \((u, v)\) or \((u, v, k, d)\) or \((u, v, k)\) or \((u, v, d)\) or \((u, v)\)

If \( G \) is a multigraph, keys indicates whether the edge key \( k \) will be reported in the third position in the edge tuple. data indicates whether the edge datadict \( d \) will appear at the end of the edge tuple.

If \( G \) is not a multigraph, the tuples are \((u, v, d)\) if data is True or \((u, v)\) if data is False.

**Return type** *iterator*

**Examples**

```python
>>> from networkx.algorithms import tree

Find minimum spanning edges by Kruskal’s algorithm

``` python
>>> G = nx.cycle_graph(4)
>>> G.add_edge(0, 3, weight=2)
>>> mst = tree.minimum_spanning_edges(G, algorithm='kruskal', data=False)
>>> edgelist = list(mst)
>>> sorted(sorted(e) for e in edgelist)
[[0, 1], [1, 2], [2, 3]]

Find minimum spanning edges by Prim’s algorithm

``` python
>>> G = nx.cycle_graph(4)
>>> G.add_edge(0, 3, weight=2)
>>> mst = tree.minimum_spanning_edges(G, algorithm='prim', data=False)
>>> edgelist = list(mst)
>>> sorted(sorted(e) for e in edgelist)
[[0, 1], [1, 2], [2, 3]]
```

**Notes**

For Borůvka’s algorithm, each edge must have a weight attribute, and each edge weight must be distinct.

For the other algorithms, if the graph edges do not have a weight attribute a default weight of 1 will be used.


**networkx.algorithms.tree.mst.maximum_spanning_edges**

**maximum_spanning_edges** *(G, algorithm='kruskal', weight='weight', keys=True, data=True, ignore_nan=False)*

Generate edges in a maximum spanning forest of an undirected weighted graph.
A maximum spanning tree is a subgraph of the graph (a tree) with the maximum possible sum of edge weights. A spanning forest is a union of the spanning trees for each connected component of the graph.

Parameters

- **G (undirected Graph)** – An undirected graph. If G is connected, then the algorithm finds a spanning tree. Otherwise, a spanning forest is found.
- **algorithm (string)** – The algorithm to use when finding a maximum spanning tree. Valid choices are ‘kruskal’, ‘prim’, or ‘boruvka’. The default is ‘kruskal’.
- **weight (string)** – Edge data key to use for weight (default ‘weight’).
- **keys (bool)** – Whether to yield edge key in multigraphs in addition to the edge. If G is not a multigraph, this is ignored.
- **data (bool, optional)** – If True yield the edge data along with the edge.
- **ignore_nan (bool (default: False))** – If a NaN is found as an edge weight normally an exception is raised. If ignore_nan is True then that edge is ignored instead.

Returns

edges – An iterator over edges in a maximum spanning tree of G. Edges connecting nodes u and v are represented as tuples: (u, v, k, d) or (u, v, k) or (u, v, d) or (u, v) if G is a multigraph, keys indicates whether the edge key k will be reported in the third position in the edge tuple. data indicates whether the edge datadict d will appear at the end of the edge tuple.

If G is not a multigraph, the tuples are (u, v, d) if data is True or (u, v) if data is False.

Return type iterator

Examples

```python
>>> from networkx.algorithms import tree

Find maximum spanning edges by Kruskal’s algorithm

```>>>` G = nx.cycle_graph(4)`
```>>>` G.add_edge(0, 3, weight=2)`
```>>>` mst = tree.maximum_spanning_edges(G, algorithm='kruskal', data=False)`
```>>>` edgelist = list(mst)`
```>>>` sorted(sorted(e) for e in edgelist)`
[[[0, 1], [0, 3], [1, 2]]

Find maximum spanning edges by Prim’s algorithm

```>>>` G = nx.cycle_graph(4)`
```>>>` G.add_edge(0, 3, weight=2) # assign weight 2 to edge 0-3`
```>>>` mst = tree.maximum_spanning_edges(G, algorithm='prim', data=False)`
```>>>` edgelist = list(mst)`
```>>>` sorted(sorted(e) for e in edgelist)`
[[[0, 1], [0, 3], [2, 3]]

3.58. Tree

509
Notes

For Borůvka’s algorithm, each edge must have a weight attribute, and each edge weight must be distinct.
For the other algorithms, if the graph edges do not have a weight attribute a default weight of 1 will be used.
Modified code from David Eppstein, April 2006 http://www.ics.uci.edu/~eppstein/PADS/

3.58.6 Exceptions

Functions for encoding and decoding trees.
Since a tree is a highly restricted form of graph, it can be represented concisely in several ways. This module includes
functions for encoding and decoding trees in the form of nested tuples and Prüfer sequences. The former requires a
rooted tree, whereas the latter can be applied to unrooted trees. Furthermore, there is a bijection from Prüfer sequences
to labeled trees.

<table>
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<tr>
<th>NotATree</th>
<th>Raised when a function expects a tree (that is, a connected undirected graph with no cycles) but gets a non-tree graph as input instead.</th>
</tr>
</thead>
</table>

networkx.algorithms.tree.coding.NotATree

except NotATree
Raised when a function expects a tree (that is, a connected undirected graph with no cycles) but gets a non-tree
graph as input instead.

3.59 Triads

Functions for analyzing triads of a graph.

<table>
<thead>
<tr>
<th>triadic_census(G)</th>
<th>Determines the triadic census of a directed graph.</th>
</tr>
</thead>
</table>

3.59.1 networkx.algorithms.triads.triadic_census

triadic_census (G)
Determines the triadic census of a directed graph.
The triadic census is a count of how many of the 16 possible types of triads are present in a directed graph.

Parameters

\[ G \text{ (digraph)} \] – A NetworkX DiGraph

Returns
census – Dictionary with triad names as keys and number of occurrences as values.

Return type
dict

Notes
This algorithm has complexity \( O(m) \) where \( m \) is the number of edges in the graph.

See also:
triad_graph()

References

3.60 Vitality

Vitality measures.

\[closeness\textunderscore vitality(G, \text{node}=None, \text{weight}=None, \text{wiener\_index}=None)\] Returns the closeness vitality for nodes in the graph.

3.60.1 networkx.algorithms.vitality.closeness_vitality

closeness_vitality\ (G, \text{node}=None, \text{weight}=None, \text{wiener\_index}=None)

Returns the closeness vitality for nodes in the graph.

The *closeness vitality* of a node, defined in Section 3.6.2 of [1], is the change in the sum of distances between all node pairs when excluding that node.

**Parameters**

- \(G\) (*NetworkX graph*) – A strongly-connected graph.
- \(\text{weight}\) (*string*) – The name of the edge attribute used as weight. This is passed directly to the \(\text{wiener\_index()}\) function.
- \(\text{node}\) (*object*) – If specified, only the closeness vitality for this node will be returned. Otherwise, a dictionary mapping each node to its closeness vitality will be returned.

**Other Parameters**

- \(\text{wiener\_index}\) (*number*) – If you have already computed the Wiener index of the graph \(G\), you can provide that value here. Otherwise, it will be computed for you.

**Returns**

If \(\text{node}\) is None, this function returns a dictionary with nodes as keys and closeness vitality as the value. Otherwise, it returns only the closeness vitality for the specified \(\text{node}\).

The closeness vitality of a node may be negative infinity if removing that node would disconnect the graph.

**Return type** dictionary or float

**Examples**

```python
>>> G = nx.cycle_graph(3)
>>> nx.closeness_vitality(G)
{0: 2.0, 1: 2.0, 2: 2.0}
```

**See also:**

closeness_centrality()
References

3.61 Voronoi cells

Functions for computing the Voronoi cells of a graph.

\texttt{voronoi\_cells}(G, center\_nodes[, weight]) Returns the Voronoi cells centered at center\_nodes with respect to the shortest-path distance metric.

3.61.1 networkx.algorithms.voronoi.voronoi_cells

\texttt{voronoi\_cells}(G, center\_nodes, weight='weight')

Returns the Voronoi cells centered at center\_nodes with respect to the shortest-path distance metric.

If \(C\) is a set of nodes in the graph and \(c\) is an element of \(C\), the Voronoi cell centered at a node \(c\) is the set of all nodes \(v\) that are closer to \(c\) than to any other center node in \(C\) with respect to the shortest-path distance metric.\(^1\)

For directed graphs, this will compute the “outward” Voronoi cells, as defined in\(^1\), in which distance is measured from the center nodes to the target node. For the “inward” Voronoi cells, use the \texttt{DiGraph.reverse()} method to reverse the orientation of the edges before invoking this function on the directed graph.

Parameters

- \(G\) (\texttt{NetworkX} graph)
- \texttt{center\_nodes} (set) – A nonempty set of nodes in the graph \(G\) that represent the center of the Voronoi cells.
- \texttt{weight} (string or function) – The edge attribute (or an arbitrary function) representing the weight of an edge. This keyword argument is as described in the documentation for \texttt{multi\_source\_dijkstra\_path()}, for example.

Returns

A mapping from center node to set of all nodes in the graph closer to that center node than to any other center node. The keys of the dictionary are the element of center\_nodes, and the values of the dictionary form a partition of the nodes of \(G\).

Return type dictionary

Examples

To get only the partition of the graph induced by the Voronoi cells, take the collection of all values in the returned dictionary:

\begin{verbatim}
>>> G = nx.path_graph(6)
>>> center_nodes = {0, 3}
>>> cells = nx.voronoi_cells(G, center_nodes)
>>> partition = set(map(frozenset, cells.values()))
>>> sorted(map(sorted, partition))
[['0', 1], [2, 3, 4, 5]]
\end{verbatim}

Raises \texttt{ValueError} – If \texttt{center\_nodes} is empty.

References

3.62 Wiener index

Functions related to the Wiener index of a graph.

\texttt{\texttt{wiener\_index}(G[, weight])} \quad \text{Returns the Wiener index of the given graph.}

3.62.1 \texttt{networkx.algorithms.wiener.wiener\_index}

\texttt{\texttt{wiener\_index}(G, weight=None)}

Returns the Wiener index of the given graph.

The \textit{Wiener index} of a graph is the sum of the shortest-path distances between each pair of reachable nodes. For pairs of nodes in undirected graphs, only one orientation of the pair is counted.

\textbf{Parameters}

- \texttt{G} (\textit{NetworkX graph})
- \texttt{weight} (\textit{object}) – The edge attribute to use as distance when computing shortest-path distances. This is passed directly to the \texttt{networkx.shortest_path_length()} function.

\textbf{Returns} The Wiener index of the graph \( G \).

\textbf{Return type} \texttt{float}

\textbf{Raises} \texttt{NetworkXError} – If the graph \( G \) is not connected.

\textbf{Notes}

If a pair of nodes is not reachable, the distance is assumed to be infinity. This means that for graphs that are not strongly-connected, this function returns \texttt{inf}.

The Wiener index is not usually defined for directed graphs, however this function uses the natural generalization of the Wiener index to directed graphs.

\textbf{Examples}

The Wiener index of the (unweighted) complete graph on \( n \) nodes equals the number of pairs of the \( n \) nodes, since each pair of nodes is at distance one:

\begin{verbatim}
>>> import networkx as nx

>>> n = 10

>>> G = nx.complete_graph(n)

>>> nx.wiener_index(G) == n * (n - 1) / 2
True

Graphs that are not strongly-connected have infinite Wiener index:

\begin{verbatim}
>>> G = nx.empty_graph(2)

>>> nx.wiener_index(G)
inf
\end{verbatim}

\end{verbatim}

3.62. Wiener index
CHAPTER
FOUR

FUNCTIONS

Functional interface to graph methods and assorted utilities.

4.1 Graph

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<tr>
<th>Function</th>
<th>Description</th>
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<td><code>degree(G[, nbunch, weight])</code></td>
<td>Returns a degree view of single node or of nbunch of nodes.</td>
</tr>
<tr>
<td><code>degree_histogram(G)</code></td>
<td>Returns a list of the frequency of each degree value.</td>
</tr>
<tr>
<td><code>density(G)</code></td>
<td>Returns the density of a graph.</td>
</tr>
<tr>
<td><code>info(G[, n])</code></td>
<td>Print short summary of information for the graph G or the node n.</td>
</tr>
<tr>
<td><code>create_empty_copy(G[, with_data])</code></td>
<td>Returns a copy of the graph G with all of the edges removed.</td>
</tr>
<tr>
<td><code>is_directed(G)</code></td>
<td>Return True if graph is directed.</td>
</tr>
<tr>
<td><code>to_directed(graph)</code></td>
<td>Returns a directed view of the graph graph.</td>
</tr>
<tr>
<td><code>to_undirected(graph)</code></td>
<td>Returns an undirected view of the graph graph.</td>
</tr>
<tr>
<td><code>is_empty(G)</code></td>
<td>Returns True if G has no edges.</td>
</tr>
<tr>
<td><code>add_star(G_to_add_to, nodes_for_star, **attr)</code></td>
<td>Add a star to Graph G_to_add_to.</td>
</tr>
<tr>
<td><code>add_path(G_to_add_to, nodes_for_path, **attr)</code></td>
<td>Add a path to the Graph G_to_add_to.</td>
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<tr>
<td><code>add_cycle(G_to_add_to, nodes_for_cycle, **attr)</code></td>
<td>Add a cycle to the Graph G_to_add_to.</td>
</tr>
<tr>
<td><code>subgraph(G, nbunch)</code></td>
<td>Returns the subgraph induced on nodes in nbunch.</td>
</tr>
<tr>
<td><code>subgraph_view(G[, filter_node, filter_edge])</code></td>
<td>View of G applying a filter on nodes and edges.</td>
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<tr>
<td><code>induced_subgraph(G, nbunch)</code></td>
<td>Returns a SubGraph view of G showing only nodes in nbunch.</td>
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<tr>
<td><code>restricted_view(G, nodes, edges)</code></td>
<td>Returns a view of G with hidden nodes and edges.</td>
</tr>
<tr>
<td><code>reverse_view(G)</code></td>
<td>View of G with edge directions reversed</td>
</tr>
<tr>
<td><code>edge_subgraph(G, edges)</code></td>
<td>Returns a view of the subgraph induced by the specified edges.</td>
</tr>
</tbody>
</table>

4.1.1 networkx.classes.function.degree

degree `(G, nbunch=None, weight=None)`

Returns a degree view of single node or of nbunch of nodes. If nbunch is omitted, then return degrees of all nodes.
4.1.2 networkx.classes.function.degree_histogram

degree_histogram(G)
Returns a list of the frequency of each degree value.

Parameters
G (Networkx graph) – A graph

Returns
hist – A list of frequencies of degrees. The degree values are the index in the list.

Return type
list

Notes
Note: the bins are width one, hence len(list) can be large (Order(number_of_edges))

4.1.3 networkx.classes.function.density

density(G)
Returns the density of a graph.

The density for undirected graphs is

\[ d = \frac{2m}{n(n - 1)} , \]

and for directed graphs is

\[ d = \frac{m}{n(n - 1)} , \]

where \( n \) is the number of nodes and \( m \) is the number of edges in \( G \).

Notes
The density is 0 for a graph without edges and 1 for a complete graph. The density of multigraphs can be higher than 1.
Self loops are counted in the total number of edges so graphs with self loops can have density higher than 1.

4.1.4 networkx.classes.function.info

info(G, n=None)
Print short summary of information for the graph G or the node n.

Parameters

• G (Networkx graph) – A graph
• n (node (any hashable)) – A node in the graph G

4.1.5 networkx.classes.function.create_empty_copy

create_empty_copy(G, with_data=True)
Returns a copy of the graph G with all of the edges removed.

Parameters
• **G** *(graph)* – A NetworkX graph

• **with_data** *(bool (default=True)) – Propagate Graph and Nodes data to the new graph.

  See also:

  `empty_graph()`

### 4.1.6 networkx.classes.function.is_directed

**is_directed**(G)

Return True if graph is directed.

### 4.1.7 networkx.classes.function.to_directed

**to_directed**(graph)

Returns a directed view of the graph `graph`.

Identical to `graph.to_directed(as_view=True)` Note that `graph.to_directed` defaults to `as_view=False` while this function always provides a view.

### 4.1.8 networkx.classes.function.to_undirected

**to_undirected**(graph)

Returns an undirected view of the graph `graph`.

Identical to `graph.to_undirected(as_view=True)` Note that `graph.to_undirected` defaults to `as_view=False` while this function always provides a view.

### 4.1.9 networkx.classes.function.is_empty

**is_empty**(G)

Returns True if `G` has no edges.

  Parameters  
  `G` *(graph)* – A NetworkX graph.

  Returns  
  True if `G` has no edges, and False otherwise.

  Return type  
  bool

**Notes**

An empty graph can have nodes but not edges. The empty graph with zero nodes is known as the null graph. This is an $O(n)$ operation where $n$ is the number of nodes in the graph.

### 4.1.10 networkx.classes.function.add_star

**add_star**(G_to_add_to, nodes_for_star, **attr)

Add a star to Graph `G_to_add_to`.

The first node in `nodes_for_star` is the middle of the star. It is connected to all other nodes.

  Parameters
• **G_to_add_to** *(graph)* – A NetworkX graph
• **nodes_for_star** *(iterable container)* – A container of nodes.
• **attr** *(keyword arguments, optional (default= no attributes))* – Attributes to add to every edge in star.

See also:

`add_path()`, `add_cycle()`

Examples

```python
>>> G = nx.Graph()
>>> nx.add_star(G, [0, 1, 2, 3])
>>> nx.add_star(G, [10, 11, 12], weight=2)
```

### 4.1.11 networkx.classes.function.add_path

**add_path** *(G_to_add_to, nodes_for_path, **attr)*

Add a path to the Graph G_to_add_to.

Parameters

• **G_to_add_to** *(graph)* – A NetworkX graph
• **nodes_for_path** *(iterable container)* – A container of nodes. A path will be constructed from the nodes (in order) and added to the graph.
• **attr** *(keyword arguments, optional (default= no attributes))* – Attributes to add to every edge in path.

See also:

`add_star()`, `add_cycle()`

Examples

```python
>>> G = nx.Graph()
>>> nx.add_path(G, [0, 1, 2, 3])
>>> nx.add_path(G, [10, 11, 12], weight=7)
```

### 4.1.12 networkx.classes.function.add_cycle

**add_cycle** *(G_to_add_to, nodes_for_cycle, **attr)*

Add a cycle to the Graph G_to_add_to.

Parameters

• **G_to_add_to** *(graph)* – A NetworkX graph
• **nodes_for_cycle** *(iterable container)* – A container of nodes. A cycle will be constructed from the nodes (in order) and added to the graph.
• **attr** *(keyword arguments, optional (default= no attributes))* – Attributes to add to every edge in cycle.
See also:

add_path(), add_star()

Examples

```python
>>> G = nx.Graph()       # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> nx.add_cycle(G, [0, 1, 2, 3])
>>> nx.add_cycle(G, [10, 11, 12], weight=7)
```

4.1.13 networkx.classes.function.subgraph

**subgraph** *(G, nbunch)*

Returns the subgraph induced on nodes in nbunch.

**Parameters**

- **G** *(graph)* – A NetworkX graph
- **nbunch** *(list, iterable)* – A container of nodes that will be iterated through once (thus it should be an iterator or be iterable). Each element of the container should be a valid node type: any hashable type except None. If nbunch is None, return all edges data in the graph. Nodes in nbunch that are not in the graph will be (quietly) ignored.

**Notes**

subgraph(G) calls G.subgraph()

4.1.14 networkx.classes.function.subgraph_view

**subgraph_view** *(G, filter_node=<function no_filter>, filter_edge=<function no_filter>)*

View of G applying a filter on nodes and edges.

**subgraph_view** provides a read-only view of the input graph that excludes nodes and edges based on the outcome of two filter functions **filter_node** and **filter_edge**.

The **filter_node** function takes one argument — the node — and returns **True** if the node should be included in the subgraph, and **False** if it should not be included.

The **filter_edge** function takes two arguments — the nodes describing an edge — and returns **True** if the edge should be included in the subgraph, and **False** if it should not be included.

Both node and edge filter functions are called on graph elements as they are queried, meaning there is no up-front cost to creating the view.

**Parameters**

- **G** *(networkx.Graph)* – A directed/undirected graph/multigraph
- **filter_node** *(callable, optional)* – A function taking a node as input, which returns **True** if the node should appear in the view.
- **filter_edge** *(callable, optional)* – A function taking as input the two nodes describing an edge, which returns **True** if the edge should appear in the view.

**Returns**  **graph** – A read-only graph view of the input graph.
Return type *networkx.Graph*

**Examples**

```python
>>> import networkx as nx
>>> G = nx.path_graph(6)
```

Filter functions operate on the node, and return `True` if the node should appear in the view:

```python
>>> def filter_node(n1):
...     return n1 != 5
...
>>> view = nx.subgraph_view(G, filter_node=filter_node)
>>> view.nodes()
NodeView((0, 1, 2, 3, 4))
```

We can use a closure pattern to filter graph elements based on additional data — for example, filtering on edge data attached to the graph:

```python
>>> G[3][4]['cross_me'] = False
>>> def filter_edge(n1, n2):
...     return G[n1][n2].get('cross_me', True)
...
>>> view = nx.subgraph_view(G, filter_edge=filter_edge)
>>> view.edges()
EdgeView([(0, 1), (1, 2), (2, 3), (4, 5)])
```

```python
>>> view = nx.subgraph_view(G, filter_node=filter_node, filter_edge=filter_edge)
>>> view.nodes()
NodeView((0, 1, 2, 3))
>>> view.edges()
EdgeView([(0, 1), (1, 2), (2, 3)])
```

### 4.1.15 `networkx.classes.function.induced_subgraph`

**induced_subgraph**(*G*, *nbunch*)

Returns a SubGraph view of *G* showing only nodes in *nbunch*.

The induced subgraph of a graph on a set of nodes *N* is the graph with nodes *N* and edges from *G* which have both ends in *N*.

**Parameters**

- *G* (*NetworkX Graph*)
- *nbunch* (*node, container of nodes or None (for all nodes)*)
Returns subgraph – A read-only view of the subgraph in \( G \) induced by the nodes. Changes to the graph \( G \) will be reflected in the view.

Return type SubGraph View

Notes

To create a mutable subgraph with its own copies of nodes edges and attributes use \( \text{subgraph.copy()} \) or \( \text{Graph(subgraph)} \)

For an inplace reduction of a graph to a subgraph you can remove nodes: \( G\).remove_nodes_from(\( n \) in \( G \) if \( n \) not in set(nbunch))

If you are going to compute subgraphs of your subgraphs you could end up with a chain of views that can be very slow once the chain has about 15 views in it. If they are all induced subgraphs, you can short-cut the chain by making them all subgraphs of the original graph. The graph class method \( G\).subgraph does this when \( G \) is a subgraph. In contrast, this function allows you to choose to build chains or not, as you wish. The returned subgraph is a view on \( G \).

Examples

```python
>>> import networkx as nx
>>> G = nx.path_graph(4)  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> H = G.subgraph([0, 1, 2])
>>> list(H.edges)
[(0, 1), (1, 2)]
```

4.1.16 networkx.classes.function.restricted_view

restricted_view \((G, \text{nodes}, \text{edges})\)

Returns a view of \( G \) with hidden nodes and edges.

The resulting subgraph filters out node \( \text{nodes} \) and edges \( \text{edges} \). Filtered out nodes also filter out any of their edges.

Parameters

- \( G \) (NetworkX Graph)
- \( \text{nodes} \) (iterable) – An iterable of nodes. Nodes not present in \( G \) are ignored.
- \( \text{edges} \) (iterable) – An iterable of edges. Edges not present in \( G \) are ignored.

Returns subgraph – A read-only restricted view of \( G \) filtering out nodes and edges. Changes to \( G \) are reflected in the view.

Return type SubGraph View

Notes

To create a mutable subgraph with its own copies of nodes edges and attributes use \( \text{subgraph.copy()} \) or \( \text{Graph(subgraph)} \)

If you create a subgraph of a subgraph recursively you may end up with a chain of subgraph views. Such chains can get quite slow for lengths near 15. To avoid long chains, try to make your subgraph based on the
original graph. We do not rule out chains programmatically so that odd cases like an edge_subgraph of a restricted_view can be created.

Examples

```python
>>> import networkx as nx
>>> G = nx.path_graph(5)
>>> H = nx.restricted_view(G, [0], [(1, 2), (3, 4)])
>>> list(H.nodes)
[1, 2, 3, 4]
>>> list(H.edges)
[(2, 3)]
```

4.1.17 networkx.classes.function.reverse_view

reverse_view(G)

View of G with edge directions reversed

reverse_view returns a read-only view of the input graph where edge directions are reversed.

Identical to digraph.reverse(copy=False)

Parameters

- G (networkx.DiGraph)

Returns graph

Return type networkx.DiGraph

Examples

```python
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_edge(1, 2)
>>> G.add_edge(2, 3)
>>> G.edges()
OutEdgeView([(1, 2), (2, 3)])

>>> view = nx.reverse_view(G)
>>> view.edges()
OutEdgeView([(2, 1), (3, 2)])
```

4.1.18 networkx.classes.function.edge_subgraph

edge_subgraph(G, edges)

Returns a view of the subgraph induced by the specified edges.

The induced subgraph contains each edge in edges and each node incident to any of those edges.

Parameters

- G (NetworkX Graph)

- edges (iterable) – An iterable of edges. Edges not present in G are ignored.
Returns subgraph – A read-only edge-induced subgraph of G. Changes to G are reflected in the view.

Return type SubGraph View

Notes

To create a mutable subgraph with its own copies of nodes edges and attributes use subgraph.copy() or Graph(subgraph)

If you create a subgraph of a subgraph recursively you can end up with a chain of subgraphs that becomes very slow with about 15 nested subgraph views. Luckily the edge_subgraph filter nests nicely so you can use the original graph as G in this function to avoid chains. We do not rule out chains programmatically so that odd cases like an edge_subgraph of a restricted_view can be created.

Examples

```python
>>> import networkx as nx
>>> G = nx.path_graph(5)
>>> H = G.edge_subgraph([(0, 1), (3, 4)])
>>> list(H.nodes)
[0, 1, 3, 4]
>>> list(H.edges)
[(0, 1), (3, 4)]
```

4.2 Nodes

<table>
<thead>
<tr>
<th>nodes(G)</th>
<th>Returns an iterator over the graph nodes.</th>
</tr>
</thead>
<tbody>
<tr>
<td>number_of_nodes(G)</td>
<td>Returns the number of nodes in the graph.</td>
</tr>
<tr>
<td>neighbors(G, n)</td>
<td>Returns a list of nodes connected to node n.</td>
</tr>
<tr>
<td>all_neighbors(graph, node)</td>
<td>Returns all of the neighbors of a node in the graph.</td>
</tr>
<tr>
<td>non_neighbors(graph, node)</td>
<td>Returns the non-neighbors of the node in the graph.</td>
</tr>
<tr>
<td>common_neighbors(G, u, v)</td>
<td>Returns the common neighbors of two nodes in a graph.</td>
</tr>
</tbody>
</table>

4.2.1 networkx.classes.function.nodes

nodes(G)

Returns an iterator over the graph nodes.

4.2.2 networkx.classes.function.number_of_nodes

number_of_nodes(G)

Returns the number of nodes in the graph.

4.2.3 networkx.classes.function.neighbors

neighbors(G, n)

Returns a list of nodes connected to node n.
4.2.4 networkx.classes.function.all_neighbors

all_neighbors (graph, node)
Returns all of the neighbors of a node in the graph.
If the graph is directed returns predecessors as well as successors.

Parameters
- graph (NetworkX graph) – Graph to find neighbors.
- node (node) – The node whose neighbors will be returned.

Returns neighbors – Iterator of neighbors
Return type iterator

4.2.5 networkx.classes.function.non_neighbors

non_neighbors (graph, node)
Returns the non-neighbors of the node in the graph.

Parameters
- graph (NetworkX graph) – Graph to find neighbors.
- node (node) – The node whose neighbors will be returned.

Returns non_neighbors – Iterator of nodes in the graph that are not neighbors of the node.
Return type iterator

4.2.6 networkx.classes.function.common_neighbors

common_neighbors (G, u, v)
Returns the common neighbors of two nodes in a graph.

Parameters
- G (graph) – A NetworkX undirected graph.
- u, v (nodes) – Nodes in the graph.

Returns cnbors – Iterator of common neighbors of u and v in the graph.
Return type iterator

Raises NetworkXError – If u or v is not a node in the graph.

Examples

```python
>>> G = nx.complete_graph(5)
>>> sorted(nx.common_neighbors(G, 0, 1))
[2, 3, 4]
```

4.3 Edges
### 4.3.1 `networkx.classes.function.edges`

edges \((G, nbunch=None)\)

Returns an edge view of edges incident to nodes in `nbunch`.

Return all edges if `nbunch` is unspecified or `nbunch=None`.

For digraphs, `edges=out_edges`

### 4.3.2 `networkx.classes.function.number_of_edges`

number_of_edges \((G)\)

Returns the number of edges in the graph.

### 4.3.3 `networkx.classes.function.non_edges`

non_edges \((graph)\)

Returns the non-existent edges in the graph.

**Parameters**

- `graph` (NetworkX graph.) – Graph to find non-existent edges.

**Returns**

- `non_edges` – Iterator of edges that are not in the graph.

**Return type** iterator

### 4.4 Self loops

#### 4.4.1 `networkx.classes.function.selfloop_edges`

selfloop_edges \((G, data=False, keys=False, default=None)\)

Returns an iterator over selfloop edges.

A selfloop edge has the same node at both ends.

**Parameters**

- `data` (string or bool, optional (default=False)) – Return selfloop edges as two tuples \((u, v)\) (data=False) or three-tuples \((u, v, datadict)\) (data=True) or three-tuples \((u, v, datavalue)\) (data='attrname')
- `keys` (bool, optional (default=False)) – If True, return edge keys with each edge.
- **default** *(value, optional (default=None)) – Value used for edges that don’t have the requested attribute. Only relevant if data is not True or False.*

  **Returns**  
  edgeiter – An iterator over all selfloop edges.

  **Return type**  
  iterator over edge tuples

  **See also:**
  
  `nodes_with_selfloops()`, `number_of_selfloops()`

**Examples**

```python
>>> G = nx.MultiGraph()  # or Graph, DiGraph, MultiDiGraph, etc
>>> ekey = G.add_edge(1, 1)
>>> ekey = G.add_edge(1, 2)
>>> list(nx.selfloop_edges(G))
[(1, 1)]
>>> list(nx.selfloop_edges(G, data=True))
[(1, 1, {})]
>>> list(nx.selfloop_edges(G, keys=True))
[(1, 1, 0)]
>>> list(nx.selfloop_edges(G, keys=True, data=True))
[(1, 1, 0, {})]
```

### 4.4.2 `networkx.classes.function.number_of_selfloops`

**number_of_selfloops** *(G)*  

Returns the number of selfloop edges.

A selfloop edge has the same node at both ends.

  **Returns** nloops – The number of selfloops.

  **Return type** int

  **See also:**
  
  `nodes_with_selfloops()`, `selfloop_edges()`

**Examples**

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge(1, 1)
>>> G.add_edge(1, 2)
>>> nx.number_of_selfloops(G)
1
```

### 4.4.3 `networkx.classes.function.nodes_with_selfloops`

**nodes_with_selfloops** *(G)*  

Returns an iterator over nodes with self loops.

A node with a self loop has an edge with both ends adjacent to that node.

  **Returns** nodelist – A iterator over nodes with self loops.
Return type: iterator

See also:

selfloop_edges(), number_of_selfloops()

Examples

```python
>>> G = nx.Graph()  # or DiGraph, MultiGraph, MultiDiGraph, etc
>>> G.add_edge(1, 1)
>>> G.add_edge(1, 2)
>>> list(nx.nodes_with_selfloops(G))
[1]
```

4.5 Attributes

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<tr>
<th>Method</th>
<th>Description</th>
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<td><code>is_weighted(G[, edge, weight])</code></td>
<td>Returns True if <code>G</code> has weighted edges.</td>
</tr>
<tr>
<td><code>is_negatively_weighted(G[, edge, weight])</code></td>
<td>Returns True if <code>G</code> has negatively weighted edges.</td>
</tr>
<tr>
<td><code>set_node_attributes(G, values[, name])</code></td>
<td>Sets node attributes from a given value or dictionary of values.</td>
</tr>
<tr>
<td><code>get_node_attributes(G, name)</code></td>
<td>Get node attributes from graph</td>
</tr>
<tr>
<td><code>set_edge_attributes(G, values[, name])</code></td>
<td>Sets edge attributes from a given value or dictionary of values.</td>
</tr>
<tr>
<td><code>get_edge_attributes(G, name)</code></td>
<td>Get edge attributes from graph</td>
</tr>
</tbody>
</table>

4.5.1 networkx.classes.function.is_weighted

`is_weighted(G, edge=None, weight='weight')`

Returns True if `G` has weighted edges.

Parameters

- G (graph) – A NetworkX graph.
- edge (tuple, optional) – A 2-tuple specifying the only edge in `G` that will be tested. If None, then every edge in `G` is tested.
- weight (string, optional) – The attribute name used to query for edge weights.

Returns A boolean signifying if `G`, or the specified edge, is weighted.

Return type: bool

Raises `NetworkXError` – If the specified edge does not exist.

Examples

```python
>>> G = nx.path_graph(4)
>>> nx.is_weighted(G)
False
>>> nx.is_weighted(G, (2, 3))
False
```
```python
>>> G = nx.DiGraph()
>>> G.add_edge(1, 2, weight=1)
>>> nx.is_weighted(G)
True
```

### 4.5.2 networkx.classes.function.is_negatively_weighted

**is_negatively_weighted** *(G, edge=None, weight='weight')*

Returns True if `G` has negatively weighted edges.

**Parameters**

- **G** *(graph)* – A NetworkX graph.
- **edge** *(tuple, optional)* – A 2-tuple specifying the only edge in `G` that will be tested. If None, then every edge in `G` is tested.
- **weight** *(string, optional)* – The attribute name used to query for edge weights.

**Returns**

A boolean signifying if `G`, or the specified edge, is negatively weighted.

**Return type** `bool`

**Raises** *NetworkXError* – If the specified edge does not exist.

#### Examples

```python
>>> G = nx.Graph()
>>> G.add_edges_from([(1, 3), (2, 4), (2, 6)])
>>> G.add_edge(1, 2, weight=4)
>>> nx.is_negatively_weighted(G, (1, 2))
False
>>> G[2][4]['weight'] = -2
>>> nx.is_negatively_weighted(G)
True
>>> G = nx.DiGraph()
>>> edges = [('0', '3', 3), ('0', '1', -5), ('1', '0', -2)]
>>> G.add_weighted_edges_from(edges)
>>> nx.is_negatively_weighted(G)
True
```

### 4.5.3 networkx.classes.function.set_node_attributes

**set_node_attributes** *(G, values, name=None)*

Sets node attributes from a given value or dictionary of values.

**Warning:** The call order of arguments `values` and `name` switched between v1.x & v2.x.

**Parameters**

- **G** *(NetworkX Graph)*
• **values** *(scalar value, dict-like)* – What the node attribute should be set to. If `values` is not a dictionary, then it is treated as a single attribute value that is then applied to every node in `G`. This means that if you provide a mutable object, like a list, updates to that object will be reflected in the node attribute for every node. The attribute name will be `name`.

If `values` is a dict or a dict of dict, it should be keyed by node to either an attribute value or a dict of attribute key/value pairs used to update the node’s attributes.

• **name** *(string (optional, default=None))* – Name of the node attribute to set if `values` is a scalar.

### Examples

After computing some property of the nodes of a graph, you may want to assign a node attribute to store the value of that property for each node:

```python
>>> G = nx.path_graph(3)
>>> bb = nx.betweenness_centrality(G)
>>> isinstance(bb, dict)
True
>>> nx.set_node_attributes(G, bb, 'betweenness')
>>> G.nodes[1]['betweenness']
1.0
```

If you provide a list as the second argument, updates to the list will be reflected in the node attribute for each node:

```python
>>> G = nx.path_graph(3)
>>> labels = []
>>> nx.set_node_attributes(G, labels, 'labels')
>>> labels.append('foo')
>>> G.nodes[0]['labels']
['foo']
>>> G.nodes[1]['labels']
['foo']
>>> G.nodes[2]['labels']
['foo']
```

If you provide a dictionary of dictionaries as the second argument, the outer dictionary is assumed to be keyed by node to an inner dictionary of node attributes for that node:

```python
>>> G = nx.path_graph(3)
>>> attrs = {0: {'attr1': 20, 'attr2': 'nothing'}, 1: {'attr2': 3}}
>>> nx.set_node_attributes(G, attrs)
>>> G.nodes[0]['attr1']
20
>>> G.nodes[0]['attr2']
'nothing'
>>> G.nodes[1]['attr2']
3
>>> G.nodes[2]
[]
```
4.5.4 networkx.classes.function.get_node_attributes

get_node_attributes \((G, \text{name})\)
Get node attributes from graph

Parameters

- \(G\) (\text{NetworkX Graph})
- \text{name} (\text{string}) – Attribute name

Returns

Return type Dictionary of attributes keyed by node.

Examples

```python
>>> G = nx.Graph()
>>> G.add_nodes_from([1, 2, 3], color='red')
>>> color = nx.get_node_attributes(G, 'color')
>>> color[1]
'red'
```

4.5.5 networkx.classes.function.set_edge_attributes

set_edge_attributes \((G, \text{values}, \text{name}={None})\)
Sets edge attributes from a given value or dictionary of values.

Warning: The call order of arguments \text{values} and \text{name} switched between v1.x & v2.x.

Parameters

- \(G\) (\text{NetworkX Graph})
- \text{values} (scalar value, dict-like) – What the edge attribute should be set to. If \text{values} is not a dictionary, then it is treated as a single attribute value that is then applied to every edge in \(G\). This means that if you provide a mutable object, like a list, updates to that object will be reflected in the edge attribute for each edge. The attribute name will be \text{name}.
- \text{name} (\text{string} (optional, default=None)) – Name of the edge attribute to set if \text{values} is a scalar.

Examples

After computing some property of the edges of a graph, you may want to assign an attribute to store the value of that property for each edge:
If you provide a list as the second argument, updates to the list will be reflected in the edge attribute for each edge:

```python
>>> labels = []
>>> nx.set_edge_attributes(G, labels, 'labels')
>>> labels.append('foo')
>>> G.edges[0, 1]['labels']
['foo']
>>> G.edges[1, 2]['labels']
['foo']
```

If you provide a dictionary of dictionaries as the second argument, the entire dictionary will be used to update edge attributes:

```python
>>> G = nx.path_graph(3)
>>> attrs = {(0, 1): {'attr1': 20, 'attr2': 'nothing'},
           (1, 2): {'attr2': 3}}
>>> nx.set_edge_attributes(G, attrs)
>>> G[0][1]['attr1']
20
>>> G[0][1]['attr2']
'nothing'
>>> G[1][2]['attr2']
3
```

### 4.5.6 networkx.classes.function.get_edge_attributes

**get_edge_attributes** *(G, name)*

Get edge attributes from graph

**Parameters**

- G *(NetworkX Graph)*
- name *(string)* – Attribute name

**Returns**

- Dictionary of attributes keyed by edge. For (di)graphs, the keys are 2-tuples of the form 
  \((u, v)\). For multi(di)graphs, the keys are 3-tuples of the form 
  \((u, v, key)\).

**Examples**

```python
>>> G = nx.Graph()
>>> nx.add_path(G, [1, 2, 3], color='red')
>>> color = nx.get_edge_attributes(G, 'color')
>>> color[(1, 2)]
'red'
```
4.6 Freezing graph structure

<table>
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<tr>
<th>Function</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td><code>freeze(G)</code></td>
<td>Modify graph to prevent further change by adding or removing nodes or edges.</td>
</tr>
<tr>
<td><code>is_frozen(G)</code></td>
<td>Returns True if graph is frozen.</td>
</tr>
</tbody>
</table>

4.6.1 networkx.classes.function.freeze

`freeze(G)`
Modify graph to prevent further change by adding or removing nodes or edges.

Node and edge data can still be modified.

**Parameters**

- `G (graph)` – A NetworkX graph

**Examples**

```python
>>> G = nx.path_graph(4)
>>> G = nx.freeze(G)
>>> try:
...     G.add_edge(4, 5)
... except nx.NetworkXError as e:
...     print(str(e))
Frozen graph can't be modified
```

**Notes**

To “unfreeze” a graph you must make a copy by creating a new graph object:

```python
>>> graph = nx.path_graph(4)
>>> frozen_graph = nx.freeze(graph)
>>> unfrozen_graph = nx.Graph(frozen_graph)
>>> nx.is_frozen(unfrozen_graph)
False
```

See also:

- `is_frozen()`

4.6.2 networkx.classes.function.is_frozen

`is_frozen(G)`
Returns True if graph is frozen.

**Parameters**

- `G (graph)` – A NetworkX graph

See also:

- `freeze()`
5.1 Atlas

Generators for the small graph atlas.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>graph_atlas(i)</td>
<td>Returns graph number ( i ) from the Graph Atlas.</td>
</tr>
<tr>
<td>graph_atlas_g()</td>
<td>Returns the list of all graphs with up to seven nodes named in the Graph Atlas.</td>
</tr>
</tbody>
</table>

5.1.1 networkx.generators.atlas.graph_atlas
def graph_atlas(i):
    
    Returns graph number \( i \) from the Graph Atlas.

    For more information, see `graph_atlas_g()`.

    Parameters i (int) – The index of the graph from the atlas to get. The graph at index 0 is assumed to be the null graph.

    Returns A list of `Graph` objects, the one at index \( i \) corresponding to the graph \( i \) in the Graph Atlas.

    Return type list

    See also:
    `graph_atlas_g()`

    Notes

    The time required by this function increases linearly with the argument \( i \), since it reads a large file sequentially in order to generate the graph\(^1\).

    References

5.1.2 networkx.generators.atlas.graph_atlas_g
def graph_atlas_g():
    
    Returns the list of all graphs with up to seven nodes named in the Graph Atlas.

    The graphs are listed in increasing order by

1. number of nodes,
2. number of edges,
3. degree sequence (for example 111223 < 112222),
4. number of automorphisms,

in that order, with three exceptions as described in the Notes section below. This causes the list to correspond with the index of the graphs in the Graph Atlas [atlas], with the first graph, \( G[0] \), being the null graph.

**Returns** A list of `Graph` objects, the one at index \( i \) corresponding to the graph \( i \) in the Graph Atlas.

**Return type** list

**See also:**

`graph_atlas()`

**Notes**

This function may be expensive in both time and space, since it reads a large file sequentially in order to populate the list.

Although the NetworkX atlas functions match the order of graphs given in the “Atlas of Graphs” book, there are (at least) three errors in the ordering described in the book. The following three pairs of nodes violate the lexicographically nondecreasing sorted degree sequence rule:

- graphs 55 and 56 with degree sequences 001111 and 000112,
- graphs 1007 and 1008 with degree sequences 333444 and 333336,
- graphs 1012 and 1213 with degree sequences 124555 and 124456.

**References**

### 5.2 Classic

Generators for some classic graphs.

The typical graph generator is called as follows:

```python
>>> G = nx.complete_graph(100)
```

returning the complete graph on \( n \) nodes labeled 0, .., 99 as a simple graph. Except for `empty_graph`, all the generators in this module return a `Graph` class (i.e. a simple, undirected graph).

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>balanced_tree(r, h[, create_using])</code></td>
<td>Returns the perfectly balanced ( r )-ary tree of height ( h ).</td>
</tr>
<tr>
<td><code>barbell_graph(m1, m2[, create_using])</code></td>
<td>Returns the Barbell Graph: two complete graphs connected by a path.</td>
</tr>
<tr>
<td><code>binomial_tree(n)</code></td>
<td>Returns the Binomial Tree of order ( n ).</td>
</tr>
<tr>
<td><code>complete_graph(n[, create_using])</code></td>
<td>Returns the complete graph ( K_n ) with ( n ) nodes.</td>
</tr>
<tr>
<td><code>complete_multipartite_graph(*subset_sizes)</code></td>
<td>Returns the complete multipartite graph with the specified subset sizes.</td>
</tr>
<tr>
<td><code>circular_ladder_graph(n[, create_using])</code></td>
<td>Returns the circular ladder graph ( C_L_n ) of length ( n ).</td>
</tr>
<tr>
<td><code>circulant_graph(n, offsets[, create_using])</code></td>
<td>Generates the circulant graph ( C_i_n(x_1, x_2, ..., x_m) ) with ( n ) vertices.</td>
</tr>
</tbody>
</table>

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</thead>
<tbody>
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<td><code>cycle_graph(n[, create_using])</code></td>
<td>Returns the cycle graph $C_n$ of cyclically connected nodes.</td>
</tr>
<tr>
<td><code>dorogovtsev_goltsev_mendes_graph(n[, ...])</code></td>
<td>Returns the hierarchically constructed Dorogovtsev-Goltsev-Mendes graph.</td>
</tr>
<tr>
<td><code>empty_graph([n, create_using, default])</code></td>
<td>Returns the empty graph with $n$ nodes and zero edges.</td>
</tr>
<tr>
<td><code>full_rary_tree(r, n[, create_using])</code></td>
<td>Creates a full $r$-ary tree of $n$ vertices.</td>
</tr>
<tr>
<td><code>ladder_graph(n[, create_using])</code></td>
<td>Returns the Ladder graph of length $n$.</td>
</tr>
<tr>
<td><code>lollipop_graph(m, n[, create_using])</code></td>
<td>Returns the Lollipop Graph; $K_m$ connected to $P_n$.</td>
</tr>
<tr>
<td><code>null_graph([create_using])</code></td>
<td>Returns the Null graph with no nodes or edges.</td>
</tr>
<tr>
<td><code>path_graph(n[, create_using])</code></td>
<td>Returns the Path graph $P_n$ of linearly connected nodes.</td>
</tr>
<tr>
<td><code>star_graph(n[, create_using])</code></td>
<td>Return the star graph</td>
</tr>
<tr>
<td><code>trivial_graph([create_using])</code></td>
<td>Return the Trivial graph with one node (with label 0) and no edges.</td>
</tr>
<tr>
<td><code>turan_graph(n[, create_using])</code></td>
<td>Return the Turan Graph</td>
</tr>
<tr>
<td><code>wheel_graph(n, r)</code></td>
<td>Return the wheel graph</td>
</tr>
</tbody>
</table>

5.2.1 networkx.generators.classic.balanced_tree

`balanced_tree (r, h, create_using=None)`

Returns the perfectly balanced $r$-ary tree of height $h$.

**Parameters**
- $r$ (**int**) – Branching factor of the tree; each node will have $r$ children.
- $h$ (**int**) – Height of the tree.
- `create_using` (**NetworkX graph constructor, optional (default=nx.Graph)**) – Graph type to create. If graph instance, then cleared before populated.

**Returns**
- $G$ – A balanced $r$-ary tree of height $h$.

**Return type**
- NetworkX graph

**Notes**

This is the rooted tree where all leaves are at distance $h$ from the root. The root has degree $r$ and all other internal nodes have degree $r + 1$.

Node labels are integers, starting from zero.

A balanced tree is also known as a complete $r$-ary tree.

5.2.2 networkx.generators.classic.barbell_graph

`barbell_graph (m1, m2, create_using=None)`

Returns the Barbell Graph: two complete graphs connected by a path.

For $m1 > 1$ and $m2 >= 0$.

Two identical complete graphs $K_{m1}$ form the left and right bells, and are connected by a path $P_{m2}$.

**The 2*m1+m2 nodes are numbered** 0, ..., $m1-1$ for the left barbell, $m1$, ..., $m1+m2-1$ for the path, and $m1+m2$, ..., $2*m1+m2-1$ for the right barbell.
The 3 subgraphs are joined via the edges \((m1-1, m1)\) and \((m1+m2-1, m1+m2)\). If \(m2=0\), this is merely two complete graphs joined together.

This graph is an extremal example in David Aldous and Jim Fill’s e-text on Random Walks on Graphs.

5.2.3 networkx.generators.classic.binomial_tree

binomial_tree\((n)\)

Returns the Binomial Tree of order \(n\).

The binomial tree of order 0 consists of a single vertex. A binomial tree of order \(k\) is defined recursively by linking two binomial trees of order \(k-1\): the root of one is the leftmost child of the root of the other.

Parameters  
\(n\) (int) – Order of the binomial tree.

Returns  
\(G\) – A binomial tree of \(2^n\) vertices and \(2^n - 1\) edges.

Return type  
NetworkX graph

5.2.4 networkx.generators.classic.complete_graph

complete_graph\((n, create_using=None)\)

Return the complete graph \(K_n\) with \(n\) nodes.

Parameters

- \(n\) (int or iterable container of nodes) – If \(n\) is an integer, nodes are from \(\text{range}(n)\). If \(n\) is a container of nodes, those nodes appear in the graph.

- create_using (NetworkX graph constructor, optional (default=nx.Graph)) – Graph type to create. If graph instance, then cleared before populated.

Examples

```python
>>> G = nx.complete_graph(9)
>>> len(G)
9
>>> G.size()
36
>>> G = nx.complete_graph(range(11, 14))
>>> list(G.nodes())
[11, 12, 13]
>>> G = nx.complete_graph(4, nx.DiGraph())
>>> G.is_directed()
True
```

5.2.5 networkx.generators.classic.complete_multipartite_graph

complete_multipartite_graph\((*subset_sizes)\)

Returns the complete multipartite graph with the specified subset sizes.

Parameters  
subset_sizes (tuple of integers or tuple of node iterables) – The arguments can either all be integer number of nodes or they can all be iterables of nodes. If integers, they represent the number of vertices in each subset of the multipartite graph. If iterables, each is used to create the nodes for that subset. The length of subset_sizes is the number of subsets.
Returns

G – Returns the complete multipartite graph with the specified subsets.

For each node, the node attribute 'subset' is an integer indicating which subset contains the node.

Return type NetworkX Graph

Examples

Creating a complete tripartite graph, with subsets of one, two, and three vertices, respectively.

```python
>>> import networkx as nx
>>> G = nx.complete_multipartite_graph(1, 2, 3)
>>> [G.nodes[u]['subset'] for u in G]
[0, 1, 1, 2, 2, 2]
>>> list(G.edges(0))
[(0, 1), (0, 2), (0, 3), (0, 4), (0, 5)]
>>> list(G.edges(2))
[(2, 0), (2, 3), (2, 4), (2, 5)]
>>> list(G.edges(4))
[(4, 0), (4, 1), (4, 2)]
```

```python
>>> G = nx.complete_multipartite_graph('a', 'bc', 'def')
>>> [G.nodes[u]['subset'] for u in sorted(G)]
[0, 1, 1, 2, 2, 2]
```

Notes

This function generalizes several other graph generator functions.

- If no subset sizes are given, this returns the null graph.
- If a single subset size $n$ is given, this returns the empty graph on $n$ nodes.
- If two subset sizes $m$ and $n$ are given, this returns the complete bipartite graph on $m + n$ nodes.
- If subset sizes 1 and $n$ are given, this returns the star graph on $n + 1$ nodes.

See also:

- `complete_bipartite_graph()`

5.2.6 networkx.generators.classic.circular_ladder_graph

circular_ladder_graph ($n$, create_using=None)

Returns the circular ladder graph $CL_n$ of length $n$.

$CL_n$ consists of two concentric $n$-cycles in which each of the $n$ pairs of concentric nodes are joined by an edge.

Node labels are the integers 0 to $n-1$

5.2.7 networkx.generators.classic.circulant_graph

circulant_graph ($n$, offsets, create_using=None)

Generates the circulant graph $Ci_n(x_1, x_2, ..., x_m)$ with $n$ vertices.
Returns

- The graph :math:`G_{i_n(x_1, \ldots, x_m)}` consisting of :math:`n` vertices :math:`0, \ldots, n-1` such that the vertex with label :math:`i` is connected to the vertices labelled :math:`(i + x)` and :math:`(i - x)`, for all :math:`x` in :math:`x_1` up to :math:`x_m`, with the indices taken modulo :math:`n`.

Parameters

- **n** (*integer*) – The number of vertices the generated graph is to contain.
- **offsets** (*list of integers*) – A list of vertex offsets, :math:`x_1` up to :math:`x_m`, as described above.
- **create_using** (*NetworkX graph constructor, optional (default=nx.Graph]*) – Graph type to create. If graph instance, then cleared before populated.

Examples

Many well-known graph families are subfamilies of the circulant graphs; for example, to generate the cycle graph on :math:`n` points, we connect every vertex to every other at offset plus or minus one. For :math:`n = 10`,

```python
>>> import networkx

>>> G = networkx.generators.classic.circulant_graph(10, [1])

>>> edges = [
... (0, 9), (0, 1), (1, 2), (2, 3), (3, 4),
... (4, 5), (5, 6), (6, 7), (7, 8), (8, 9)]

>>> sorted(edges) == sorted(G.edges())
True
```

Similarly, we can generate the complete graph on 5 points with the set of offsets [1, 2]:

```python
>>> G = networkx.generators.classic.circulant_graph(5, [1, 2])

>>> edges = [
... (0, 1), (0, 2), (0, 3), (0, 4), (1, 2),
... (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)]

>>> sorted(edges) == sorted(G.edges())
True
```

5.2.8 networkx.generators.classic.cycle_graph

cycle_graph(*n*, create_using=None)

Returns the cycle graph :math:`C_n` of cyclically connected nodes.

:math:`C_n` is a path with its two end-nodes connected.

Parameters

- **n** (*int or iterable container of nodes*) – If :math:`n` is an integer, nodes are from :math:`\text{range}(n)`. If :math:`n` is a container of nodes, those nodes appear in the graph.
- **create_using** (*NetworkX graph constructor, optional (default=nx.Graph]*) – Graph type to create. If graph instance, then cleared before populated.
Notes

If create_using is directed, the direction is in increasing order.

5.2.9 networkx.generators.classic.dorogovtsev_goltsev_mendes_graph

dorogovtsev_goltsev_mendes_graph (n, create_using=None)

Returns the hierarchically constructed Dorogovtsev-Goltsev-Mendes graph.

n is the generation. See: arXiv:/cond-mat/0112143 by Dorogovtsev, Goltsev and Mendes.

5.2.10 networkx.generators.classic.empty_graph

empty_graph (n=0, create_using=None, default=<class 'networkx.classes.graph.Graph'>)

Returns the empty graph with n nodes and zero edges.

Parameters

- n (int or iterable container of nodes (default = 0)) – If n is an integer, nodes are from range(n). If n is a container of nodes, those nodes appear in the graph.
- create_using (Graph Instance, Constructor or None) – Indicator of type of graph to return. If a Graph-type instance, then clear and use it. If None, use the default constructor. If a constructor, call it to create an empty graph.
- default (Graph constructor (optional, default = nx.Graph)) – The constructor to use if create_using is None. If None, then nx.Graph is used. This is used when passing an unknown create_using value through your home-grown function to empty_graph and you want a default constructor other than nx.Graph.

Examples

```python
>>> G = nx.empty_graph(10)
>>> G.number_of_nodes()
10
>>> G.number_of_edges()
0
>>> G = nx.empty_graph("ABC")
>>> G.number_of_nodes()
3
>>> sorted(G)
['A', 'B', 'C']
```

Notes

The variable create_using should be a Graph Constructor or a “graph”-like object. Constructors, e.g. nx.Graph or nx.MultiGraph will be used to create the returned graph. “graph”-like objects will be cleared (nodes and edges will be removed) and refitted as an empty “graph” with nodes specified in n. This capability is useful for specifying the class-nature of the resulting empty “graph” (i.e. Graph, DiGraph, MyWeirdGraphClass, etc.).

The variable create_using has three main uses: Firstly, the variable create_using can be used to create an empty digraph, multigraph, etc. For example,
will create an empty digraph on n nodes.

Secondly, one can pass an existing graph (digraph, multigraph, etc.) via create_using. For example, if G is an existing graph (resp. digraph, multigraph, etc.), then empty_graph(n, create_using=G) will empty G (i.e. delete all nodes and edges using G.clear()) and then add n nodes and zero edges, and return the modified graph.

Thirdly, when constructing your home-grown graph creation function you can use empty_graph to construct the graph by passing a user defined create_using to empty_graph. In this case, if you want the default constructor to be other than nx.Graph, specify default.

```python
>>> def mygraph(n, create_using=None):
...     G = nx.empty_graph(n, create_using, nx.MultiGraph)
...     G.add_edges_from([(0, 1), (0, 1)])
...     return G

>>> G = mygraph(3)
>>> G.is_multigraph()
True
>>> G = mygraph(3, nx.Graph)
>>> G.is_multigraph()
False
```

See also create_empty_copy(G).

### 5.2.11 networkx.generators.classic.full_rary_tree

`full_rary_tree(r, n, create_using=None)`

Creates a full r-ary tree of n vertices.

Sometimes called a k-ary, n-ary, or m-ary tree. “... all non-leaf vertices have exactly r children and all levels are full except for some rightmost position of the bottom level (if a leaf at the bottom level is missing, then so are all of the leaves to its right.”

Parameters

- `r` (int) – branching factor of the tree
- `n` (int) – Number of nodes in the tree
- `create_using` (NetworkX graph constructor, optional (default=nx.Graph)) – Graph type to create. If graph instance, then cleared before populated.

Returns G – An r-ary tree with n nodes

Return type networkx Graph

References

### 5.2.12 networkx.generators.classic.ladder_graph

`ladder_graph(n, create_using=None)`

Returns the Ladder graph of length n.

This is two paths of n nodes, with each pair connected by a single edge.

---

1 An introduction to data structures and algorithms, James Andrew Storer, Birkhauser Boston 2001, (page 225).
Node labels are the integers 0 to \( 2^n - 1 \).

### 5.2.13 networkx.generators.classic.lollipop_graph

**lollipop_graph** \((m, n, create_using=None)\)

Returns the Lollipop Graph; \( K_m \) connected to \( P_n \).

This is the Barbell Graph without the right barbell.

**Parameters**

- **m, n** (int or iterable container of nodes (default = 0)) – If an integer, nodes are from \( \text{range}(m) \) and \( \text{range}(m,m+n) \). If a container, the entries are the coordinate of the node.

  The nodes for \( m \) appear in the complete graph \( K_m \) and the nodes for \( n \) appear in the path \( P_n \).

- **create_using** (NetworkX graph constructor, optional (default=nx.Graph)) – Graph type to create. If graph instance, then cleared before populated.

**Notes**

The 2 subgraphs are joined via an edge \((m-1, m)\). If \( n=0 \), this is merely a complete graph.

(This graph is an extremal example in David Aldous and Jim Fill’s etext on Random Walks on Graphs.)

### 5.2.14 networkx.generators.classic.null_graph

**null_graph** (create_using=None)

Returns the Null graph with no nodes or edges.

See empty_graph for the use of create_using.

### 5.2.15 networkx.generators.classic.path_graph

**path_graph** \((n, create_using=None)\)

Returns the Path graph \( P_n \) of linearly connected nodes.

**Parameters**

- **n** (int or iterable) – If an integer, node labels are 0 to \( n \) with center 0. If an iterable of nodes, the center is the first.

- **create_using** (NetworkX graph constructor, optional (default=nx.Graph)) – Graph type to create. If graph instance, then cleared before populated.

### 5.2.16 networkx.generators.classic.star_graph

**star_graph** \((n, create_using=None)\)

Return the star graph

The star graph consists of one center node connected to \( n \) outer nodes.

**Parameters**
• **n** (*int or iterable*) – If an integer, node labels are 0 to n with center 0. If an iterable of nodes, the center is the first.

• **create_using** (*NetworkX graph constructor, optional (default=nx.Graph]*) – Graph type to create. If graph instance, then cleared before populated.

**Notes**

The graph has n+1 nodes for integer n. So star_graph(3) is the same as star_graph(range(4)).

### 5.2.17 networkx.generators.classic.trivial_graph

**trivial_graph** (*create_using=None*)

Return the Trivial graph with one node (with label 0) and no edges.

### 5.2.18 networkx.generators.classic.turan_graph

**turan_graph** (*n, r*)

Return the Turan Graph

The Turan Graph is a complete multipartite graph on *n* vertices with *r* disjoint subsets. It is the graph with the edges for any graph with *n* vertices and *r* disjoint subsets.

Given *n* and *r*, we generate a complete multipartite graph with \((r - (n \mod r))\) partitions of size \(n/r\), rounded down, and \(n \mod r\) partitions of size \(n/r + 1\), rounded down.

#### Parameters

• **n** (*int*) – The number of vertices.

• **r** (*int*) – The number of partitions. Must be less than or equal to *n*.

**Notes**

Must satisfy \(1 <= r <= n\). The graph has \((r - 1)(n^2)/(2r)\) edges, rounded down.

### 5.2.19 networkx.generators.classic.wheel_graph

**wheel_graph** (*n, create_using=None*)

Return the wheel graph

The wheel graph consists of a hub node connected to a cycle of \((n-1)\) nodes.

#### Parameters

• **n** (*int or iterable*) – If an integer, node labels are 0 to n with center 0. If an iterable of nodes, the center is the first.

• **create_using** (*NetworkX graph constructor, optional (default=nx.Graph]*) – Graph type to create. If graph instance, then cleared before populated.

• **Node labels are the integers 0 to n - 1**.
5.3 Expanders

Provides explicit constructions of expander graphs.

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<thead>
<tr>
<th>NetworkX Reference, Release 2.5rc1.dev20191214022633</th>
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<td><strong>5.3 Expanders</strong></td>
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<td>Provides explicit constructions of expander graphs.</td>
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5.4 Lattice

Functions for generating grid graphs and lattices

The `grid_2d_graph()`, `triangular_lattice_graph()`, and `hexagonal_lattice_graph()` functions correspond to the three regular tilings of the plane, the square, triangular, and hexagonal tilings, respectively. `grid_graph()` and `hypercube_graph()` are similar for arbitrary dimensions. Useful relevant discussion can be found about Triangular Tiling, and Square, Hex and Triangle Grids.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
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<tbody>
<tr>
<td><code>grid_2d_graph(m, n[, periodic, create_using])</code></td>
<td>Returns the two-dimensional grid graph.</td>
</tr>
<tr>
<td><code>grid_graph(dim[, periodic])</code></td>
<td>Returns the n-dimensional grid graph.</td>
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<tr>
<td><code>hexagonal_lattice_graph(m, n[, periodic,...])</code></td>
<td>Returns an m by n hexagonal lattice graph.</td>
</tr>
<tr>
<td><code>hypercube_graph(n)</code></td>
<td>Returns the n-dimensional hypercube graph.</td>
</tr>
<tr>
<td><code>triangular_lattice_graph(m, n[, periodic,...])</code></td>
<td>Returns the m by n triangular lattice graph.</td>
</tr>
</tbody>
</table>

5.4.1 networkx.generators.lattice.grid_2d_graph

`grid_2d_graph(m, n, periodic=False, create_using=None)`

Returns the two-dimensional grid graph.

The grid graph has each node connected to its four nearest neighbors.

Parameters

- `m, n (int or iterable container of nodes)` – If an integer, nodes are from `range(n)`. If a container, elements become the coordinate of the nodes.
- `periodic (bool (default: False))` – If this is `True` the nodes on the grid boundaries are joined to the corresponding nodes on the opposite grid boundaries.
- `create_using (NetworkX graph constructor, optional (default=nx.Graph))` – Graph type to create. If graph instance, then cleared before populated.

Returns The (possibly periodic) grid graph of the specified dimensions.

Return type NetworkX graph

5.4.2 networkx.generators.lattice.grid_graph

`grid_graph(dim, periodic=False)`

Returns the n-dimensional grid graph.

The dimension n is the length of the list `dim` and the size in each dimension is the value of the corresponding list element.

Parameters

- `dim (list or tuple of numbers or iterables of nodes)` – ‘dim’ is a tuple or list with, for each dimension, either a number that is the size of that dimension or an iterable of nodes for that dimension. The dimension of the grid_graph is the length of `dim`. 
• periodic (bool) – If periodic is True the nodes on the grid boundaries are joined to the corresponding nodes on the opposite grid boundaries.

Returns The (possibly periodic) grid graph of the specified dimensions.

Return type NetworkX graph

Examples

To produce a 2 by 3 by 4 grid graph, a graph on 24 nodes:

```python
>>> from networkx import grid_graph
>>> G = grid_graph(dim=[2, 3, 4])
>>> len(G)
24
>>> G = grid_graph(dim=[range(7, 9), range(3, 6)])
>>> len(G)
6
```

5.4.3 networkx.generators.lattice.hexagonal_lattice_graph

hexagonal_lattice_graph (m, n, periodic=False, with_positions=True, create_using=None)

Returns an m by n hexagonal lattice graph.

The hexagonal lattice graph is a graph whose nodes and edges are the hexagonal tiling of the plane.

The returned graph will have m rows and n columns of hexagons. Odd numbered columns are shifted up relative to even numbered columns.

Positions of nodes are computed by default or with_positions is True. Node positions creating the standard embedding in the plane with sidelength 1 and are stored in the node attribute ‘pos’. pos = nx.get_node_attributes(G, 'pos') creates a dict ready for drawing.

Parameters

• m (int) – The number of rows of hexagons in the lattice.

• n (int) – The number of columns of hexagons in the lattice.

• periodic (bool) – Whether to make a periodic grid by joining the boundary vertices. For this to work n must be odd and both n > 1 and m > 1. The periodic connections create another row and column of hexagons so these graphs have fewer nodes as boundary nodes are identified.

• with_positions (bool (default: True)) – Store the coordinates of each node in the graph node attribute ‘pos’. The coordinates provide a lattice with vertical columns of hexagons offset to interleave and cover the plane. Periodic positions shift the nodes vertically in a nonlinear way so the edges don’t overlap so much.

• create_using (NetworkX graph constructor, optional (default=nx.Graph)) – Graph type to create. If graph instance, then cleared before populated. If graph is directed, edges will point up or right.

Returns The m by n hexagonal lattice graph.

Return type NetworkX graph

5.4. Lattice 545
5.4.4 networkx.generators.lattice.hypercube_graph

hypercube_graph \((n)\)
Returns the \(n\)-dimensional hypercube graph.

The nodes are the integers between 0 and \(2^n - 1\), inclusive.

For more information on the hypercube graph, see the Wikipedia article Hypercube graph.

Parameters  
\(n\) (int) – The dimension of the hypercube. The number of nodes in the graph will be \(2^n\).

Returns  
The hypercube graph of dimension \(n\).

Return type  
NetworkX graph

5.4.5 networkx.generators.lattice.triangular_lattice_graph

triangular_lattice_graph \((m, n, periodic=False, with_positions=True, create_using=None)\)
Returns the \(m\) by \(n\) triangular lattice graph.

The triangular lattice graph is a two-dimensional grid graph in which each square unit has a diagonal edge (each grid unit has a chord).

The returned graph has \(m\) rows and \(n\) columns of triangles. Rows and columns include both triangles pointing up and down. Rows form a strip of constant height. Columns form a series of diamond shapes, staggered with the columns on either side. Another way to state the size is that the nodes form a grid of \(m+1\) rows and \((n + 1) / 2\) columns. The odd row nodes are shifted horizontally relative to the even rows.

Directed graph types have edges pointed up or right.
Positions of nodes are computed by default or with_positions is True. The position of each node (embedded in a euclidean plane) is stored in the graph using equilateral triangles with sidelength 1. The height between rows of nodes is thus \(\sqrt{3}/2\). Nodes lie in the first quadrant with the node \((0, 0)\) at the origin.

Parameters  
- \(m\) (int) – The number of rows in the lattice.
- \(n\) (int) – The number of columns in the lattice.
- periodic (bool (default: False)) – If True, join the boundary vertices of the grid using periodic boundary conditions. The join between boundaries is the final row and column of triangles. This means there is one row and one column fewer nodes for the periodic lattice. Periodic lattices require \(m \geq 3, n \geq 5\) and are allowed but misaligned if \(m\) or \(n\) are odd
- with_positions (bool (default: True)) – Store the coordinates of each node in the graph node attribute ‘pos’. The coordinates provide a lattice with equilateral triangles. Periodic positions shift the nodes vertically in a nonlinear way so the edges don’t overlap so much.
- create_using (NetworkX graph constructor, optional (default=nx.Graph)) – Graph type to create. If graph instance, then cleared before populated.

Returns  
The \(m\) by \(n\) triangular lattice graph.

Return type  
NetworkX graph

5.5 Small

Various small and named graphs, together with some compact generators.
5.5.1 `networkx.generators.small.make_small_graph`

`make_small_graph(graph_description[, ...])`  
Return the small graph described by `graph_description`.  

`graph_description` is a list of the form `[ltype, name, n, xlist]`  
Here `ltype` is one of “adjacencylist” or “edgelist”, `name` is the name of the graph and `n` the number of nodes. This constructs a graph of `n` nodes with integer labels 0...n-1.

If `ltype=“adjacencylist”` then `xlist` is an adjacency list with exactly `n` entries, in with the `j`’th entry (which can be empty) specifies the nodes connected to vertex `j`. e.g. the “square” graph C_4 can be obtained by

```python
>>> G=nx.make_small_graph(["adjacencylist","C_4",4,[[2,4],[1,3],[2,4],[1,3]]])
```

or, since we do not need to add edges twice,

```python
>>> G=nx.make_small_graph(["adjacencylist","C_4",4,[[2,4],[3],[4],[]]])
```

If `ltype=“edgelist”` then `xlist` is an edge list written as `[[v1,w2],[v2,w2],...,[vk,wk]]`, where `vj` and `wj` integers in the range `1...n` e.g. the “square” graph C_4 can be obtained by

```python
>>> G=nx.make_small_graph(["edgelist","C_4",4,[[1,2],[3,4],[2,3],[4,1]]])
```

Use the `create_using` argument to choose the graph class/type.
5.5.2 networkx.generators.small.LCF_graph

**LCF_graph** (*n*, *shift_list*, *repeats*, *create_using=None*)

Return the cubic graph specified in LCF notation.

LCF notation (LCF=Lederberg-Coxeter-Fruchte) is a compressed notation used in the generation of various cubic Hamiltonian graphs of high symmetry. See, for example, dodecahedral_graph, desargues_graph, heawood_graph and pappus_graph below.

**n (number of nodes)** The starting graph is the n-cycle with nodes 0,...,n-1. (The null graph is returned if n < 0.)

*shift_list* = [s1,s2,...,sk], a list of integer shifts mod n,

**repeats** integer specifying the number of times that shifts in *shift_list* are successively applied to each v_current in the n-cycle to generate an edge between v_current and v_current+shift mod n.

For v1 cycling through the n-cycle a total of k*repeats with shift cycling through shiftlist repeats times connect v1 with v1+shift mod n

The utility graph $K_{3,3}$

```python
>>> G = nx.LCF_graph(6, [3, -3], 3)
```

The Heawood graph

```python
>>> G = nx.LCF_graph(14, [5, -5], 7)
```

See [http://mathworld.wolfram.com/LCFNotation.html](http://mathworld.wolfram.com/LCFNotation.html) for a description and references.

5.5.3 networkx.generators.small.bull_graph

**bull_graph** (*create_using=None*)

Returns the Bull graph.

5.5.4 networkx.generators.small.chvatal_graph

**chvatal_graph** (*create_using=None*)

Returns the Chvátal graph.

5.5.5 networkx.generators.small.cubical_graph

**cubical_graph** (*create_using=None*)

Returns the 3-regular Platonic Cubical graph.

5.5.6 networkx.generators.small.desargues_graph

**desargues_graph** (*create_using=None*)

Return the Desargues graph.
5.5.7 networkx.generators.small.diamond_graph

diamond_graph (create_using=None)
Returns the Diamond graph.

5.5.8 networkx.generators.small.dodecahedral_graph

dodecahedral_graph (create_using=None)
Returns the Platonic Dodecahedral graph.

5.5.9 networkx.generators.small.frucht_graph

frucht_graph (create_using=None)
Returns the Frucht Graph.

The Frucht Graph is the smallest cubical graph whose automorphism group consists only of the identity element.

5.5.10 networkx.generators.small.heawood_graph

heawood_graph (create_using=None)
Returns the Heawood graph, a (3,6) cage.

5.5.11 networkx.generators.small.hoffman_singleton_graph

hoffman_singleton_graph()
Returns the Hoffman-Singleton Graph.

5.5.12 networkx.generators.small.house_graph

house_graph (create_using=None)
Returns the House graph (square with triangle on top).

5.5.13 networkx.generators.small.house_x_graph

house_x_graph (create_using=None)
Returns the House graph with a cross inside the house square.

5.5.14 networkx.generators.small.icosahedral_graph

icosahedral_graph (create_using=None)
Returns the Platonic Icosahedral graph.
5.5.15 networkx.generators.small.krackhardt_kite_graph

krackhardt_kite_graph(create_using=None)
Return the Krackhardt Kite Social Network.
A 10 actor social network introduced by David Krackhardt to illustrate: degree, betweenness, centrality, closeness, etc. The traditional labeling is: Andre=1, Beverley=2, Carol=3, Diane=4, Ed=5, Fernando=6, Garth=7, Heather=8, Ike=9, Jane=10.

5.5.16 networkx.generators.small.moebius_kantor_graph

moebius_kantor_graph(create_using=None)
Returns the Moebius-Kantor graph.

5.5.17 networkx.generators.small.octahedral_graph

octahedral_graph(create_using=None)
Returns the Platonic Octahedral graph.

5.5.18 networkx.generators.small.pappus_graph

pappus_graph()
Return the Pappus graph.

5.5.19 networkx.generators.small.petersen_graph

petersen_graph(create_using=None)
Returns the Petersen graph.

5.5.20 networkx.generators.small.sedgewick_maze_graph

sedgewick_maze_graph(create_using=None)
Return a small maze with a cycle.
This is the maze used in Sedgewick, 3rd Edition, Part 5, Graph Algorithms, Chapter 18, e.g. Figure 18.2 and following. Nodes are numbered 0,...,7.

5.5.21 networkx.generators.small.tetrahedral_graph

tetrahedral_graph(create_using=None)
Return the 3-regular Platonic Tetrahedral graph.

5.5.22 networkx.generators.small.truncated_cube_graph

truncated_cube_graph(create_using=None)
Returns the skeleton of the truncated cube.
5.5.23 networkx.generators.small.truncated_tetrahedron_graph

documented_tetrahedron_graph(create_using=None)

Returns the skeleton of the truncated Platonic tetrahedron.

5.5.24 networkx.generators.small.tutte_graph

documented_tutte_graph(create_using=None)

Returns the Tutte graph.

5.6 Random Graphs

Generators for random graphs.

<table>
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<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fast_gnp_random_graph(n, p[, seed, directed])</td>
<td>Returns a $G_{n,p}$ random graph, also known as an Erdős-Rényi graph or a binomial graph.</td>
</tr>
<tr>
<td>gnp_random_graph(n, p[, seed, directed])</td>
<td>Returns a $G_{n,p}$ random graph, also known as an Erdős-Rényi graph or a binomial graph.</td>
</tr>
<tr>
<td>dense_gnm_random_graph(n, m[, seed])</td>
<td>Returns a $G_{n,m}$ random graph.</td>
</tr>
<tr>
<td>gnm_random_graph(n, m[, seed, directed])</td>
<td>Returns a $G_{n,m}$ random graph.</td>
</tr>
<tr>
<td>erdos_renyi_graph(n, p[, seed, directed])</td>
<td>Returns a $G_{n,p}$ random graph, also known as an Erdős-Rényi graph or a binomial graph.</td>
</tr>
<tr>
<td>binomial_graph(n, p[, seed, directed])</td>
<td>Returns a $G_{n,p}$ random graph, also known as an Erdős-Rényi graph or a binomial graph.</td>
</tr>
<tr>
<td>newman_watts_strogatz_graph(n, k, p[, seed])</td>
<td>Returns a Newman–Watts–Strogatz small-world graph.</td>
</tr>
<tr>
<td>watts_strogatz_graph(n, k, p[, seed])</td>
<td>Returns a Watts–Strogatz small-world graph.</td>
</tr>
<tr>
<td>connected_watts_strogatz_graph(n, k, p[, ...])</td>
<td>Returns a connected Watts–Strogatz small-world graph.</td>
</tr>
<tr>
<td>random_regular_graph(d, n[, seed])</td>
<td>Returns a random $d$-regular graph on $n$ nodes.</td>
</tr>
<tr>
<td>barabasi_albert_graph(n, m[, seed])</td>
<td>Returns a random graph according to the Barabási–Albert preferential attachment model.</td>
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<tr>
<td>dual_barabasi_albert_graph(n, m1, m2, p[, seed])</td>
<td>Returns a random graph according to the dual Barabási–Albert preferential attachment model.</td>
</tr>
<tr>
<td>extended_barabasi_albert_graph(n, m, p, q[, ...])</td>
<td>Returns an extended Barabási–Albert model graph.</td>
</tr>
<tr>
<td>powerlaw_cluster_graph(n, m, p[, seed])</td>
<td>Holme and Kim algorithm for growing graphs with powerlaw degree distribution and approximate average clustering.</td>
</tr>
<tr>
<td>random_kernel_graph(n, kernel_integral[...])</td>
<td>Returns a random graph based on the specified kernel.</td>
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<tr>
<td>random_lobster(n, p1, p2[, seed])</td>
<td>Returns a random lobster graph.</td>
</tr>
<tr>
<td>random_shell_graph(constructor[, seed])</td>
<td>Returns a random shell graph for the constructor given.</td>
</tr>
<tr>
<td>random_powerlaw_tree(n[, gamma, seed, tries])</td>
<td>Returns a tree with a power law degree distribution.</td>
</tr>
<tr>
<td>random_powerlaw_tree_sequence(n[, gamma, ...])</td>
<td>Returns a degree sequence for a tree with a power law distribution.</td>
</tr>
<tr>
<td>random_kernel_graph(n, kernel_integral[...])</td>
<td>Returns a random graph based on the specified kernel.</td>
</tr>
</tbody>
</table>
5.6.1 networkx.generators.random_graphs.fast_gnp_random_graph

`fast_gnp_random_graph(n, p, seed=None, directed=False)`

Returns a $G_{n,p}$ random graph, also known as an Erdős-Rényi graph or a binomial graph.

**Parameters**
- `n` (*int*) – The number of nodes.
- `p` (*float*) – Probability for edge creation.
- `seed` (*integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.
- `directed` (*bool, optional (default=False)) – If True, this function returns a directed graph.

**Notes**

The $G_{n,p}$ graph algorithm chooses each of the $\frac{n(n-1)}{2}$ (undirected) or $n(n-1)$ (directed) possible edges with probability $p$.

This algorithm\(^1\) runs in $O(n + m)$ time, where $m$ is the expected number of edges, which equals $pn(n - 1)/2$. This should be faster than `gnp_random_graph()` when $p$ is small and the expected number of edges is small (that is, the graph is sparse).

**See also:**
- `gnp_random_graph()`

**References**

5.6.2 networkx.generators.random_graphs.gnp_random_graph

`gnp_random_graph(n, p, seed=None, directed=False)`

Returns a $G_{n,p}$ random graph, also known as an Erdős-Rényi graph or a binomial graph.

The $G_{n,p}$ model chooses each of the possible edges with probability $p$.

The functions `binomial_graph()` and `erdos_renyi_graph()` are aliases of this function.

**Parameters**
- `n` (*int*) – The number of nodes.
- `p` (*float*) – Probability for edge creation.
- `seed` (*integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.
- `directed` (*bool, optional (default=False)) – If True, this function returns a directed graph.

**See also:**
- `fast_gnp_random_graph()`

---

Notes

This algorithm\(^2\) runs in \(O(n^2)\) time. For sparse graphs (that is, for small values of \(p\)), \texttt{fast_gnp_random_graph()} is a faster algorithm.

References

5.6.3 networkx.generators.random_graphs.dense_gnm_random_graph

dense_gnm_random_graph \((n, m, seed=None)\)

Returns a \(G_{n,m}\) random graph.

In the \(G_{n,m}\) model, a graph is chosen uniformly at random from the set of all graphs with \(n\) nodes and \(m\) edges.

This algorithm should be faster than \texttt{gnm_random_graph()} for dense graphs.

Parameters

- \(n\) (int) – The number of nodes.
- \(m\) (int) – The number of edges.
- \texttt{seed} (integer, random_state, or None (default)) – Indicator of random number generation state. See \textit{Randomness}.

See also:

\texttt{gnm_random_graph()}

Notes

Algorithm by Keith M. Briggs Mar 31, 2006. Inspired by Knuth’s Algorithm S (Selection sampling technique), in section 3.4.2 of\(^1\).

References

5.6.4 networkx.generators.random_graphs.gnm_random_graph

gnm_random_graph \((n, m, seed=None, directed=False)\)

Returns a \(G_{n,m}\) random graph.

In the \(G_{n,m}\) model, a graph is chosen uniformly at random from the set of all graphs with \(n\) nodes and \(m\) edges.

This algorithm should be faster than \texttt{dense_gnm_random_graph()} for sparse graphs.

Parameters

- \(n\) (int) – The number of nodes.
- \(m\) (int) – The number of edges.
- \texttt{seed} (integer, random_state, or None (default)) – Indicator of random number generation state. See \textit{Randomness}.
- \texttt{directed} (bool, optional (default=False)) – If True return a directed graph


See also:

dense_gnm_random_graph()

5.6.5 networkx.generators.random_graphs.erdos_renyi_graph

erdos_renyi_graph(n, p, seed=None, directed=False)

Returns a $G_{n,p}$ random graph, also known as an Erdős-Rényi graph or a binomial graph.

The $G_{n,p}$ model chooses each of the possible edges with probability $p$.

The functions binomial_graph() and erdos_renyi_graph() are aliases of this function.

Parameters

• n (int) – The number of nodes.
• p (float) – Probability for edge creation.
• seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.
• directed (bool, optional (default=False)) – If True, this function returns a directed graph.

See also:

fast_gnp_random_graph()

Notes

This algorithm\(^2\) runs in $O(n^2)$ time. For sparse graphs (that is, for small values of $p$), fast_gnp_random_graph() is a faster algorithm.

References

5.6.6 networkx.generators.random_graphs.binomial_graph

binomial_graph(n, p, seed=None, directed=False)

Returns a $G_{n,p}$ random graph, also known as an Erdős-Rényi graph or a binomial graph.

The $G_{n,p}$ model chooses each of the possible edges with probability $p$.

The functions binomial_graph() and erdos_renyi_graph() are aliases of this function.

Parameters

• n (int) – The number of nodes.
• p (float) – Probability for edge creation.
• seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.
• directed (bool, optional (default=False)) – If True, this function returns a directed graph.

See also:

fast_gnp_random_graph()

Notes

This algorithm\(^2\) runs in $O(n^2)$ time. For sparse graphs (that is, for small values of $p$), `fast_gnp_random_graph()` is a faster algorithm.

References

5.6.7 networkx.generators.random_graphs.newman_watts_strogatz_graph

`newman_watts_strogatz_graph(n, k, p, seed=None)`


Parameters

- `n` (*int*) – The number of nodes.
- `k` (*int*) – Each node is joined with its $k$ nearest neighbors in a ring topology.
- `p` (*float*) – The probability of adding a new edge for each edge.
- `seed` (*integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Notes

First create a ring over $n$ nodes\(^1\). Then each node in the ring is connected with its $k$ nearest neighbors (or $k - 1$ neighbors if $k$ is odd). Then shortcuts are created by adding new edges as follows: for each edge $(u, v)$ in the underlying “$n$-ring with $k$ nearest neighbors” with probability $p$ add a new edge $(u, w)$ with randomly-chosen existing node $w$. In contrast with `watts_strogatz_graph()`, no edges are removed.

See also:

`watts_strogatz_graph()`

References

5.6.8 networkx.generators.random_graphs.watts_strogatz_graph

`watts_strogatz_graph(n, k, p, seed=None)`

Returns a Watts–Strogatz small-world graph.

Parameters

- `n` (*int*) – The number of nodes
- `k` (*int*) – Each node is joined with its $k$ nearest neighbors in a ring topology.
- `p` (*float*) – The probability of rewiring each edge
- `seed` (*integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.


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See also:

newman_watts_strogatz_graph(), connected_watts_strogatz_graph()

Notes

First create a ring over \( n \) nodes\(^1\). Then each node in the ring is joined to its \( k \) nearest neighbors (or \( k - 1 \) neighbors if \( k \) is odd). Then shortcuts are created by replacing some edges as follows: for each edge \((u, v)\) in the underlying “\( n \)-ring with \( k \) nearest neighbors” with probability \( p \) replace it with a new edge \((u, w)\) with uniformly random choice of existing node \( w \).

In contrast with newman_watts_strogatz_graph(), the random rewiring does not increase the number of edges. The rewired graph is not guaranteed to be connected as in connected_watts_strogatz_graph().

References

5.6.9 networkx.generators.random_graphs.connected_watts_strogatz_graph

connected_watts_strogatz_graph \((n, k, p, \text{tries}=100, \text{seed}=\text{None})\)

Returns a connected Watts–Strogatz small-world graph.

Attempts to generate a connected graph by repeated generation of Watts–Strogatz small-world graphs. An exception is raised if the maximum number of tries is exceeded.

Parameters

- \( n \) (int) – The number of nodes
- \( k \) (int) – Each node is joined with its \( k \) nearest neighbors in a ring topology.
- \( p \) (float) – The probability of rewiring each edge
- \( \text{tries} \) (int) – Number of attempts to generate a connected graph.
- \( \text{seed} \) (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Notes

First create a ring over \( n \) nodes\(^1\). Then each node in the ring is joined to its \( k \) nearest neighbors (or \( k - 1 \) neighbors if \( k \) is odd). Then shortcuts are created by replacing some edges as follows: for each edge \((u, v)\) in the underlying “\( n \)-ring with \( k \) nearest neighbors” with probability \( p \) replace it with a new edge \((u, w)\) with uniformly random choice of existing node \( w \). The entire process is repeated until a connected graph results.

See also:

newman_watts_strogatz_graph(), watts_strogatz_graph()


References

5.6.10 networkx.generators.random_graphs.random_regular_graph

random_regular_graph \((d, n, seed=None)\)

Returns a random \(d\)-regular graph on \(n\) nodes.

The resulting graph has no self-loops or parallel edges.

Parameters

- **\(d\) (int)** – The degree of each node.
- **\(n\) (integer)** – The number of nodes. The value of \(n \times d\) must be even.
- **\(seed\) (integer, random_state, or None (default))** – Indicator of random number generation state. See Randomness.

Notes

The nodes are numbered from 0 to \(n - 1\).

Kim and Vu’s paper\(^2\) shows that this algorithm samples in an asymptotically uniform way from the space of random graphs when \(d = O(n^{1/3-\epsilon})\).

Raises **NetworkXError** – If \(n \times d\) is odd or \(d\) is greater than or equal to \(n\).

References

5.6.11 networkx.generators.random_graphs.barabasi_albert_graph

barabasi_albert_graph \((n, m, seed=None)\)

Returns a random graph according to the Barabási–Albert preferential attachment model.

A graph of \(n\) nodes is grown by attaching new nodes each with \(m\) edges that are preferentially attached to existing nodes with high degree.

Parameters

- **\(n\) (int)** – Number of nodes
- **\(m\) (int)** – Number of edges to attach from a new node to existing nodes
- **\(seed\) (integer, random_state, or None (default))** – Indicator of random number generation state. See Randomness.

Returns **\(G\)**

Return type **Graph**

Raises **NetworkXError** – If \(m\) does not satisfy \(1 \leq m < n\).

---

References

5.6.12 networkx.generators.random_graphs.dual_barabasi_albert_graph

dual_barabasi_albert_graph(n, m1, m2, p, seed=None)

Returns a random graph according to the dual Barabási–Albert preferential attachment model.

A graph of $n$ nodes is grown by attaching new nodes each with either $m_1$ edges (with probability $p$) or $m_2$ edges (with probability $1 - p$) that are preferentially attached to existing nodes with high degree.

Parameters

- n (int) – Number of nodes
- m1 (int) – Number of edges to attach from a new node to existing nodes with probability $p$
- m2 (int) – Number of edges to attach from a new node to existing nodes with probability $1 - p$
- p (float) – The probability of attaching $m_1$ edges (as opposed to $m_2$ edges)
- seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns G

Return type Graph

Raises NetworkXError – If $m_1$ and $m_2$ do not satisfy $1 <= m_1, m_2 < n$ or $p$ does not satisfy $0 <= p <= 1$.

References

5.6.13 networkx.generators.random_graphs.extended_barabasi_albert_graph

extended_barabasi_albert_graph(n, m, p, q, seed=None)

Returns an extended Barabási–Albert model graph.

An extended Barabási–Albert model graph is a random graph constructed using preferential attachment. The extended model allows new edges, rewired edges or new nodes. Based on the probabilities $p$ and $q$ with $p + q < 1$, the growing behavior of the graph is determined as:

1) With $p$ probability, $m$ new edges are added to the graph, starting from randomly chosen existing nodes and attached preferentially at the other end.
2) With $q$ probability, $m$ existing edges are rewired by randomly choosing an edge and rewiring one end to a preferentially chosen node.
3) With $(1 - p - q)$ probability, $m$ new nodes are added to the graph with edges attached preferentially.

When $p = q = 0$, the model behaves just like the Barabási–Alber mo

Parameters

- n (int) – Number of nodes
- m (int) – Number of edges with which a new node attaches to existing nodes
- p (float) – Probability value for adding an edge between existing nodes. $p + q < 1$
- q (float) – Probability value of rewiring of existing edges. $p + q < 1$
• **seed** *(integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.*

Returns G

Return type *Graph*

Raises **NetworkXError** – If $m$ does not satisfy $1 \leq m < n$ or $1 \geq p + q$

**References**

5.6.14 networkx.generators.random_graphs.powerlaw_cluster_graph

**powerlaw_cluster_graph** *(n, m, p, seed=None)*

Holme and Kim algorithm for growing graphs with powerlaw degree distribution and approximate average clustering.

Parameters

- **n** *(int)* – the number of nodes
- **m** *(int)* – the number of random edges to add for each new node
- **p** *(float,)* – Probability of adding a triangle after adding a random edge
- **seed** *(integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.*

Notes

The average clustering has a hard time getting above a certain cutoff that depends on $m$. This cutoff is often quite low. The transitivity (fraction of triangles to possible triangles) seems to decrease with network size.

It is essentially the Barabási–Albert (BA) growth model with an extra step that each random edge is followed by a chance of making an edge to one of its neighbors too (and thus a triangle).

This algorithm improves on BA in the sense that it enables a higher average clustering to be attained if desired.

It seems possible to have a disconnected graph with this algorithm since the initial $m$ nodes may not be all linked to a new node on the first iteration like the BA model.

Raises **NetworkXError** – If $m$ does not satisfy $1 \leq m \leq n$ or $p$ does not satisfy $0 \leq p \leq 1$.

**References**

5.6.15 networkx.generators.random_graphs.random_kernel_graph

**random_kernel_graph** *(n, kernel_integral, kernel_root=None, seed=None)*

Returns an random graph based on the specified kernel.

The algorithm chooses each of the $\frac{n(n - 1)}{2}$ possible edges with probability specified by a kernel $\kappa(x, y)^1$.

The kernel $\kappa(x, y)$ must be a symmetric (in $x, y$), non-negative, bounded function.

Parameters

- **n** *(int)* – The number of nodes

---

**kernal_integral** *(function)* – Function that returns the definite integral of the kernel $\kappa(x, y)$,

$$F(y, a, b) := \int_a^b \kappa(x, y) dx$$

**kernel_root** *(function (optional))* – Function that returns the root $b$ of the equation $F(y, a, b) = r$. If None, the root is found using scipy.optimize.brentq() (this requires SciPy).

**seed** *(integer, random_state, or None (default))* – Indicator of random number generation state. See Randomness.

Notes

The kernel is specified through its definite integral which must be provided as one of the arguments. If the integral and root of the kernel integral can be found in $O(1)$ time then this algorithm runs in time $O(n + m)$ where $m$ is the expected number of edges\(^2\).

The nodes are set to integers from 0 to $n - 1$.

Examples

Generate an Erdős–Rényi random graph $G(n, c/n)$, with kernel $\kappa(x, y) = c$ where $c$ is the mean expected degree.

```python
>>> def integral(u, w, z):
...     return c * (z - w)

>>> def root(u, w, r):
...     return r / c + w

>>> c = 1

>>> graph = nx.random_kernel_graph(1000, integral, root)
```

See also:

gnp_random_graph(), expected_degree_graph()

References

5.6.16 networkx.generators.random_graphs.random_lobster

**random_lobster** *(n, p1, p2, seed=None)*

Returns a random lobster graph.

A lobster is a tree that reduces to a caterpillar when pruning all leaf nodes. A caterpillar is a tree that reduces to a path graph when pruning all leaf nodes; setting $p_2$ to zero produces a caterpillar.

Parameters

- **n** *(int)* – The expected number of nodes in the backbone
- **p1** *(float)* – Probability of adding an edge to the backbone
- **p2** *(float)* – Probability of adding an edge one level beyond backbone
- **seed** *(integer, random_state, or None (default))* – Indicator of random number generation state. See Randomness.

5.6.17 networkx.generators.random_graphs.random_shell_graph

random_shell_graph(constructor, seed=None)

Returns a random shell graph for the constructor given.

Parameters

- constructor (list of three-tuples) – Represents the parameters for a shell, starting at the center shell. Each element of the list must be of the form (n, m, d), where n is the number of nodes in the shell, m is the number of edges in the shell, and d is the ratio of inter-shell (next) edges to intra-shell edges. If d is zero, there will be no intra-shell edges, and if d is one there will be all possible intra-shell edges.

- seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Examples

```python
>>> constructor = [(10, 20, 0.8), (20, 40, 0.8)]
>>> G = nx.random_shell_graph(constructor)
```

5.6.18 networkx.generators.random_graphs.random_powerlaw_tree

random_powerlaw_tree(n, gamma=3, seed=None, tries=100)

Returns a tree with a power law degree distribution.

Parameters

- n (int) – The number of nodes.

- gamma (float) – Exponent of the power law.

- seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

- tries (int) – Number of attempts to adjust the sequence to make it a tree.

Raises NetworkXError – If no valid sequence is found within the maximum number of attempts.

Notes

A trial power law degree sequence is chosen and then elements are swapped with new elements from a powerlaw distribution until the sequence makes a tree (by checking, for example, that the number of edges is one smaller than the number of nodes).

5.6.19 networkx.generators.random_graphs.random_powerlaw_tree_sequence

random_powerlaw_tree_sequence(n, gamma=3, seed=None, tries=100)

Returns a degree sequence for a tree with a power law degree distribution.

Parameters

- n (int) – The number of nodes.

- gamma (float) – Exponent of the power law.
• **seed** (*integer, random_state, or None (default)) – Indicator of random number generation state. See `Randomness`.

• **tries** (*int*) – Number of attempts to adjust the sequence to make it a tree.

**Raises** `NetworkXError` – If no valid sequence is found within the maximum number of attempts.

**Notes**

A trial power law degree sequence is chosen and then elements are swapped with new elements from a power law distribution until the sequence makes a tree (by checking, for example, that the number of edges is one smaller than the number of nodes).

### 5.7 Duplication Divergence

Functions for generating graphs based on the “duplication” method.

These graph generators start with a small initial graph then duplicate nodes and (partially) duplicate their edges. These functions are generally inspired by biological networks.

<table>
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<tr>
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<tbody>
<tr>
<td><code>duplication_divergence_graph</code></td>
<td>Returns an undirected graph using the duplication-divergence model.</td>
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<tr>
<td><code>partial_duplication_graph</code></td>
<td>Returns a random graph using the partial duplication model.</td>
</tr>
</tbody>
</table>

#### 5.7.1 `networkx.generators.duplication.duplication_divergence_graph`

`duplication_divergence_graph(n, p[, seed])` Returns an undirected graph using the duplication-divergence model.

A graph of `n` nodes is created by duplicating the initial nodes and retaining edges incident to the original nodes with a retention probability `p`.

**Parameters**

• **n** (*int*) – The desired number of nodes in the graph.

• **p** (*float*) – The probability for retaining the edge of the replicated node.

• **seed** (*integer, random_state, or None (default)) – Indicator of random number generation state. See `Randomness`.

**Returns** `G`  

**Return type** `Graph`  

**Raises** `NetworkXError` – If `p` is not a valid probability. If `n` is less than 2.

**Notes**

This algorithm appears in [1].

This implementation disallows the possibility of generating disconnected graphs.
5.7.2 networkx.generators.duplication.partial_duplication_graph

partial_duplication_graph \((N, n, p, q, seed=None)\)

Returns a random graph using the partial duplication model.

**Parameters**

- \(N\) (int) – The total number of nodes in the final graph.
- \(n\) (int) – The number of nodes in the initial clique.
- \(p\) (float) – The probability of joining each neighbor of a node to the duplicate node. Must be a number in the between zero and one, inclusive.
- \(q\) (float) – The probability of joining the source node to the duplicate node. Must be a number in the between zero and one, inclusive.
- \(seed\) (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

**Notes**

A graph of nodes is grown by creating a fully connected graph of size \(n\). The following procedure is then repeated until a total of \(N\) nodes have been reached.

1. A random node, \(u\), is picked and a new node, \(v\), is created.
2. For each neighbor of \(u\) an edge from the neighbor to \(v\) is created with probability \(p\).
3. An edge from \(u\) to \(v\) is created with probability \(q\).

This algorithm appears in [1].

This implementation allows the possibility of generating disconnected graphs.

**References**

5.8 Degree Sequence

Generate graphs with a given degree sequence or expected degree sequence.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
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<tbody>
<tr>
<td>configuration_model(deg_sequence[...])</td>
<td>Returns a random graph with the given degree sequence.</td>
</tr>
<tr>
<td>directed_configuration_model(...[...])</td>
<td>Returns a directed_random graph with the given degree sequences.</td>
</tr>
<tr>
<td>expected_degree_graph(w[, seed, selfloops])</td>
<td>Returns a random graph with given expected degrees.</td>
</tr>
<tr>
<td>havel_hakimi_graph(deg_sequence[, create_using])</td>
<td>Returns a simple graph with given degree sequence constructed using the Havel-Hakimi algorithm.</td>
</tr>
<tr>
<td>directed_havel_hakimi_graph(in_deg_sequence,...)</td>
<td>Returns a directed graph with the given degree sequences.</td>
</tr>
<tr>
<td>degree_sequence_tree(deg_sequence[...])</td>
<td>Make a tree for the given degree sequence.</td>
</tr>
<tr>
<td>random_degree_sequence_graph(sequence[...])</td>
<td>Returns a simple random graph with the given degree sequence.</td>
</tr>
</tbody>
</table>
5.8.1 networkx.generators.degree_seq.configuration_model

configuration_model (deg_sequence, create_using=None, seed=None)

Returns a random graph with the given degree sequence.

The configuration model generates a random pseudograph (graph with parallel edges and self-loops) by randomly assigning edges to match the given degree sequence.

Parameters

• deg_sequence (list of nonnegative integers) – Each list entry corresponds to the degree of a node.

• create_using (NetworkX graph constructor, optional (default MultiGraph)) – Graph type to create. If graph instance, then cleared before populated.

• seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns G – A graph with the specified degree sequence. Nodes are labeled starting at 0 with an index corresponding to the position in deg_sequence.

Return type MultiGraph

Raises NetworkXError – If the degree sequence does not have an even sum.

See also is_graphical()

Notes

As described by Newman\(^1\).

A non-graphical degree sequence (not realizable by some simple graph) is allowed since this function returns graphs with self-loops and parallel edges. An exception is raised if the degree sequence does not have an even sum.

This configuration model construction process can lead to duplicate edges and loops. You can remove the self-loops and parallel edges (see below) which will likely result in a graph that doesn’t have the exact degree sequence specified.

The density of self-loops and parallel edges tends to decrease as the number of nodes increases. However, typically the number of self-loops will approach a Poisson distribution with a nonzero mean, and similarly for the number of parallel edges. Consider a node with \(k\) stubs. The probability of being joined to another stub of the same node is basically \((k - 1) / N\), where \(k\) is the degree and \(N\) is the number of nodes. So the probability of a self-loop scales like \(c / N\) for some constant \(c\). As \(N\) grows, this means we expect \(c\) self-loops. Similarly for parallel edges.

References

Examples

You can create a degree sequence following a particular distribution by using the one of the distribution functions in random_sequence (or one of your own). For example, to create an undirected multigraph on one hundred nodes with degree sequence chosen from the power law distribution:

>>> sequence = nx.random_powerlaw_tree_sequence(100, tries=5000)
>>> G = nx.configuration_model(sequence)
>>> len(G)
100
>>> actual_degrees = [d for v, d in G.degree()]
>>> actual_degrees == sequence
True

The returned graph is a multigraph, which may have parallel edges. To remove any parallel edges from the returned graph:

>>> G = nx.Graph(G)

Similarly, to remove self-loops:

>>> G.remove_edges_from(nx.selfloop_edges(G))

5.8.2 networkx.generators.degree_seq.directed_configuration_model

directed_configuration_model (in_degree_sequence, out_degree_sequence, create_using=None, seed=None)

Returns a directed_random graph with the given degree sequences.

The configuration model generates a random directed pseudograph (graph with parallel edges and self loops) by randomly assigning edges to match the given degree sequences.

Parameters

- **in_degree_sequence** (list of nonnegative integers) – Each list entry corresponds to the in-degree of a node.
- **out_degree_sequence** (list of nonnegative integers) – Each list entry corresponds to the out-degree of a node.
- **create_using** (NetworkX graph constructor, optional (default MultiDiGraph)) – Graph type to create. If graph instance, then cleared before populated.
- **seed** (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns G – A graph with the specified degree sequences. Nodes are labeled starting at 0 with an index corresponding to the position in deg_sequence.

Return type MultiDiGraph

Raises NetworkXError – If the degree sequences do not have the same sum.

See also:
configuration_model()

Notes

Algorithm as described by Newman¹.

A non-graphical degree sequence (not realizable by some simple graph) is allowed since this function returns graphs with self loops and parallel edges. An exception is raised if the degree sequences does not have the same sum.

This configuration model construction process can lead to duplicate edges and loops. You can remove the self-loops and parallel edges (see below) which will likely result in a graph that doesn’t have the exact degree sequence specified. This “finite-size effect” decreases as the size of the graph increases.

References

Examples

One can modify the in- and out-degree sequences from an existing directed graph in order to create a new directed graph. For example, here we modify the directed path graph:

```python
>>> D = nx.DiGraph([(0, 1), (1, 2), (2, 3)])
>>> din = list(d for n, d in D.in_degree())
>>> dout = list(d for n, d in D.out_degree())
>>> din.append(1)
>>> dout[0] = 2
>>> # We now expect an edge from node 0 to a new node, node 3.
... D = nx.directed_configuration_model(din, dout)
```

The returned graph is a directed multigraph, which may have parallel edges. To remove any parallel edges from the returned graph:

```python
>>> D = nx.DiGraph(D)
```

Similarly, to remove self-loops:

```python
>>> D.remove_edges_from(nx.selfloop_edges(D))
```

5.8.3 networkx.generators.degree_seq.expected_degree_graph

expected_degree_graph(w, seed=None, selfloops=True)

Returns a random graph with given expected degrees.

Given a sequence of expected degrees \( W = (w_0, w_1, \ldots, w_{n-1}) \) of length \( n \) this algorithm assigns an edge between node \( u \) and node \( v \) with probability

\[
P_{uv} = \frac{w_u w_v}{\sum_k w_k}
\]

Parameters

- \( w \) (list) – The list of expected degrees.
- selfloops (bool (default=True)) – Set to False to remove the possibility of self-loop edges.
- seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns

Return type Graph
Examples

```python
>>> z=[10 for i in range(100)]
>>> G=nx.expected_degree_graph(z)
```

Notes

The nodes have integer labels corresponding to index of expected degrees input sequence.

The complexity of this algorithm is $O(n + m)$ where $n$ is the number of nodes and $m$ is the expected number of edges.

The model in\(^1\) includes the possibility of self-loop edges. Set selfloops=False to produce a graph without self loops.

For finite graphs this model doesn’t produce exactly the given expected degree sequence. Instead the expected degrees are as follows.

For the case without self loops (selfloops=False),

$$E[\text{deg}(u)] = \sum_{v \neq u} p_{uv} = w_u \left(1 - \frac{w_u}{\sum_k w_k}\right).$$

NetworkX uses the standard convention that a self-loop edge counts 2 in the degree of a node, so with self loops (selfloops=True),

$$E[\text{deg}(u)] = \sum_{v \neq u} p_{uv} + 2p_{uu} = w_u \left(1 + \frac{w_u}{\sum_k w_k}\right).$$

References

5.8.4 networkx.generators.degree_seq.havel_hakimi_graph

**havel_hakimi_graph** *(deg_sequence, create_using=None)*

Returns a simple graph with given degree sequence constructed using the Havel-Hakimi algorithm.

**Parameters**

- **deg_sequence** (*list of integers*) – Each integer corresponds to the degree of a node (need not be sorted).
- **create_using** (*NetworkX graph constructor, optional (default=nx.Graph)*) – Graph type to create. If graph instance, then cleared before populated. Directed graphs are not allowed.

**Raises** *NetworkXException* – For a non-graphical degree sequence (i.e. one not realizable by some simple graph).

Notes

The Havel-Hakimi algorithm constructs a simple graph by successively connecting the node of highest degree to other nodes of highest degree, resorting remaining nodes by degree, and repeating the process. The resulting graph has a high degree-associativity. Nodes are labeled 1,..., len(deg_sequence), corresponding to their position in deg_sequence.

The basic algorithm is from Hakimi\(^1\) and was generalized by Kleitman and Wang\(^2\).

**References**

5.8.5 networkx.generators.degree_seq.directed_havel_hakimi_graph

**directed_havel_hakimi_graph** (*in_deg_sequence*, *out_deg_sequence*, *create_using=None*)

Returns a directed graph with the given degree sequences.

**Parameters**

- *in_deg_sequence* (list of integers) – Each list entry corresponds to the in-degree of a node.
- *out_deg_sequence* (list of integers) – Each list entry corresponds to the out-degree of a node.
- *create_using* (NetworkX graph constructor, optional (default DiGraph)) – Graph type to create. If graph instance, then cleared before populated.

**Returns** G – A graph with the specified degree sequences. Nodes are labeled starting at 0 with an index corresponding to the position in deg_sequence

**Return type** DiGraph

** Raises** NetworkXError – If the degree sequences are not digraphical.

**See also:** configuration_model()

**Notes**

Algorithm as described by Kleitman and Wang\(^1\).

**References**

5.8.6 networkx.generators.degree_seq.degree_sequence_tree

**degree_sequence_tree** (*deg_sequence*, *create_using=None*)

Make a tree for the given degree sequence.

A tree has \#nodes-\#edges=1 so the degree sequence must have len(deg_sequence)-sum(deg_sequence)/2=1

5.8.7 networkx.generators.degree_seq.random_degree_sequence_graph

**random_degree_sequence_graph** (*sequence*, *seed=None*, *tries=10*)

Returns a simple random graph with the given degree sequence.

If the maximum degree \(d_m\) in the sequence is \(O(m^{1/4})\) then the algorithm produces almost uniform random graphs in \(O(md_m)\) time where \(m\) is the number of edges.

**Parameters**


• **sequence** (*list of integers*) – Sequence of degrees

• **seed** (*integer, random_state, or None (default)) – Indicator of random number generation state. See *Randomness*.  

• **tries** (*int, optional*) – Maximum number of tries to create a graph

**Returns** G – A graph with the specified degree sequence. Nodes are labeled starting at 0 with an index corresponding to the position in the sequence.

**Return type** *Graph*

**Raises**

• *NetworkXUnfeasible* – If the degree sequence is not graphical.

• *NetworkXError* – If a graph is not produced in specified number of tries

**See also:**

*is_graphical*, *configuration_model()*

**Notes**

The generator algorithm\(^1\) is not guaranteed to produce a graph.

**References**

**Examples**

```python
>>> sequence = [1, 2, 2, 3]
>>> G = nx.random_degree_sequence_graph(sequence, seed=42)
>>> sorted(d for n, d in G.degree())
[1, 2, 2, 3]
```

### 5.9 Random Clustered

Generate graphs with given degree and triangle sequence.

#### 5.9.1 networkx.generators.random_clustered.random_clustered_graph

**random_clustered_graph** (*joint_degree_sequence*) Generate a random graph with the given joint independent edge degree and triangle degree sequence.

---

\(^1\) Moshen Bayati, Jeong Han Kim, and Amin Saberi, A sequential algorithm for generating random graphs. Algorithmica, Volume 58, Number 4, 860-910, DOI: 10.1007/s00453-009-9340-1
of \( u \) and the number \( d_{u,i} \) is the independent edge degree.

**Parameters**

- **joint_degree_sequence** *(list of integer pairs)* – Each list entry corresponds to the independent edge degree and triangle degree of a node.
- **create_using** *(NetworkX graph constructor, optional (default MultiGraph))* – Graph type to create. If graph instance, then cleared before populated.
- **seed** *(integer, random_state, or None (default))* – Indicator of random number generation state. See **Randomness**.

**Returns** G – A graph with the specified degree sequence. Nodes are labeled starting at 0 with an index corresponding to the position in deg_sequence.

**Return type** MultiGraph

**Raises** NetworkXError – If the independent edge degree sequence sum is not even or the triangle degree sequence sum is not divisible by 3.

**Notes**

As described by Miller\(^1\) (see also Newman\(^2\) for an equivalent description).

A non-graphical degree sequence (not realizable by some simple graph) is allowed since this function returns graphs with self loops and parallel edges. An exception is raised if the independent degree sequence does not have an even sum or the triangle degree sequence sum is not divisible by 3.

This configuration model-like construction process can lead to duplicate edges and loops. You can remove the self-loops and parallel edges (see below) which will likely result in a graph that doesn’t have the exact degree sequence specified. This “finite-size effect” decreases as the size of the graph increases.

**References**

**Examples**

```python
>>> deg = [(1, 0), (1, 0), (1, 0), (2, 0), (1, 0), (2, 1), (0, 1), (0, 1)]
>>> G = nx.random_clustered_graph(deg)
```

To remove parallel edges:

```python
>>> G = nx.Graph(G)
```

To remove self loops:

```python
>>> G.remove_edges_from(nx.selfloop_edges(G))
```

**5.10 Directed**

Generators for some directed graphs, including growing network (GN) graphs and scale-free graphs.

---

\(^1\) Joel C. Miller. “Percolation and epidemics in random clustered networks”. In: Physical review. E, Statistical, nonlinear, and soft matter physics 80 (2 Part 1 August 2009).

gn_graph(n[, kernel, create_using, seed]) Returns the growing network (GN) digraph with n nodes.

gnr_graph(n, p[, create_using, seed]) Returns the growing network with redirection (GNR) digraph with n nodes and redirection probability p.

gnc_graph(n[, create_using, seed]) Returns the growing network with copying (GNC) digraph with n nodes.

random_k_out_graph(n, k, alpha[,...]) Returns a random k-out graph with preferential attachment.

scale_free_graph(n[, alpha, beta, gamma, ...]) Returns a scale-free directed graph.

5.10.1 networkx.generators.directed.gn_graph

**gn_graph** (n, kernel=None, create_using=None, seed=None)
Returns the growing network (GN) digraph with n nodes.

The GN graph is built by adding nodes one at a time with a link to one previously added node. The target node for the link is chosen with probability based on degree. The default attachment kernel is a linear function of the degree of a node.

The graph is always a (directed) tree.

**Parameters**

- **n** (*int*) – The number of nodes for the generated graph.
- **kernel** (*function*) – The attachment kernel.
- **create_using** (*NetworkX graph constructor, optional (default DiGraph]*) – Graph type to create. If graph instance, then cleared before populated.
- **seed** (*integer, random_state, or None (default]*) – Indicator of random number generation state. See Randomness.

**Examples**

To create the undirected GN graph, use the to_directed() method:

```python
>>> D = nx.gn_graph(10)  # the GN graph
>>> G = D.to_undirected()  # the undirected version
```

To specify an attachment kernel, use the kernel keyword argument:

```python
>>> D = nx.gn_graph(10, kernel=lambda x: x ** 1.5)  # A_k = k^1.5
```

**References**

5.10.2 networkx.generators.directed.gnr_graph

**gnr_graph** (n, p, create_using=None, seed=None)
Returns the growing network with redirection (GNR) digraph with n nodes and redirection probability p.

The GNR graph is built by adding nodes one at a time with a link to one previously added node. The previous target node is chosen uniformly at random. With probability p the link is instead “redirected” to the successor node of the target.
The graph is always a (directed) tree.

Parameters

- n (int) – The number of nodes for the generated graph.
- p (float) – The redirection probability.
- create_using (NetworkX graph constructor, optional (default DiGraph)) – Graph type to create. If graph instance, then cleared before populated.
- seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Examples

To create the undirected GNR graph, use the to_directed() method:

```python
>>> D = nx.gnr_graph(10, 0.5)  # the GNR graph
>>> G = D.to_undirected()  # the undirected version
```

References

5.10.3 networkx.generators.directed.gnc_graph

gnc_graph(n, create_using=None, seed=None)

Returns the growing network with copying (GNC) digraph with n nodes.

The GNC graph is built by adding nodes one at a time with a link to one previously added node (chosen uniformly at random) and to all of that node’s successors.

Parameters

- n (int) – The number of nodes for the generated graph.
- create_using (NetworkX graph constructor, optional (default DiGraph)) – Graph type to create. If graph instance, then cleared before populated.
- seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

References

5.10.4 networkx.generators.directed.random_k_out_graph

random_k_out_graph(n, k, alpha, self_loops=True, seed=None)

Returns a random k-out graph with preferential attachment.

A random k-out graph with preferential attachment is a multidigraph generated by the following algorithm.

1. Begin with an empty digraph, and initially set each node to have weight alpha.
2. Choose a node \( u \) with out-degree less than \( k \) uniformly at random.
3. Choose a node \( v \) from with probability proportional to its weight.
4. Add a directed edge from \( u \) to \( v \), and increase the weight of \( v \) by one.
5. If each node has out-degree \( k \), halt, otherwise repeat from step 2.
For more information on this model of random graph, see [1].

Parameters

- **n** (*int*) – The number of nodes in the returned graph.
- **k** (*int*) – The out-degree of each node in the returned graph.
- **alpha** (*float*) – A positive float representing the initial weight of each vertex. A higher number means that in step 3 above, nodes will be chosen more like a true uniformly random sample, and a lower number means that nodes are more likely to be chosen as their in-degree increases. If this parameter is not positive, a ValueError is raised.
- **self_loops** (*bool*) – If True, self-loops are allowed when generating the graph.
- **seed** (*integer, random_state, or None (default)*) – Indicator of random number generation state. See Randomness.

Returns

A k-out-regular multidigraph generated according to the above algorithm.

Return type

MultiDiGraph

Raises

ValueError – If alpha is not positive.

Notes

The returned multidigraph may not be strongly connected, or even weakly connected.

References


5.10.5 networkx.generators.directed.scale_free_graph

scale_free_graph(*n*, **alpha=0.41, beta=0.54, gamma=0.05, delta_in=0.2, delta_out=0, create_using=None, seed=None)*

Returns a scale-free directed graph.

Parameters

- **n** (*integer*) – Number of nodes in graph
- **alpha** (*float*) – Probability for adding a new node connected to an existing node chosen randomly according to the in-degree distribution.
- **beta** (*float*) – Probability for adding an edge between two existing nodes. One existing node is chosen randomly according the in-degree distribution and the other chosen randomly according to the out-degree distribution.
- **gamma** (*float*) – Probability for adding a new node connected to an existing node chosen randomly according to the out-degree distribution.
- **delta_in** (*float*) – Bias for choosing nodes from in-degree distribution.
- **delta_out** (*float*) – Bias for choosing nodes from out-degree distribution.
- **create_using** (*NetworkX graph constructor, optional*) – The default is a MultiDiGraph 3-cycle. If a graph instance, use it without clearing first. If a graph constructor, call it to construct an empty graph.
• **seed** *(integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.*

### Examples

Create a scale-free graph on one hundred nodes:

```python
>>> G = nx.scale_free_graph(100)
```

### Notes

The sum of alpha, beta, and gamma must be 1.

### References

#### 5.11 Geometric

Generators for geometric graphs.

<table>
<thead>
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<th>Function</th>
<th>Description</th>
</tr>
</thead>
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<td><code>random_geometric_graph(n, radius[, dim, ...])</code></td>
<td>Returns a random geometric graph in the unit cube of dimensions dim.</td>
</tr>
<tr>
<td><code>soft_random_geometric_graph(n[, radius[, ...]])</code></td>
<td>Returns a soft random geometric graph in the unit cube.</td>
</tr>
<tr>
<td><code>geographical_threshold_graph(n[, theta[, ...]])</code></td>
<td>Returns a geographical threshold graph.</td>
</tr>
<tr>
<td><code>waxman_graph(n[, beta, alpha, L, domain, ...])</code></td>
<td>Returns a Waxman random graph.</td>
</tr>
<tr>
<td><code>navigable_small_world_graph(n[, p, q, r,...])</code></td>
<td>Returns a navigable small-world graph.</td>
</tr>
<tr>
<td><code>thresholded_random_geometric_graph(n[, ...])</code></td>
<td>Returns a thresholded random geometric graph in the unit cube.</td>
</tr>
</tbody>
</table>

#### 5.11.1 networkx.generators.geometric.random_geometric_graph

`random_geometric_graph (n, radius, dim=2, pos=None, p=2, seed=None)`

Returns a random geometric graph in the unit cube of dimensions dim.

The random geometric graph model places n nodes uniformly at random in the unit cube. Two nodes are joined by an edge if the distance between the nodes is at most radius.

Edges are determined using a KDTree when SciPy is available. This reduces the time complexity from $O(n^2)$ to $O(n)$.

**Parameters**

- **n** *(int or iterable) – Number of nodes or iterable of nodes*
- **radius** *(float) – Distance threshold value*
- **dim** *(int, optional) – Dimension of graph*
- **pos** *(dict, optional) – A dictionary keyed by node with node positions as values.*
• **p** (*float*, *optional*) – Which Minkowski distance metric to use. *p* has to meet the condition \(1 \leq p \leq \infty\).

   If this argument is not specified, the \(L^2\) metric (the Euclidean distance metric), \(p = 2\) is used. This should not be confused with the *p* of an Erdős-Rényi random graph, which represents probability.

• **seed** (*integer, random_state, or None (default)*) – Indicator of random number generation state. See *Randomness*.

**Returns** A random geometric graph, undirected and without self-loops. Each node has a node attribute 'pos' that stores the position of that node in Euclidean space as provided by the *pos* keyword argument or, if *pos* was not provided, as generated by this function.

**Return type** *Graph*

**Examples**

Create a random geometric graph on twenty nodes where nodes are joined by an edge if their distance is at most 0.1:

```python
>>> G = nx.random_geometric_graph(20, 0.1)
```

**Notes**

This uses a *k*-d tree to build the graph.

The *pos* keyword argument can be used to specify node positions so you can create an arbitrary distribution and domain for positions.

For example, to use a 2D Gaussian distribution of node positions with mean (0, 0) and standard deviation 2:

```python
>>> import random

>>> n = 20

>>> pos = {i: (random.gauss(0, 2), random.gauss(0, 2)) for i in range(n)}

>>> G = nx.random_geometric_graph(n, 0.2, pos=pos)
```

**References**

5.11.2 *networkx*.*generators.geometric*.soft_random_geometric_graph

**soft_random_geometric_graph** (*n*, *radius*, *dim=2*, *pos=None*, *p=2*, *p_dist=None*, *seed=None*)

Returns a soft random geometric graph in the unit cube.

The soft random geometric graph [1] model places *n* nodes uniformly at random in the unit cube in dimension *dim*. Two nodes of distance, *dist*, computed by the *p*-Minkowski distance metric are joined by an edge with probability *p_dist* if the computed distance metric value of the nodes is at most *radius*, otherwise they are not joined.

Edges within *radius* of each other are determined using a KDTree when SciPy is available. This reduces the time complexity from \(O(n^2)\) to \(O(n)\).

**Parameters**

• **n** (*int or iterable*) – Number of nodes or iterable of nodes

• **radius** (*float*) – Distance threshold value
• **dim** (*int, optional*) – Dimension of graph

• **pos** (*dict, optional*) – A dictionary keyed by node with node positions as values.

• **p** (*float, optional*) – Which Minkowski distance metric to use. \( p \) has to meet the condition \( 1 \leq p \leq \infty \).

  If this argument is not specified, the \( L^2 \) metric (the Euclidean distance metric), \( p = 2 \) is used.

  This should not be confused with the \( p \) of an Erdős-Rényi random graph, which represents probability.

• **p_dist** (*function, optional*) – A probability density function computing the probability of connecting two nodes that are of distance, \( \text{dist} \), computed by the Minkowski distance metric. The probability density function, \( \text{p_dist} \), must be any function that takes the metric value as input and outputs a single probability value between 0-1. The scipy.stats package has many probability distribution functions implemented and tools for custom probability distribution definitions [2], and passing the .pdf method of scipy.stats distributions can be used here. If the probability function, \( \text{p_dist} \), is not supplied, the default function is an exponential distribution with rate parameter \( \lambda = 1 \).

• **seed** (*integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns A soft random geometric graph, undirected and without self-loops. Each node has a node attribute 'pos' that stores the position of that node in Euclidean space as provided by the pos keyword argument or, if pos was not provided, as generated by this function.

Return type **Graph**

Examples

Default Graph:

\[ G = nx.soft_random_geometric_graph(50, 0.2) \]

Custom Graph:

Create a soft random geometric graph on 100 uniformly distributed nodes where nodes are joined by an edge with probability computed from an exponential distribution with rate parameter \( \lambda = 1 \) if their Euclidean distance is at most 0.2.

Notes

This uses a \( k \)-d tree to build the graph.

The pos keyword argument can be used to specify node positions so you can create an arbitrary distribution and domain for positions.

For example, to use a 2D Gaussian distribution of node positions with mean \((0, 0)\) and standard deviation 2

The scipy.stats package can be used to define the probability distribution with the .pdf method used as p_dist.

```python
>>> import random
>>> import math

>>> n = 100
>>> pos = {i: (random.gauss(0, 2), random.gauss(0, 2)) for i in range(n)}
>>> p_dist = lambda dist : math.exp(-dist)
>>> G = nx.soft_random_geometric_graph(n, 0.2, pos=pos, p_dist=p_dist)
```
5.11.3 networkx.generators.geometric.geographical_threshold_graph

def geographical_threshold_graph(n, theta, dim=2, pos=None, weight=None, metric=None, p_dist=None, seed=None):
    Returns a geographical threshold graph.

    The geographical threshold graph model places $n$ nodes uniformly at random in a rectangular domain. Each
    node $u$ is assigned a weight $w_u$. Two nodes $u$ and $v$ are joined by an edge if
    $$(w_u + w_v)h(r) \geq \theta$$
    where $r$ is the distance between $u$ and $v$, $h(r)$ is a probability of connection as a function of $r$, and $\theta$ as the
    threshold parameter. $h(r)$ corresponds to the p_dist parameter.

    Parameters
    
    - **n** (*int or iterable*) – Number of nodes or iterable of nodes
    - **theta** (*float*) – Threshold value
    - **dim** (*int, optional*) – Dimension of graph
    - **pos** (*dict*) – Node positions as a dictionary of tuples keyed by node.
    - **weight** (*dict*) – Node weights as a dictionary of numbers keyed by node.
    - **metric** (*function*) – A metric on vectors of numbers (represented as lists or tuples). This
      must be a function that accepts two lists (or tuples) as input and yields a number as output.
      The function must also satisfy the four requirements of a metric. Specifically, if $d$ is the
      function and $x, y, z$ are vectors in the graph, then $d$ must satisfy
      1. $d(x, y) \geq 0$,
      2. $d(x, y) = 0$ if and only if $x = y$,
      3. $d(x, y) = d(y, x)$,
      4. $d(x, z) \leq d(x, y) + d(y, z)$.
      If this argument is not specified, the Euclidean distance metric is used.
    - **p_dist** (*function, optional*) – A probability density function computing the probability of
      connecting two nodes that are of distance, $r$, computed by metric. The probability density
      function, p_dist, must be any function that takes the metric value as input and outputs a
      single probability value between 0-1. The scipy.stats package has many probability distri-
      bution functions implemented and tools for custom probability distribution definitions [2],
      and passing the .pdf method of scipy.stats distributions can be used here. If the probability
      function, p_dist, is not supplied, the default exponential function :math: $r^{-2}$ is used.
    - **seed** (*integer, random_state, or None (default)) – Indicator of random number generation
      state. See Randomness.

    Returns

    A random geographic threshold graph, undirected and without self-loops.

    Each node has a node attribute pos that stores the position of that node in Euclidean space
    as provided by the pos keyword argument or, if pos was not provided, as generated by this
    function. Similarly, each node has a node attribute weight that stores the weight of that node
    as provided or as generated.

    Return type **Graph**
Examples

Specify an alternate distance metric using the `metric` keyword argument. For example, to use the taxicab metric instead of the default Euclidean metric:

```python
>>> dist = lambda x, y: sum(abs(a - b) for a, b in zip(x, y))
>>> G = nx.geographical_threshold_graph(10, 0.1, metric=dist)
```

Notes

If weights are not specified they are assigned to nodes by drawing randomly from the exponential distribution with rate parameter $\lambda = 1$. To specify weights from a different distribution, use the `weight` keyword argument:

```python
>>> import random

>>> n = 20

>>> w = {i: random.expovariate(5.0) for i in range(n)}
>>> G = nx.geographical_threshold_graph(20, 50, weight=w)
```

If node positions are not specified they are randomly assigned from the uniform distribution.

References

5.11.4 networkx.generators.geometric.waxman_graph

`waxman_graph(n, beta=0.4, alpha=0.1, L=None, domain=(0, 0, 1, 1), metric=None, seed=None)`

Returns a Waxman random graph.

The Waxman random graph model places $n$ nodes uniformly at random in a rectangular domain. Each pair of nodes at distance $d$ is joined by an edge with probability

$$p = \beta \exp(-d/\alpha L).$$

This function implements both Waxman models, using the `L` keyword argument.

- Waxman-1: if $L$ is not specified, it is set to be the maximum distance between any pair of nodes.
- Waxman-2: if $L$ is specified, the distance between a pair of nodes is chosen uniformly at random from the interval $[0, L]$.

Parameters

- `n (int or iterable)` – Number of nodes or iterable of nodes
- `beta (float)` – Model parameter
- `alpha (float)` – Model parameter
- `L (float, optional)` – Maximum distance between nodes. If not specified, the actual distance is calculated.
- `domain (four-tuple of numbers, optional)` – Domain size, given as a tuple of the form $(x_{\text{min}}, y_{\text{min}}, x_{\text{max}}, y_{\text{max}})$.
- `metric (function)` – A metric on vectors of numbers (represented as lists or tuples). This must be a function that accepts two lists (or tuples) as input and yields a number as output. The function must also satisfy the four requirements of a metric. Specifically, if $d$ is the function and $x, y, z$ are vectors in the graph, then $d$ must satisfy

$$(x - y) - (x - z) \leq d(x, y) + d(y, z).$$
1. \(d(x, y) \geq 0\),
2. \(d(x, y) = 0\) if and only if \(x = y\),
3. \(d(x, y) = d(y, x)\),
4. \(d(x, z) \leq d(x, y) + d(y, z)\).

If this argument is not specified, the Euclidean distance metric is used.

- **seed** (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

**Returns** A random Waxman graph, undirected and without self-loops. Each node has a node attribute 'pos' that stores the position of that node in Euclidean space as generated by this function.

**Return type** Graph

**Examples**

Specify an alternate distance metric using the `metric` keyword argument. For example, to use the “taxicab metric” instead of the default Euclidean metric:

```python
>>> dist = lambda x, y: sum(abs(a - b) for a, b in zip(x, y))
>>> G = nx.waxman_graph(10, 0.5, 0.1, metric=dist)
```

**Notes**

Starting in NetworkX 2.0 the parameters alpha and beta align with their usual roles in the probability distribution. In earlier versions their positions in the expression were reversed. Their position in the calling sequence reversed as well to minimize backward incompatibility.

**References**

5.11.5 `networkx.generators.geometric.navigable_small_world_graph`

`navigable_small_world_graph (n, p=1, q=1, r=2, dim=2, seed=None)`

Returns a navigable small-world graph.

A navigable small-world graph is a directed grid with additional long-range connections that are chosen randomly.

[...] we begin with a set of nodes [...] that are identified with the set of lattice points in an \(n \times n\) square, \(\{(i, j) : i \in \{1, 2, \ldots, n\}, j \in \{1, 2, \ldots, n\}\}\), and we define the lattice distance between two nodes \((i, j)\) and \((k, l)\) to be the number of “lattice steps” separating them: \(d((i, j), (k, l)) = |k - i| + |l - j|\).

For a universal constant \(p \geq 1\), the node \(u\) has a directed edge to every other node within lattice distance \(p\)—these are its local contacts. For universal constants \(q \geq 0\) and \(r \geq 0\) we also construct directed edges from \(u\) to \(q\) other nodes (the long-range contacts) using independent random trials; the \(i\) has endpoint \(v\) with probability proportional to \([d(u, v)]^{-r}\).

---

Parameters

- \( n \) (int) – The length of one side of the lattice; the number of nodes in the graph is therefore \( n^2 \).
- \( p \) (int) – The diameter of short range connections. Each node is joined with every other node within this lattice distance.
- \( q \) (int) – The number of long-range connections for each node.
- \( r \) (float) – Exponent for decaying probability of connections. The probability of connecting to a node at lattice distance \( d \) is \( 1/d^r \).
- \( \text{dim} \) (int) – Dimension of grid
- \( \text{seed} \) (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

References

5.11.6 networkx.generators.geometric.thresholded_random_geometric_graph

thresholded_random_geometric_graph \((n, radius, theta, \text{dim}=2, \text{pos}=None, \text{weight}=None, p=2, \text{seed}=None)\)

Returns a thresholded random geographic graph in the unit cube.

The thresholded random geographic graph [1] model places \( n \) nodes uniformly at random in the unit cube of dimensions \( \text{dim} \). Each node \( u \) is assigned a weight \( w_u \). Two nodes \( u \) and \( v \) are joined by an edge if they are within the maximum connection distance, \( \text{radius} \) computed by the \( p \)-Minkowski distance and the summation of weights \( w_u + w_v \) is greater than or equal to the threshold parameter \( \theta \).

Edges within \( \text{radius} \) of each other are determined using a KDTree when SciPy is available. This reduces the time complexity from \( O(n^2) \) to \( O(n) \).

Parameters

- \( n \) (int or iterable) – Number of nodes or iterable of nodes
- \( \text{radius} \) (float) – Distance threshold value
- \( \text{theta} \) (float) – Threshold value
- \( \text{dim} \) (int, optional) – Dimension of graph
- \( \text{pos} \) (dict, optional) – A dictionary keyed by node with node positions as values.
- \( \text{weight} \) (dict, optional) – Node weights as a dictionary of numbers keyed by node.
- \( p \) (float, optional) – Which Minkowski distance metric to use. \( p \) has to meet the condition \( 1 \leq p \leq \infty \).

If this argument is not specified, the \( L^2 \) metric (the Euclidean distance metric), \( p = 2 \) is used. This should not be confused with the \( p \) of an Erdős-Rényi random graph, which represents probability.
- \( \text{seed} \) (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns

A thresholded random geographic graph, undirected and without self-loops.
Each node has a node attribute 'pos' that stores the position of that node in Euclidean space as provided by the pos keyword argument or, if pos was not provided, as generated by this function. Similarly, each node has a node attribute 'weight' that stores the weight of that node as provided or as generated.

**Return type**  
*Graph*

**Examples**

Default Graph:

G = nx.thresholded_random_geometric_graph(50, 0.2, 0.1)

Custom Graph:

Create a thresholded random geometric graph on 50 uniformly distributed nodes where nodes are joined by an edge if their sum weights drawn from an exponential distribution with rate = 5 are >= theta = 0.1 and their Euclidean distance is at most 0.2.

**Notes**

This uses a k-d tree to build the graph.

The pos keyword argument can be used to specify node positions so you can create an arbitrary distribution and domain for positions.

For example, to use a 2D Gaussian distribution of node positions with mean (0, 0) and standard deviation 2

If weights are not specified they are assigned to nodes by drawing randomly from the exponential distribution with rate parameter \( \lambda = 1 \). To specify weights from a different distribution, use the weight keyword argument:

```python
>>> import random
>>> import math

>>> n = 50
>>> pos = {i: (random.gauss(0, 2), random.gauss(0, 2)) for i in range(n)}
>>> w = {i: random.expovariate(5.0) for i in range(n)}
>>> G = nx.thresholded_random_geometric_graph(n, 0.2, 0.1, 2, pos, w)
```

**References**

5.12 Line Graph

Functions for generating line graphs.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
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<tr>
<td><code>line_graph(G[, create_using])</code></td>
<td>Returns the line graph of the graph or digraph G.</td>
</tr>
<tr>
<td><code>inverse_line_graph(G)</code></td>
<td>Returns the inverse line graph of graph G.</td>
</tr>
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</table>

5.12.1 networkx.generators.line.line_graph

`line_graph(G, create_using=None)`  
Returns the line graph of the graph or digraph G.
The line graph of a graph $G$ has a node for each edge in $G$ and an edge joining those nodes if the two edges in $G$ share a common node. For directed graphs, nodes are adjacent exactly when the edges they represent form a directed path of length two.

The nodes of the line graph are 2-tuples of nodes in the original graph (or 3-tuples for multigraphs, with the key of the edge as the third element).

For information about self-loops and more discussion, see the Notes section below.

**Parameters**

- **$G$ (graph)** – A NetworkX Graph, DiGraph, MultiGraph, or MultiDiGraph.

- **create_using (NetworkX graph constructor, optional (default=nx.Graph))** – Graph type to create. If graph instance, then cleared before populated.

**Returns** $L$ – The line graph of $G$.

**Return type** graph

**Examples**

```python
>>> import networkx as nx
>>> G = nx.star_graph(3)
>>> L = nx.line_graph(G)
>>> print(sorted(map(sorted, L.edges())))  # makes a 3-clique, K3
[(0, 1), (0, 2), (0, 3)]
```

**Notes**

Graph, node, and edge data are not propagated to the new graph. For undirected graphs, the nodes in $G$ must be sortable, otherwise the constructed line graph may not be correct.

**Self-loops in undirected graphs**

For an undirected graph $G$ without multiple edges, each edge can be written as a set $\{u, v\}$. Its line graph $L$ has the edges of $G$ as its nodes. If $x$ and $y$ are two nodes in $L$, then $(x, y)$ is an edge in $L$ if and only if the intersection of $x$ and $y$ is nonempty. Thus, the set of all edges is determined by the set of all pairwise intersections of edges in $G$.

Trivially, every edge in $G$ would have a nonzero intersection with itself, and so every node in $L$ should have a self-loop. This is not so interesting, and the original context of line graphs was with simple graphs, which had no self-loops or multiple edges. The line graph was also meant to be a simple graph and thus, self-loops in $L$ are not part of the standard definition of a line graph. In a pairwise intersection matrix, this is analogous to excluding the diagonal entries from the line graph definition.

Self-loops and multiple edges in $G$ add nodes to $L$ in a natural way, and do not require any fundamental changes to the definition. It might be argued that the self-loops we excluded before should now be included. However, the self-loops are still “trivial” in some sense and thus, are usually excluded.

**Self-loops in directed graphs**

For a directed graph $G$ without multiple edges, each edge can be written as a tuple $(u, v)$. Its line graph $L$ has the edges of $G$ as its nodes. If $x$ and $y$ are two nodes in $L$, then $(x, y)$ is an edge in $L$ if and only if the tail of $x$ matches the head of $y$, for example, if $x = (a, b)$ and $y = (b, c)$ for some vertices $a, b,$ and $c$ in $G$.

Due to the directed nature of the edges, it is no longer the case that every edge in $G$ should have a self-loop in $L$. Now, the only time self-loops arise is if a node in $G$ itself has a self-loop. So such self-loops are no longer “trivial” but instead, represent essential features of the topology of $G$. For this reason, the historical development
of line digraphs is such that self-loops are included. When the graph $G$ has multiple edges, once again only superficial changes are required to the definition.

**References**


### 5.12.2 networkx.generators.line.inverse_line_graph

**inverse_line_graph** $(G)$

Returns the inverse line graph of graph $G$.

If $H$ is a graph, and $G$ is the line graph of $H$, such that $H = L(G)$. Then $H$ is the inverse line graph of $G$.

Not all graphs are line graphs and these do not have an inverse line graph. In these cases this generator returns a NetworkXError.

**Parameters**

- $G$ *(graph)* – A NetworkX Graph

**Returns**

- $H$ – The inverse line graph of $G$.

**Return type**

- graph

**Raises**

- `NetworkXNotImplemented` – If $G$ is directed or a multigraph
- `NetworkXError` – If $G$ is not a line graph

**Notes**

This is an implementation of the Roussopolous algorithm.

If $G$ consists of multiple components, then the algorithm doesn’t work. You should invert every component separately:

```python
>>> K5 = nx.complete_graph(5)
>>> P4 = nx.Graph([('a', 'b'), ('b', 'c'), ('c', 'd')])
>>> G = nx.union(K5, P4)
>>> root_graphs = []
>>> for comp in nx.connected_components(G):
...    root_graphs.append(nx.inverse_line_graph(G.subgraph(comp)))
>>> len(root_graphs)
2
```

**References**

5.13 Ego Graph

Ego graph.

\texttt{ego\_graph(G, n[, radius, center, ...])} Returns induced subgraph of neighbors centered at node \(n\) within a given radius.

5.13.1 \texttt{networkx.generators.ego.ego\_graph}

\texttt{ego\_graph(G, n, radius=1, center=True, undirected=False, distance=None)}

Returns induced subgraph of neighbors centered at node \(n\) within a given radius.

**Parameters**

- \(G\) (graph) – A NetworkX Graph or DiGraph
- \(n\) (node) – A single node
- \(radius\) (number, optional) – Include all neighbors of distance\(<=\)radius from \(n\).
- \(center\) (bool, optional) – If False, do not include center node in graph
- \(undirected\) (bool, optional) – If True use both in- and out-neighbors of directed graphs.
- \(distance\) (key, optional) – Use specified edge data key as distance. For example, setting distance='weight' will use the edge weight to measure the distance from the node \(n\).

**Notes**

For directed graphs \(D\) this produces the “out” neighborhood or successors. If you want the neighborhood of predecessors first reverse the graph with \(D\).reverse(). If you want both directions use the keyword argument undirected=True.

Node, edge, and graph attributes are copied to the returned subgraph.

5.14 Stochastic

Functions for generating stochastic graphs from a given weighted directed graph.

\texttt{stochastic\_graph(G[, copy, weight])} Returns a right-stochastic representation of directed graph \(G\).

5.14.1 \texttt{networkx.generators.stochastic.stochastic\_graph}

\texttt{stochastic\_graph(G, copy=True, weight='weight')} Returns a right-stochastic representation of directed graph \(G\).

A right-stochastic graph is a weighted digraph in which for each node, the sum of the weights of all the out-edges of that node is 1. If the graph is already weighted (for example, via a ‘weight’ edge attribute), the reweighting takes that into account.

**Parameters**
• **G** *(directed graph)* – A `DiGraph` or `MultiDiGraph`.

• **copy** *(boolean, optional)* – If this is True, then this function returns a new graph with the stochastic reweighting. Otherwise, the original graph is modified in-place (and also returned, for convenience).

• **weight** *(edge attribute key (optional, default=’weight’))* – Edge attribute key used for reading the existing weight and setting the new weight. If no attribute with this key is found for an edge, then the edge weight is assumed to be 1. If an edge has a weight, it must be a a positive number.

### 5.15 AS graph

Generates graphs resembling the Internet Autonomous System network

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<th><code>random_internet_as_graph(n[, seed])</code></th>
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#### 5.15.1 networkx.generators.internet_as_graphs.random_internet_as_graph

`random_internet_as_graph(n, seed=None)`

Generates a random undirected graph resembling the Internet AS network

**Parameters**

• **n** *(integer in [1000, 10000])* – Number of graph nodes

• **seed** *(integer, random_state, or None (default))* – Indicator of random number generation state. See `Randomness`.

**Returns**

• **G** – A randomly generated undirected graph

**Return type**

Networkx Graph object

**Notes**

This algorithm returns an undirected graph resembling the Internet Autonomous System (AS) network, it uses the approach by Elmokashfi et al. [1] and it grants the properties described in the related paper [1].

Each node models an autonomous system, with an attribute ‘type’ specifying its kind; tier-1 (T), mid-level (M), customer (C) or content-provider (CP). Each edge models an ADV communication link (hence, bidirectional) with .. attribute:: - `type`  
- `transit`, the kind of commercial agreement between nodes;  
- `customer`  
  `<node id>`, the identifier of the node acting as customer (‘none’ if type is peer).

**References**

5.16 Intersection

Generators for random intersection graphs.

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<td><code>uniform_random_intersection_graph(n, m, p[, . . .])</code></td>
<td>Returns a uniform random intersection graph.</td>
</tr>
<tr>
<td><code>k_random_intersection_graph(n, m, k[, seed])</code></td>
<td>Returns a intersection graph with randomly chosen attribute sets for each node that are of equal size (k).</td>
</tr>
<tr>
<td><code>general_random_intersection_graph(n, m, p[, . . .])</code></td>
<td>Returns a random intersection graph with independent probabilities for connections between node and attribute sets.</td>
</tr>
</tbody>
</table>

5.16.1 networkx.generators.intersection.uniform_random_intersection_graph

`uniform_random_intersection_graph(n, m, p, seed=None)`

Returns a uniform random intersection graph.

Parameters

- `n (int)` – The number of nodes in the first bipartite set (nodes)
- `m (int)` – The number of nodes in the second bipartite set (attributes)
- `p (float)` – Probability of connecting nodes between bipartite sets
- `seed (integer, random_state, or None (default))` – Indicator of random number generation state. See Randomness.

See also:

- `gnp_random_graph()`

References

5.16.2 networkx.generators.intersection.k_random_intersection_graph

`k_random_intersection_graph(n, m, k, seed=None)`

Returns a intersection graph with randomly chosen attribute sets for each node that are of equal size (k).

Parameters

- `n (int)` – The number of nodes in the first bipartite set (nodes)
- `m (int)` – The number of nodes in the second bipartite set (attributes)
- `k (float)` – Size of attribute set to assign to each node.
- `seed (integer, random_state, or None (default))` – Indicator of random number generation state. See Randomness.

See also:

- `gnp_random_graph()`, `uniform_random_intersection_graph()`
5.16.3 networkx.generators.intersection.general_random_intersection_graph

`general_random_intersection_graph(n, m, p, seed=None)`

Returns a random intersection graph with independent probabilities for connections between node and attribute sets.

**Parameters**

- `n` *(int)* – The number of nodes in the first bipartite set (nodes)
- `m` *(int)* – The number of nodes in the second bipartite set (attributes)
- `p` *(list of floats of length m)* – Probabilities for connecting nodes to each attribute
- `seed` *(integer, random_state, or None (default)*) – Indicator of random number generation state. See *Randomness*.

**See also:**

`gnp_random_graph()`, `uniform_random_intersection_graph()`

---

5.17 Social Networks

Famous social networks.

<table>
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<td><code>karate_club_graph()</code></td>
<td>Returns Zachary’s Karate Club graph.</td>
</tr>
<tr>
<td><code>davis_southern_women_graph()</code></td>
<td>Returns Davis Southern women social network.</td>
</tr>
<tr>
<td><code>florentine_families_graph()</code></td>
<td>Returns Florentine families graph.</td>
</tr>
<tr>
<td><code>les_miserables_graph()</code></td>
<td>Returns coappearance network of characters in the novel Les Miserables.</td>
</tr>
</tbody>
</table>

5.17.1 networkx.generators.social.karate_club_graph

`karate_club_graph()`

Returns Zachary’s Karate Club graph.

Each node in the returned graph has a node attribute `club` that indicates the name of the club to which the member represented by that node belongs, either ‘Mr. Hi’ or ‘Officer’.

**Examples**

To get the name of the club to which a node belongs:

```python
>>> import networkx as nx
>>> G = nx.karate_club_graph()
>>> G.nodes[5]['club']
'Mr. Hi'
>>> G.nodes[9]['club']
'Officer'
```
References

5.17.2 networkx.generators.social.davis_southern_women_graph
davis_southern_women_graph()
Returns Davis Southern women social network.
This is a bipartite graph.

References

5.17.3 networkx.generators.social.florentine_families_graph
florentine_families_graph()
Returns Florentine families graph.

References

5.17.4 networkx.generators.social.les_miserables_graph
les_miserables_graph()
Returns coappearance network of characters in the novel Les Miserables.

References

5.18 Community

Generators for classes of graphs used in studying social networks.

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<td>Generate a Gaussian random partition graph.</td>
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<td>LFR_benchmark_graph(n, tau1, tau2, mu[, ...])</td>
<td>Returns the LFR benchmark graph.</td>
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<td>Defines a “ring of cliques” graph.</td>
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<td>windmill_graph(n, k)</td>
<td>Generate a windmill graph.</td>
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</table>

5.18.1 networkx.generators.community.caveman_graph
caveman_graph(l, k)
Returns a caveman graph of l cliques of size k.
Parameters

• \( l (\text{int}) \) – Number of cliques
• \( k (\text{int}) \) – Size of cliques

Returns \( G \) – caveman graph

Returns \( G \) – connected caveman graph

Notes

This returns an undirected graph, it can be converted to a directed graph using `nx.to_directed()`, or a multigraph using `nx.MultiGraph(nx.caveman_graph(l, k))`. Only the undirected version is described in\(^1\) and it is unclear which of the directed generalizations is most useful.

Examples

```python
>>> G = nx.caveman_graph(3, 3)
```

See also:

`connected_caveman_graph()`

References

5.18.2 networkx.generators.community.connected_caveman_graph

`connected_caveman_graph(l, k)`

Returns a connected caveman graph of \( l \) cliques of size \( k \).

The connected caveman graph is formed by creating \( n \) cliques of size \( k \), then a single edge in each clique is rewired to a node in an adjacent clique.

Parameters

• \( l (\text{int}) \) – number of cliques
• \( k (\text{int}) \) – size of cliques

Returns \( G \) – connected caveman graph

Returns \( G \) – connected caveman graph

Notes

This returns an undirected graph, it can be converted to a directed graph using `nx.to_directed()`, or a multigraph using `nx.MultiGraph(nx.caveman_graph(l, k))`. Only the undirected version is described in\(^1\) and it is unclear which of the directed generalizations is most useful.

---


Examples

```python
>>> G = nx.connected_caveman_graph(3, 3)
```

References

5.18.3 networkx.generators.community.gaussian_random_partition_graph

gaussian_random_partition_graph \((n, s, v, p_{in}, p_{out}, \text{directed}=False, \text{seed}=None)\)

Generate a Gaussian random partition graph.

A Gaussian random partition graph is created by creating \(k\) partitions each with a size drawn from a normal distribution with mean \(s\) and variance \(s/v\). Nodes are connected within clusters with probability \(p_{in}\) and between clusters with probability \(p_{out}\)\(^1\)

Parameters

- \(n\) \((\text{int})\) – Number of nodes in the graph
- \(s\) \((\text{float})\) – Mean cluster size
- \(v\) \((\text{float})\) – Shape parameter. The variance of cluster size distribution is \(s/v\).
- \(p_{in}\) \((\text{float})\) – Probability of intra cluster connection.
- \(p_{out}\) \((\text{float})\) – Probability of inter cluster connection.
- \(\text{directed}\) \((\text{boolean}, \text{optional } \text{default}=False)\) – Whether to create a directed graph or not
- \(\text{seed}\) \((\text{integer}, \text{random_state}, \text{or } \text{None (default)})\) – Indicator of random number generation state. See Randomness.

Returns \(G\) – gaussian random partition graph

Return type \ NetworkX Graph or DiGraph

Raises \ NetworkXError \ – If \(s\) is \(> n\) \ If \(p_{in}\) or \(p_{out}\) is not in \([0,1]\)

Notes

Note the number of partitions is dependent on \(s,v\) and \(n\), and that the last partition may be considerably smaller, as it is sized to simply fill out the nodes \(^1\)

See also:

\ random_partition_graph() 

Examples

```python
>>> G = nx.gaussian_random_partition_graph(100,10,10,.25,.1)
>>> len(G)
100
```
5.18.4 networkx.generators.community.LFR_benchmark_graph

LFR_benchmark_graph\( (n, \tau_1, \tau_2, \mu, \text{average_degree}=\text{None}, \text{min_degree}=\text{None}, \text{max_degree}=\text{None}, \text{min_community}=\text{None}, \text{max_community}=\text{None}, \text{tol}=1e-07, \text{max_iters}=500, \text{seed}=\text{None}) \)

Returns the LFR benchmark graph.

This algorithm proceeds as follows:

1) Find a degree sequence with a power law distribution, and minimum value \( \text{min_degree} \), which has approximate average degree \( \text{average_degree} \). This is accomplished by either
   a) specifying \( \text{min_degree} \) and not \( \text{average_degree} \),
   b) specifying \( \text{average_degree} \) and not \( \text{min_degree} \), in which case a suitable minimum degree will be found.

\( \text{max_degree} \) can also be specified, otherwise it will be set to \( n \). Each node \( u \) will have \( \mu \mathbb{m}_\text{deg}(u) \) edges joining it to nodes in communities other than its own and \( (1 - \mu) \mathbb{m}_\text{deg}(u) \) edges joining it to nodes in its own community.

2) Generate community sizes according to a power law distribution with exponent \( \tau_2 \). If \( \text{min_community} \) and \( \text{max_community} \) are not specified they will be selected to be \( \text{min_degree} \) and \( \text{max_degree} \), respectively. Community sizes are generated until the sum of their sizes equals \( n \).

3) Each node will be randomly assigned a community with the condition that the community is large enough for the node’s intra-community degree, \( (1 - \mu) \mathbb{m}_\text{deg}(u) \) as described in step 2. If a community grows too large, a random node will be selected for reassignment to a new community, until all nodes have been assigned a community.

4) Each node \( u \) then adds \( (1 - \mu) \mathbb{m}_\text{deg}(u) \) intra-community edges and \( \mu \mathbb{m}_\text{deg}(u) \) inter-community edges.

Parameters

- \( n \ (\text{int}) \) – Number of nodes in the created graph.
- \( \tau_1 \ (\text{float}) \) – Power law exponent for the degree distribution of the created graph. This value must be strictly greater than one.
- \( \tau_2 \ (\text{float}) \) – Power law exponent for the community size distribution in the created graph. This value must be strictly greater than one.
- \( \mu \ (\text{float}) \) – Fraction of intra-community edges incident to each node. This value must be in the interval \([0, 1]\).
- \( \text{average_degree} \ (\text{float}) \) – Desired average degree of nodes in the created graph. This value must be in the interval \([0, n]\). Exactly one of this and \( \text{min_degree} \) must be specified, otherwise a NetworkXError is raised.
- \( \text{min_degree} \ (\text{int}) \) – Minimum degree of nodes in the created graph. This value must be in the interval \([0, n]\). Exactly one of this and \( \text{average_degree} \) must be specified, otherwise a NetworkXError is raised.
- \( \text{max_degree} \ (\text{int}) \) – Maximum degree of nodes in the created graph. If not specified, this is set to \( n \), the total number of nodes in the graph.
- \( \text{min_community} \ (\text{int}) \) – Minimum size of communities in the graph. If not specified, this is set to \( \text{min_degree} \).
• **max_community** (*int*) – Maximum size of communities in the graph. If not specified, this is set to \(n\), the total number of nodes in the graph.

• **tol** (*float*) – Tolerance when comparing floats, specifically when comparing average degree values.

• **max_iters** (*int*) – Maximum number of iterations to try to create the community sizes, degree distribution, and community affiliations.

• **seed** (*integer, random_state, or None (default)) – Indicator of random number generation state. See *Randomness*.

**Returns**

\(G\) – The LFR benchmark graph generated according to the specified parameters.

Each node in the graph has a node attribute ‘community’ that stores the community (that is, the set of nodes) that includes it.

**Return type** NetworkX graph

**Raises**

• **NetworkXError** – If any of the parameters do not meet their upper and lower bounds:
  – \(\tau_1\) and \(\tau_2\) must be strictly greater than 1.
  – \(\mu\) must be in \([0, 1]\).
  – \(\text{max\_degree}\) must be in \([1, \ldots, n]\).
  – \(\text{min\_community}\) and \(\text{max\_community}\) must be in \([0, \ldots, n]\).

If not exactly one of \(\text{average\_degree}\) and \(\text{min\_degree}\) is specified.

If \(\text{min\_degree}\) is not specified and a suitable \(\text{min\_degree}\) cannot be found.

• **ExceededMaxIterations** – If a valid degree sequence cannot be created within \(\text{max\_iters}\) number of iterations.

If a valid set of community sizes cannot be created within \(\text{max\_iters}\) number of iterations.

If a valid community assignment cannot be created within \(\text{10 \times n \times max\_iters}\) number of iterations.

**Examples**

Basic usage:

```python
>>> from networkx.generators.community import LFR_benchmark_graph
>>> n = 250
>>> tau1 = 3
>>> tau2 = 1.5
>>> mu = 0.1
>>> G = LFR_benchmark_graph(n, tau1, tau2, mu, average_degree=5,
...                          min_community=20, seed=10)
```

Continuing the example above, you can get the communities from the node attributes of the graph:

```python
>>> communities = {frozenset(G.nodes[v]['community']) for v in G}
```
Notes

This algorithm differs slightly from the original way it was presented in [1].

1) Rather than connecting the graph via a configuration model then rewiring to match the intra-community and inter-community degrees, we do this wiring explicitly at the end, which should be equivalent.

2) The code posted on the author's website [2] calculates the random power law distributed variables and their average using continuous approximations, whereas we use the discrete distributions here as both degree and community size are discrete.

Though the authors describe the algorithm as quite robust, testing during development indicates that a somewhat narrower parameter set is likely to successfully produce a graph. Some suggestions have been provided in the event of exceptions.

References

5.18.5 networkx.generators.community.planted_partition_graph

planted_partition_graph(l, k, p_in, p_out, seed=None, directed=False)

Returns the planted l-partition graph.

This model partitions a graph with n=l*k vertices in l groups with k vertices each. Vertices of the same group are linked with a probability p_in, and vertices of different groups are linked with probability p_out.

Parameters

- l (int) – Number of groups
- k (int) – Number of vertices in each group
- p_in (float) – probability of connecting vertices within a group
- p_out (float) – probability of connected vertices between groups
- seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.
- directed (bool,optional (default=False)) – If True return a directed graph

Returns G – planted l-partition graph

Return type NetworkX Graph or DiGraph

Raises NetworkXError – If p_in,p_out are not in [0,1] or

Examples

>>> G = nx.planted_partition_graph(4, 3, 0.5, 0.1, seed=42)

See also:

random_partition_model()
References

5.18.6 networkx.generators.community.random_partition_graph

random_partition_graph \(\text{(sizes, p\_in, p\_out, seed=None, directed=False)}\)

Returns the random partition graph with a partition of sizes.

A partition graph is a graph of communities with sizes defined by s in sizes. Nodes in the same group are connected with probability p\_in and nodes of different groups are connected with probability p\_out.

Parameters

- sizes \((\text{list of ints})\) – Sizes of groups
- p\_in \((\text{float})\) – probability of edges within groups
- p\_out \((\text{float})\) – probability of edges between groups
- directed \((\text{boolean optional, default=False})\) – Whether to create a directed graph
- seed \((\text{integer, random_state, or None (default)})\) – Indicator of random number generation state. See Randomness.

Returns G – random partition graph of size sum(gs)

Return type NetworkX Graph or DiGraph

Raises NetworkXError – If p\_in or p\_out is not in \([0,1]\)

Examples

```python
>>> G = nx.random_partition_graph([10,10,10],.25,.01)
>>> len(G)
30
>>> partition = G.graph['partition']
>>> len(partition)
3
```

Notes

This is a generalization of the planted-l-partition described in\(^1\). It allows for the creation of groups of any size. The partition is stored as a graph attribute ‘partition’.

References

5.18.7 networkx.generators.community.relaxed_caveman_graph

relaxed_caveman_graph \(\text{(l, k, p, seed=None)}\)

Returns a relaxed caveman graph.

A relaxed caveman graph starts with \(l\) cliques of size \(k\). Edges are then randomly rewired with probability \(p\) to link different cliques.

Parameters

- \(l\) \((\text{int})\) – Number of groups

---

• \( k \) (int) – Size of cliques
• \( p \) (float) – Probability of rewiring each edge.
• seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns  \( G \) – Relaxed Caveman Graph
Return type  NetworkX Graph
Raises  NetworkXError – If \( p \) is not in [0,1]

Examples

```python
>>> G = nx.relaxed_caveman_graph(2, 3, 0.1, seed=42)
```

References

5.18.8 networkx.generators.community.ring_of_cliques

ring_of_cliques \((num\_cliques, clique\_size)\)
Defines a “ring of cliques” graph.
A ring of cliques graph is consisting of cliques, connected through single links. Each clique is a complete graph.

Parameters
• num_cliques (int) – Number of cliques
• clique_size (int) – Size of cliques

Returns  \( G \) – ring of cliques graph
Return type  NetworkX Graph
 Raises  NetworkXError – If the number of cliques is lower than 2 or if the size of cliques is smaller than 2.

Examples

```python
>>> G = nx.ring_of_cliques(8, 4)
```

See also:
connected_caveman_graph()

Notes
The connected_caveman_graph graph removes a link from each clique to connect it with the next clique. Instead, the ring_of_cliques graph simply adds the link without removing any link from the cliques.
5.18.9 networkx.generators.community.stochastic_block_model

stochastic_block_model(sizes, p, nodelist=None, seed=None, directed=False, selfloops=False, sparse=True)

Returns a stochastic block model graph.

This model partitions the nodes in blocks of arbitrary sizes, and places edges between pairs of nodes independently, with a probability that depends on the blocks.

Parameters

- **sizes** *(list of ints)* – Sizes of blocks
- **p** *(list of list of floats)* – Element \((r, s)\) gives the density of edges going from the nodes of group \(r\) to nodes of group \(s\). \(p\) must match the number of groups \((\text{len}(\text{sizes}) == \text{len}(p))\), and it must be symmetric if the graph is undirected.
- **nodelist** *(list, optional)* – The block tags are assigned according to the node identifiers in nodelist. If nodelist is None, then the ordering is the range \([0, \text{sum}(\text{sizes})-1]\).
- **seed** *(integer, random_state, or None (default))* – Indicator of random number generation state. See Randomness.
- **directed** *(boolean optional, default=False)* – Whether to create a directed graph or not.
- **selfloops** *(boolean optional, default=False)* – Whether to include self-loops or not.
- **sparse** *(boolean optional, default=True)* – Use the sparse heuristic to speed up the generator.

Returns **g** – Stochastic block model graph of size \(\text{sum}(\text{sizes})\)

Return type  NetworkX Graph or DiGraph

Raises **NetworkXError** – If probabilities are not in \([0,1] \). If the probability matrix is not square (directed case). If the probability matrix is not symmetric (undirected case). If the sizes list does not match nodelist or the probability matrix. If nodelist contains duplicate.

Examples

```python
>>> sizes = [75, 75, 300]
>>> probs = [[0.25, 0.05, 0.02],
... [0.05, 0.35, 0.07],
... [0.02, 0.07, 0.40]]
>>> g = nx.stochastic_block_model(sizes, probs, seed=0)
>>> len(g)
450
>>> H = nx.quotient_graph(g, g.graph['partition'], relabel=True)
>>> for v in H.nodes(data=True):
...    print(round(v[1]['density'], 3))
... 0.245
... 0.348
... 0.405
>>> for v in H.edges(data=True):
...    print(round(1.0 * v[2]['weight'] / (sizes[v[0]] * sizes[v[1]]), 3))
... 0.051
... 0.022
... 0.07
```
See also:

`random_partition_graph()`, `planted_partition_graph()`, `gaussian_random_partition_graph()`, `gnp_random_graph()`

References

5.18.10 networkx.generators.community.windmill_graph

`windmill_graph(n, k)`

Generate a windmill graph. A windmill graph is a graph of \( n \) cliques each of size \( k \) that are all joined at one node. It can be thought of as taking a disjoint union of \( n \) cliques of size \( k \), selecting one point from each, and contracting all of the selected points. Alternatively, one could generate \( n \) cliques of size \( k-1 \) and one node that is connected to all other nodes in the graph.

Parameters

- \( n \) (int) – Number of cliques
- \( k \) (int) – Size of cliques

Returns \( G \) – windmill graph with \( n \) cliques of size \( k \)

Return type: NetworkX Graph

 Raises `NetworkXError` – If the number of cliques is less than two If the size of the cliques are less than two

Examples

```python
>>> G = nx.windmill_graph(4, 5)
```

Notes

The node labeled 0 will be the node connected to all other nodes. Note that windmill graphs are usually denoted \( W_d(k,n) \), so the parameters are in the opposite order as the parameters of this method.

5.19 Spectral

Generates graphs with a given eigenvector structure

`spectral_graph_forge(G, alpha[, ...])` Returns a random simple graph with spectrum resembling that of \( G \)

5.19.1 networkx.generators.spectral_graph_forge.spectral_graph_forge

`spectral_graph_forge(G, alpha, transformation='identity', seed=None)`

Returns a random simple graph with spectrum resembling that of \( G \)

This algorithm, called Spectral Graph Forge (SGF), computes the eigenvectors of a given graph adjacency matrix, filters them and builds a random graph with a similar eigenstructure. SGF has been proved to be particularly useful for synthesizing realistic social networks and it can also be used to anonymize graph sensitive data.
Parameters

- **G** (*Graph*)
- **alpha** (*float*) – Ratio representing the percentage of eigenvectors of G to consider, values in [0,1].
- **transformation** (*string, optional*) – Represents the intended matrix linear transformation, possible values are ‘identity’ and ‘modularity’
- **seed** (*integer, random_state, or None (default)*) – Indicator of numpy random number generation state. See Randomness.

Returns **H** – A graph with a similar eigenvector structure of the input one.

Return type **Graph**

Raises **NetworkXError** – If transformation has a value different from ‘identity’ or ‘modularity’

Notes

Spectral Graph Forge (SGF) generates a random simple graph resembling the global properties of the given one. It leverages the low-rank approximation of the associated adjacency matrix driven by the alpha precision parameter. SGF preserves the number of nodes of the input graph and their ordering. This way, nodes of output graphs resemble the properties of the input one and attributes can be directly mapped.

It considers the graph adjacency matrices which can optionally be transformed to other symmetric real matrices (currently transformation options include identity and modularity). The modularity transformation, in the sense of Newman’s modularity matrix allows the focusing on community structure related properties of the graph.

SGF applies a low-rank approximation whose fixed rank is computed from the ratio alpha of the input graph adjacency matrix dimension. This step performs a filtering on the input eigenvectors similar to the low pass filtering common in telecommunications.

The filtered values (after truncation) are used as input to a Bernoulli sampling for constructing a random adjacency matrix.

References

Examples

```python
>>> import networkx as nx
>>> G = nx.karate_club_graph()
>>> H = nx.spectral_graph_forge(G, 0.3)
```

5.20 Trees

Functions for generating trees.

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<th>Function</th>
<th>Description</th>
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</thead>
<tbody>
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<td>random_tree(n, seed)</td>
<td>Returns a uniformly random tree on n nodes.</td>
</tr>
<tr>
<td>prefix_tree(paths)</td>
<td>Creates a directed prefix tree from the given list of iterables.</td>
</tr>
</tbody>
</table>
5.20.1 networkx.generators.trees.random_tree

**random_tree** (*n, seed=None*)

Returns a uniformly random tree on *n* nodes.

**Parameters**

- **n** (*int*) – A positive integer representing the number of nodes in the tree.
- **seed** (*integer, random_state, or None (default)*) – Indicator of random number generation state. See *Randomness*.

**Returns**

A tree, given as an undirected graph, whose nodes are numbers in the set \(\{0, \ldots, n - 1\}\).

**Return type**

NetworkX graph

**Raises**

*NetworkXPointlessConcept* – If *n* is zero (because the null graph is not a tree).

**Notes**

The current implementation of this function generates a uniformly random Prüfer sequence then converts that to a tree via the *from_prufer_sequence()* function. Since there is a bijection between Prüfer sequences of length *n* - 2 and trees on *n* nodes, the tree is chosen uniformly at random from the set of all trees on *n* nodes.

5.20.2 networkx.generators.trees.prefix_tree

**prefix_tree** (*paths*)

Creates a directed prefix tree from the given list of iterables.

**Parameters**

- **paths** (*iterable of lists*) – An iterable over “paths”, which are themselves lists of nodes. Common prefixes among these paths are converted into common initial segments in the generated tree.

Most commonly, this may be an iterable over lists of integers, or an iterable over Python strings.

**Returns**

- **T** (*DiGraph*) – A directed graph representing an arborescence consisting of the prefix tree generated by *paths*. Nodes are directed “downward”, from parent to child. A special “synthetic” root node is added to be the parent of the first node in each path. A special “synthetic” leaf node, the “nil” node, is added to be the child of all nodes representing the last element in a path. (The addition of this nil node technically makes this not an arborescence but a directed acyclic graph; removing the nil node makes it an arborescence.)

Each node has an attribute ‘source’ whose value is the original element of the path to which this node corresponds. The ‘source’ of the root node is None, and the ‘source’ of the nil node is NIL.

The root node is the only node of in-degree zero in the graph, and the nil node is the only node of out-degree zero. For convenience, the nil node can be accessed via the NIL attribute; for example:

```python
>>> from networkx.generators.trees import NIL
>>> paths = ['ab', 'abs', 'ad']
>>> T, root = nx.prefix_tree(paths)
>>> T.predecessors(NIL)
<dict_keyiterator object at 0x...>
```

- **root** (*string*) – The randomly generated uuid of the root node.
Notes

The prefix tree is also known as a trie.

Examples

Create a prefix tree from a list of strings with some common prefixes:

```python
>>> strings = ['ab', 'abs', 'ad']
>>> T, root = nx.prefix_tree(strings)
```

Continuing the above example, to recover the original paths that generated the prefix tree, traverse up the tree from the NIL node to the root:

```python
>>> from networkx.generators.trees import NIL
>>> strings = ['ab', 'abs', 'ad']
>>> T, root = nx.prefix_tree(strings)
>>> recovered = []
>>> for v in T.predecessors(NIL):
...     s = ''
...     while v != root:
...         # Prepend the character `v` to the accumulator `s`.
...         s = str(T.nodes[v]['source']) + s
...         # Each non-nil, non-root node has exactly one parent.
...         v = next(T.predecessors(v))
...     recovered.append(s)
>>> sorted(recovered)
['ab', 'abs', 'ad']
```

5.21 Non Isomorphic Trees

Implementation of the Wright, Richmond, Odlyzko and McKay (WROM) algorithm for the enumeration of all non-isomorphic free trees of a given order. Rooted trees are represented by level sequences, i.e., lists in which the i-th element specifies the distance of vertex i to the root.

<table>
<thead>
<tr>
<th>nonisomorphic_trees(order[, create])</th>
<th>Returns a list of nonisomorphic trees</th>
</tr>
</thead>
<tbody>
<tr>
<td>number_of_nonisomorphic_trees(order)</td>
<td>Returns the number of nonisomorphic trees</td>
</tr>
</tbody>
</table>

5.21.1 networkx.generators.nonisomorphic_trees.nonisomorphic_trees

nonisomorphic_trees(order, create='graph')

Returns a list of nonisomorphic trees

Parameters

- order (int) – order of the desired tree(s)
- create (graph or matrix (default="Graph")) – If graph is selected a list of trees will be returned, if matrix is selected a list of adjacency matrix will be returned

Returns

- G (List of NetworkX Graphs)
• M (List of Adjacency matrices)

References

5.21.2 networkx.generators.nonisomorphic_trees.number_of_nonisomorphic_trees

**number_of_nonisomorphic_trees** *(order)*

Returns the number of nonisomorphic trees

**Parameters**

- order *(int)* – order of the desired tree(s)

**Returns**

- length

**Return type**

Number of nonisomorphic graphs for the given order

References

5.22 Triads

Functions that generate the triad graphs, that is, the possible digraphs on three nodes.

**triad_graph**(triad_name)

Returns the triad graph with the given name.

5.22.1 networkx.generators.triads.triad_graph

**triad_graph**(triad_name)

Returns the triad graph with the given name.

Each string in the following tuple is a valid triad name:

- ('003', '012', '102', '021D', '021U', '021C', '111D', '111U',
  '030T', '030C', '201', '120D', '120U', '120C', '210', '300')

Each triad name corresponds to one of the possible valid digraph on three nodes.

**Parameters**

- triad_name *(string)* – The name of a triad, as described above.

**Returns**

The digraph on three nodes with the given name. The nodes of the graph are the single-character strings ‘a’, ‘b’, and ‘c’.

**Return type**

DiGraph

**Raises**

ValueError – If triad_name is not the name of a triad.

See also:

triadic_census()

5.23 Joint Degree Sequence

Generate graphs with a given joint degree and directed joint degree
is_valid_joint_degree(joint_degrees) Checks whether the given joint degree dictionary is realizable.

joint_degree_graph(joint_degrees[, seed]) Generates a random simple graph with the given joint degree dictionary.

is_valid_directed_joint_degree(in_degrees, ...) Checks whether the given directed joint degree input is realizable

directed_joint_degree_graph(in_degrees, ...) Generates a random simple directed graph with the joint degree.

5.23.1 networkx.generators.joint_degree_seq.is_valid_joint_degree

is_valid_joint_degree(joint_degrees)
Checks whether the given joint degree dictionary is realizable.

A joint degree dictionary is a dictionary of dictionaries, in which entry joint_degrees[k][l] is an integer representing the number of edges joining nodes of degree k with nodes of degree l. Such a dictionary is realizable as a simple graph if and only if the following conditions are satisfied.

• each entry must be an integer,

• the total number of nodes of degree k, computed by \( \frac{\text{sum}(\text{joint_degrees}[k].values())}{k} \), must be an integer,

• the total number of edges joining nodes of degree k with nodes of degree l cannot exceed the total number of possible edges,

• each diagonal entry joint_degrees[k][k] must be even (this is a convention assumed by the joint_degree_graph() function).

Parameters
joint_degrees (dictionary of dictionary of integers) – A joint degree dictionary in which entry joint_degrees[k][l] is the number of edges joining nodes of degree k with nodes of degree l.

Returns Whether the given joint degree dictionary is realizable as a simple graph.

Return type bool

References

5.23.2 networkx.generators.joint_degree_seq.joint_degree_graph

joint_degree_graph (joint_degrees, seed=None)
Generates a random simple graph with the given joint degree dictionary.

Parameters

• joint_degrees (dictionary of dictionary of integers) – A joint degree dictionary in which entry joint_degrees[k][l] is the number of edges joining nodes of degree k with nodes of degree l.

• seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns G – A graph with the specified joint degree dictionary.

Return type Graph
Raises `NetworkXError` – If `joint_degrees` dictionary is not realizable.

Notes

In each iteration of the “while loop” the algorithm picks two disconnected nodes \( v \) and \( w \), of degree \( k \) and \( l \) correspondingly, for which \( \text{joint_degrees}[k][l] \) has not reached its target yet. It then adds edge \((v, w)\) and increases the number of edges in graph \( G \) by one.

The intelligence of the algorithm lies in the fact that it is always possible to add an edge between such disconnected nodes \( v \) and \( w \), even if one or both nodes do not have free stubs. That is made possible by executing a “neighbor switch”, an edge rewiring move that releases a free stub while keeping the joint degree of \( G \) the same.

The algorithm continues for \( E \) (number of edges) iterations of the “while loop”, at the which point all entries of the given \( \text{joint_degrees}[k][l] \) have reached their target values and the construction is complete.

References

Examples

```python
>>> import networkx as nx
>>> joint_degrees = {1: {4: 1},
                  ...  2: {2: 2, 3: 2, 4: 2},
                  ...  3: {2: 2, 4: 1},
                  ...  4: {1: 1, 2: 2, 3: 1}}
>>> G=nx.joint_degree_graph(joint_degrees)
```

5.23.3 `networkx.generators.joint_degree_seq.is_valid_directed_joint_degree`

`is_valid_directed_joint_degree` \((\text{in_degrees}, \text{out_degrees}, \text{nkk})\)

Checks whether the given directed joint degree input is realizable

Parameters

- `in_degrees` (list of integers) – in degree sequence contains the in degrees of nodes.
- `out_degrees` (list of integers) – out degree sequence contains the out degrees of nodes.
- `nkk` (dictionary of dictionary of integers) – directed joint degree dictionary. for nodes of out degree \( k \) (first level of dict) and nodes of in degree \( l \) (second level of dict) describes the number of edges.

Returns returns true if given input is realizable, else returns false.

Return type boolean

Notes

Here is the list of conditions that the inputs (in/out degree sequences, nkk) need to satisfy for simple directed graph realizability:

- Condition 0: in_degrees and out_degrees have the same length
- Condition 1: nkk[k][l] is integer for all k,l
• Condition 2: \( \text{sum}(\text{nkk}[k])/k = \text{number of nodes with partition id } k, \text{ is an integer and matching degree sequence} \)

• Condition 3: number of edges and non-chords between \( k \) and \( l \) cannot exceed maximum possible number of edges

References


5.23.4 networkx.generators.joint_degree_seq.directed_joint_degree_graph

directed_joint_degree_graph \( (\text{in}\_\text{degrees}, \text{out}\_\text{degrees}, \text{nkk}, \text{seed}=\text{None}) \)
Generates a random simple directed graph with the joint degree.

Parameters

• degree_seq \( (\text{list of tuples of size } 3) \) – degree sequence contains tuples of nodes with node id, in degree and out degree.

• nkk \( (\text{dictionary of dictionary of integers}) \) – directed joint degree dictionary, for nodes of out degree \( k \) (first level of dict) and nodes of in degree \( l \) (second level of dict) describes the number of edges.

• seed \( (\text{hashable object, optional}) \) – Seed for random number generator.

Returns G – A directed graph with the specified inputs.

Return type Graph

Raises NetworkXError – If degree_seq and nkk are not realizable as a simple directed graph.

Notes

Similarly to the undirected version: In each iteration of the “while loop” the algorithm picks two disconnected nodes \( v \) and \( w \), of degree \( k \) and \( l \) correspondingly, for which \( \text{nkk}[k][l] \) has not reached its target yet i.e. (for given \( k, l \)): \( \text{n\_edges\_add} < \text{nkk}[k][l] \). It then adds edge \( (v, w) \) and always increases the number of edges in graph \( G \) by one.

The intelligence of the algorithm lies in the fact that it is always possible to add an edge between disconnected nodes \( v \) and \( w \), for which \( \text{nkk}[[\text{degree}(v)][\text{degree}(w)] \) has not reached its target, even if one or both nodes do not have free stubs. If either node \( v \) or \( w \) does not have a free stub, we perform a “neighbor switch”, an edge rewiring move that releases a free stub while keeping nkk the same.

The difference for the directed version lies in the fact that neighbor switches might not be able to rewire, but in these cases unsaturated nodes can be reassigned to use instead, see [1] for detailed description and proofs.

The algorithm continues for \( E \) (number of edges in the graph) iterations of the “while loop”, at which point all entries of the given \( \text{nkk}[k][l] \) have reached their target values and the construction is complete.

References

Examples

```python
>>> import networkx as nx
>>> in_degrees = [0, 1, 1, 2]
>>> out_degrees = [1, 1, 1, 1]
>>> nkk = {1:{1:2,2:2}}
>>> G=nx.directed_joint_degree_graph(in_degrees, out_degrees, nkk)
```
5.24.2 networkx.generators.mycielski.mycielski_graph

mycielski_graph(n)
Generator for the n_th Mycielski Graph.

The Mycielski family of graphs is an infinite set of graphs. \( M_1 \) is the singleton graph, \( M_2 \) is two vertices with an edge, and, for \( i > 2 \), \( M_i \) is the Mycielskian of \( M_{i-1} \).

More information can be found at http://mathworld.wolfram.com/MycielskiGraph.html

Parameters n (int) – The desired Mycielski Graph.

Returns M – The n_th Mycielski Graph

Return type graph

Notes
The first graph in the Mycielski sequence is the singleton graph. The Mycielskian of this graph is not the \( P_2 \) graph, but rather the \( P_2 \) graph with an extra, isolated vertex. The second Mycielski graph is the \( P_2 \) graph, so the first two are hard coded. The remaining graphs are generated using the Mycielski operation.

5.25 Harary Graph

Generators for Harary graphs

This module gives two generators for the Harary graph, which was introduced by the famous mathematician Frank Harary in his 1962 work [H]. The first generator gives the Harary graph that maximizes the node connectivity with given number of nodes and given number of edges. The second generator gives the Harary graph that minimizes the number of edges in the graph with given node connectivity and number of nodes.

References

<table>
<thead>
<tr>
<th>hnm_harary_graph(n, m[, create_using])</th>
<th>Returns the Harary graph with given numbers of nodes and edges.</th>
</tr>
</thead>
<tbody>
<tr>
<td>hkn_harary_graph(k, n[, create_using])</td>
<td>Returns the Harary graph with given node connectivity and node number.</td>
</tr>
</tbody>
</table>

5.25.1 networkx.generators.harary_graph.hnm_harary_graph

hnm_harary_graph(n, m, create_using=None)
Returns the Harary graph with given numbers of nodes and edges.

The Harary graph \( H_{n,m} \) is the graph that maximizes node connectivity with \( n \) nodes and \( m \) edges.

This maximum node connectivity is known to be \( \text{floor}(2m/n) \).\(^1\)

Parameters

- **n (integer)** – The number of nodes the generated graph is to contain
- **m (integer)** – The number of edges the generated graph is to contain

create_using (NetworkX graph constructor, optional Graph type) – to create (default=nx.Graph). If graph instance, then cleared before populated.

Returns The Harary graph $H_{n,m}$.

Return type NetworkX graph

See also:

hkn_harary_graph()

Notes

This algorithm runs in $O(m)$ time. It is implemented by following the Reference$^2$.

References

5.25.2 networkx.generators.harary_graph.hkn_harary_graph

hkn_harary_graph ($k$, $n$, create_using=None)

Returns the Harary graph with given node connectivity and node number.

The Harary graph $H_{k,n}$ is the graph that minimizes the number of edges needed with given node connectivity $k$ and node number $n$.

This smallest number of edges is known to be $\text{ceil}(kn/2)^1$.

Parameters

- $k$ (integer) – The node connectivity of the generated graph
- $n$ (integer) – The number of nodes the generated graph is to contain
- create_using (NetworkX graph constructor, optional Graph type) – to create (default=nx.Graph). If graph instance, then cleared before populated.

Returns The Harary graph $H_{k,n}$.

Return type NetworkX graph

See also:

hnm_harary_graph()

Notes

This algorithm runs in $O(kn)$ time. It is implemented by following the Reference$^2$.

References

5.26 Cographs

Generators for cographs

---


A cograph is a graph containing no path on four vertices. Cographs or $P_4$-free graphs can be obtained from a single vertex by disjoint union and complementation operations.

**References**

```
random_cograph(n, seed=None) Returns a random cograph with $2^n$ nodes.
```

### 5.26.1 NetworkX Generators: Cographs

**random_cograph**

```python
random_cograph(n, seed=None)
```

Returns a random cograph with $2^n$ nodes.

A cograph is a graph containing no path on four vertices. Cographs or $P_4$-free graphs can be obtained from a single vertex by disjoint union and complementation operations.

This generator starts off from a single vertex and performs disjoint union and full join operations on itself. The decision on which operation will take place is random.

**Parameters**

- **n** (*int*) – The order of the cograph.
- **seed** (*integer, random_state, or None (default)) – Indicator of random number generation state. See *Randomness*.

**Returns**

- **G**

**Return type**

A random graph containing no path on four vertices.

**See also:**

- `full_join()`, `union()`

**References**
6.1 Graph Matrix

Adjacency matrix and incidence matrix of graphs.

```python
adjacency_matrix(G[, nodelist, weight])
```

Returns adjacency matrix of G.

**Parameters**

- `G` ([graph]) – A NetworkX graph
- `nodelist` ([list, optional]) – The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- `weight` ([string or None, optional (default='weight')) – The edge data key used to provide each value in the matrix. If None, then each edge has weight 1.

**Returns**

- `A` – Adjacency matrix representation of G.

**Return type** SciPy sparse matrix

**Notes**

For directed graphs, entry i,j corresponds to an edge from i to j.

If you want a pure Python adjacency matrix representation try networkx.convert.to_dict_of_dicts which will return a dictionary-of-dictionaries format that can be addressed as a sparse matrix.

For MultiGraph/MultiDiGraph with parallel edges the weights are summed. See to_numpy_matrix for other options.

The convention used for self-loop edges in graphs is to assign the diagonal matrix entry value to the edge weight attribute (or the number 1 if the edge has no weight attribute). If the alternate convention of doubling the edge weight is desired the resulting Scipy sparse matrix can be modified as follows:
```python
>>> import scipy as sp

>>> G = nx.Graph([(1,1)])
>>> A = nx.adjacency_matrix(G)
>>> print(A.todense())
[[1]]

>>>

>>> A.setdiag(A.diagonal() * 2)

>>> print(A.todense())
[[2]]
```

See also:

- `to_numpy_matrix()`
- `to_scipy_sparse_matrix()`
- `to_dict_of_dicts()`
- `adjacency_spectrum()`

### 6.1.2 networkx.linalg.graphmatrix.incidence_matrix

`incidence_matrix(G, nodelist=None, edgelist=None, oriented=False, weight=None)`  
Returns incidence matrix of G.

The incidence matrix assigns each row to a node and each column to an edge. For a standard incidence matrix a 1 appears wherever a row’s node is incident on the column’s edge. For an oriented incidence matrix each edge is assigned an orientation (arbitrarily for undirected and aligning to direction for directed). A -1 appears for the tail of an edge and 1 for the head of the edge. The elements are zero otherwise.

**Parameters**

- **G** (*graph*) – A NetworkX graph
- **nodelist** (*list*, *optional (default= all nodes in G)*) – The rows are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- **edgelist** (*list*, *optional (default= all edges in G)*) – The columns are ordered according to the edges in edgelist. If edgelist is None, then the ordering is produced by G.edges().
- **oriented** (*bool*, *optional (default=False)*) – If True, matrix elements are +1 or -1 for the head or tail node respectively of each edge. If False, +1 occurs at both nodes.
- **weight** (*string or None*, *optional (default=None)*) – The edge data key used to provide each value in the matrix. If None, then each edge has weight 1. Edge weights, if used, should be positive so that the orientation can provide the sign.

**Returns**

- **A** – The incidence matrix of G.

  **Return type** SciPy sparse matrix

**Notes**

For MultiGraph/MultiDiGraph, the edges in edgelist should be (u,v,key) 3-tuples.

“Networks are the best discrete model for so many problems in applied mathematics”\(^1\).

---

\(^1\) Gil Strang, Network applications: A = incidence matrix, [http://academicearth.org/lectures/network-applications-incidence-matrix](http://academicearth.org/lectures/network-applications-incidence-matrix)
6.2 Laplacian Matrix

Laplacian matrix of graphs.

**References**

### 6.2.1 `networkx.linalg.laplacianmatrix.laplacian_matrix`

`laplacian_matrix(G[, nodelist, weight])` Returns the Laplacian matrix of G.

The graph Laplacian is the matrix $L = D - A$, where $A$ is the adjacency matrix and $D$ is the diagonal matrix of node degrees.

**Parameters**

- **G** *(graph)* – A NetworkX graph
- **nodelist** *(list, optional)* – The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- **weight** *(string or None, optional (default='weight'))* – The edge data key used to compute each value in the matrix. If None, then each edge has weight 1.

**Returns** $L$ – The Laplacian matrix of G.

**Return type** SciPy sparse matrix

**Notes**

For MultiGraph/MultiDiGraph, the edges weights are summed.

**See also:**
- `to_numpy_matrix()`, `normalized_laplacian_matrix()`, `laplacian_spectrum()`

### 6.2.2 `networkx.linalg.laplacianmatrix.normalized_laplacian_matrix`

`normalized_laplacian_matrix(G[, nodelist, weight])` Returns the normalized Laplacian matrix of G.

The normalized graph Laplacian is the matrix

$$N = D^{-1/2}LD^{-1/2}$$

where $L$ is the graph Laplacian and $D$ is the diagonal matrix of node degrees.

**Parameters**

- **G** *(graph)* – A NetworkX graph
• **nodelist** *(list, optional)* – The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().

• **weight** *(string or None, optional (default='weight'))* – The edge data key used to compute each value in the matrix. If None, then each edge has weight 1.

**Returns** N – The normalized Laplacian matrix of G.

**Return type** NumPy matrix

**Notes**

For MultiGraph/MultiDiGraph, the edges weights are summed. See to_numpy_matrix for other options.

If the Graph contains selfloops, D is defined as diag(sum(A,1)), where A is the adjacency matrix².

**See also:**
laplacian_matrix(), normalized_laplacian_spectrum()

**References**

6.2.3 networkx.linalg.laplacianmatrix.directed_laplacian_matrix

directed_laplacian_matrix *(G, nodelist=None, weight='weight', walk_type=None, alpha=0.95)*

Returns the directed Laplacian matrix of G.

The graph directed Laplacian is the matrix

\[
L = I - (\Phi^{1/2} P \Phi^{-1/2} + \Phi^{-1/2} P^T \Phi^{1/2})/2
\]

where I is the identity matrix, P is the transition matrix of the graph, and \(\Phi\) a matrix with the Perron vector of \(P\) in the diagonal and zeros elsewhere.

Depending on the value of \(walk\_type\), \(P\) can be the transition matrix induced by a random walk, a lazy random walk, or a random walk with teleportation (PageRank).

**Parameters**

• **G** *(DiGraph)* – A NetworkX graph

• **nodelist** *(list, optional)* – The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().

• **weight** *(string or None, optional (default='weight'))* – The edge data key used to compute each value in the matrix. If None, then each edge has weight 1.

• **walk_type** *(string or None, optional (default=None))* – If None, \(P\) is selected depending on the properties of the graph. Otherwise is one of ‘random’, ‘lazy’, or ‘pagerank’

• **alpha** *(real)* – (1 - alpha) is the teleportation probability used with pagerank

**Returns** L – Normalized Laplacian of G.

**Return type** NumPy array

---

Notes

Only implemented for DiGraphs

See also:

laplacian_matrix()

References

6.3 Bethe Hessian Matrix

Bethe Hessian or deformed Laplacian matrix of graphs.

\[ \text{bethe_hessian_matrix}(G[, r, nodelist]) \]

Returns the Bethe Hessian matrix of G.

The Bethe Hessian is a family of matrices parametrized by \( r \), defined as \[ H(r) = (r^2 - 1) I - r A + D \] where \( A \) is the adjacency matrix, \( D \) is the diagonal matrix of node degrees, and \( I \) is the identity matrix. It is equal to the graph laplacian when the regularizer \( r = 1 \).

The default choice of regularizer should be the ratio \[ r_m = \frac{\sum k_i}{\sum k_i^2} - 1 \]

Parameters

- \( G \) (Graph) – A NetworkX graph
- \( r \) (float) – Regularizer parameter
- \( nodelist \) (list, optional) – The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().

Returns \( H \) – The Bethe Hessian matrix of \( G \), with paramter \( r \).

Return type Numpy matrix

Examples

```python
>>> import networkx as nx
>>> k = [3, 2, 2, 1, 0]
>>> G = nx.havel_hakimi_graph(k)
>>> H = nx.modularity_matrix(G)
```

See also:

bethe_hessian_spectrum(),

adjacency_matrix(),

to_numpy_matrix(),

laplacian_matrix()
6.4 Algebraic Connectivity

Algebraic connectivity and Fiedler vectors of undirected graphs.

algebraic_connectivity(G[, weight, ...]) Returns the algebraic connectivity of an undirected graph.

fiedler_vector(G[, weight, normalized, tol, ...]) Returns the Fiedler vector of a connected undirected graph.

spectral_ordering(G[, weight, normalized, ...]) Compute the spectral_ordering of a graph.

6.4.1 networkx.linalg.algebraicconnectivity.algebraic_connectivity

algebraic_connectivity (G, weight=’weight’, normalized=False, tol=1e-08, method=’tracemin_pcg’, seed=None)

Returns the algebraic connectivity of an undirected graph.

The algebraic connectivity of a connected undirected graph is the second smallest eigenvalue of its Laplacian matrix.

Parameters

- G (NetworkX graph) – An undirected graph.
- weight (object, optional (default: None)) – The data key used to determine the weight of each edge. If None, then each edge has unit weight.
- normalized (bool, optional (default: False)) – Whether the normalized Laplacian matrix is used.
- tol (float, optional (default: 1e-8)) – Tolerance of relative residual in eigenvalue computation.
- method (string, optional (default: ‘tracemin_pcg’)) – Method of eigenvalue computation. It must be one of the tracemin options shown below (TraceMIN), ‘lanczos’ (Lanczos iteration) or ‘lobpcg’ (LOBPCG).

The TraceMIN algorithm uses a linear system solver. The following values allow specifying the solver to be used.

<table>
<thead>
<tr>
<th>Value</th>
<th>Solver</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘tracemin_pcg’</td>
<td>Preconditioned conjugate gradient method</td>
</tr>
<tr>
<td>‘tracemin_chol’</td>
<td>Cholesky factorization</td>
</tr>
<tr>
<td>‘tracemin_lu’</td>
<td>LU factorization</td>
</tr>
</tbody>
</table>

- seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns algebraic_connectivity – Algebraic connectivity.

Return type float

Raises

- NetworkXNotImplemented – If G is directed.
- NetworkXError – If G has less than two nodes.
Notes

Edge weights are interpreted by their absolute values. For MultiGraph’s, weights of parallel edges are summed. Zero-weighted edges are ignored.

To use Cholesky factorization in the TraceMIN algorithm, the `scikit.sparse` package must be installed.

See also:

```
laplacian_matrix()
```

6.4.2 networkx.linalg.algebraicconnectivity.fiedler_vector

```
fiedler_vector(G, weight='weight', normalized=False, tol=1e-08, method='tracemin_pcg', seed=None)
```

Returns the Fiedler vector of a connected undirected graph.

The Fiedler vector of a connected undirected graph is the eigenvector corresponding to the second smallest eigenvalue of the Laplacian matrix of the graph.

Parameters

- `G` (*NetworkX graph*) – An undirected graph.
- `weight` (*object, optional (default: None)*) – The data key used to determine the weight of each edge. If None, then each edge has unit weight.
- `normalized` (*bool, optional (default: False)*) – Whether the normalized Laplacian matrix is used.
- `method` (*string, optional (default: 'tracemin_pcg')*) – Method of eigenvalue computation. It must be one of the tracemin options shown below (TraceMIN), ‘lanczos’ (Lanczos iteration) or ‘lobpcg’ (LOBPCG).

The TraceMIN algorithm uses a linear system solver. The following values allow specifying the solver to be used.

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<td>Cholesky factorization</td>
</tr>
<tr>
<td>‘tracemin_lu’</td>
<td>LU factorization</td>
</tr>
</tbody>
</table>

- `seed` (*integer, random_state, or None (default)*) – Indicator of random number generation state. See *Randomness*.

Returns `fiedler_vector` – Fiedler vector.

Return type NumPy array of floats.

Raises

- `NetworkXNotImplemented` – If G is directed.
- `NetworkXError` – If G has less than two nodes or is not connected.
Notes

Edge weights are interpreted by their absolute values. For MultiGraph’s, weights of parallel edges are summed. Zero-weighted edges are ignored.

To use Cholesky factorization in the TraceMIN algorithm, the `scikit.sparse` package must be installed.

See also:

`laplacian_matrix()`

6.4.3 networkx.linalg.algebraicconnectivity.spectral_ordering

`spectral_ordering` \((G, \text{weight='weight', normalized=False, tol=1e-08, method='tracemin_pcg', seed=None})\)

Compute the spectral ordering of a graph.

The spectral ordering of a graph is an ordering of its nodes where nodes in the same weakly connected components appear contiguous and ordered by their corresponding elements in the Fiedler vector of the component.

Parameters

- \(G\) (NetworkX graph) – A graph.
- weight (object, optional (default: None)) – The data key used to determine the weight of each edge. If None, then each edge has unit weight.
- normalized (bool, optional (default: False)) – Whether the normalized Laplacian matrix is used.
- tol (float, optional (default: 1e-8)) – Tolerance of relative residual in eigenvalue computation.
- method (string, optional (default: ‘tracemin_pcg’)) – Method of eigenvalue computation. It must be one of the tracemin options shown below (TraceMIN), ‘lanczos’ (Lanczos iteration) or ‘lobpcg’ (LOBPCG).

The TraceMIN algorithm uses a linear system solver. The following values allow specifying the solver to be used.

<table>
<thead>
<tr>
<th>Value</th>
<th>Solver</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘tracemin_pcg’</td>
<td>Preconditioned conjugate gradient method</td>
</tr>
<tr>
<td>‘tracemin_chol’</td>
<td>Cholesky factorization</td>
</tr>
<tr>
<td>‘tracemin_lu’</td>
<td>LU factorization</td>
</tr>
</tbody>
</table>

- seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns spectral_ordering – Spectral ordering of nodes.

Return type NumPy array of floats.

Raises `NetworkXError` – If G is empty.

Notes

Edge weights are interpreted by their absolute values. For MultiGraph’s, weights of parallel edges are summed. Zero-weighted edges are ignored.
To use Cholesky factorization in the TraceMIN algorithm, the scikit.sparse package must be installed.

See also:
laplacian_matrix()

6.5 Attribute Matrices

Functions for constructing matrix-like objects from graph attributes.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>attr_matrix(G[, edge_attr, node_attr, ...])</td>
<td>Returns a NumPy matrix using attributes from G.</td>
</tr>
<tr>
<td>attr_sparse_matrix(G[, edge_attr, ...])</td>
<td>Returns a SciPy sparse matrix using attributes from G.</td>
</tr>
</tbody>
</table>

6.5.1 networkx.linalg.attrmatrix.attr_matrix

attr_matrix(G, edge_attr=None, node_attr=None, normalized=False, rc_order=None, dtype=None, order=None)

Returns a NumPy matrix using attributes from G.

If only G is passed in, then the adjacency matrix is constructed.

Let A be a discrete set of values for the node attribute node_attr. Then the elements of A represent the rows and columns of the constructed matrix. Now, iterate through every edge e=(u,v) in G and consider the value of the edge attribute edge_attr. If ua and va are the values of the node attribute node_attr for u and v, respectively, then the value of the edge attribute is added to the matrix element at (ua, va).

Parameters

- **G (graph)** – The NetworkX graph used to construct the NumPy matrix.
- **edge_attr** (str, optional) – Each element of the matrix represents a running total of the specified edge attribute for edges whose node attributes correspond to the rows/cols of the matrix. The attribute must be present for all edges in the graph. If no attribute is specified, then we just count the number of edges whose node attributes correspond to the matrix element.
- **node_attr** (str, optional) – Each row and column in the matrix represents a particular value of the node attribute. The attribute must be present for all nodes in the graph. Note, the values of this attribute should be reliably hashable. So, float values are not recommended. If no attribute is specified, then the rows and columns will be the nodes of the graph.
- **normalized** (bool, optional) – If True, then each row is normalized by the summation of its values.
- **rc_order** (list, optional) – A list of the node attribute values. This list specifies the ordering of rows and columns of the array. If no ordering is provided, then the ordering will be random (and also, a return value).

Other Parameters

- **dtype** (NumPy data-type, optional) – A valid NumPy dtype used to initialize the array. Keep in mind certain dtypes can yield unexpected results if the array is to be normalized. The parameter is passed to numpy.zeros(). If unspecified, the NumPy default is used.
- **order** (‘C’, ‘F’, optional) – Whether to store multidimensional data in C- or Fortran-contiguous (row- or column-wise) order in memory. This parameter is passed to numpy.zeros(). If unspecified, the NumPy default is used.
Returns

- \( M \) (NumPy matrix) – The attribute matrix.
- \( \text{ordering} \) (list) – If \( \text{rc\_order} \) was specified, then only the matrix is returned. However, if \( \text{rc\_order} \) was None, then the ordering used to construct the matrix is returned as well.

Examples

Construct an adjacency matrix:

```python
>>> G = nx.Graph()
>>> G.add_edge(0, 1, thickness=1, weight=3)
>>> G.add_edge(0, 2, thickness=2)
>>> G.add_edge(1, 2, thickness=3)
>>> nx.attr_matrix(G, rc_order=[0, 1, 2])
matrix([[0., 1., 1.],
        [1., 0., 1.],
        [1., 1., 0.]])
```

Alternatively, we can obtain the matrix describing edge thickness.

```python
>>> nx.attr_matrix(G, edge_attr='thickness', rc_order=[0, 1, 2])
matrix([[0., 1., 2.],
        [1., 0., 3.],
        [2., 3., 0.]])
```

We can also color the nodes and ask for the probability distribution over all edges \((u,v)\) describing:

\[
\Pr(v \text{ has color } Y \mid u \text{ has color } X)
\]

```python
>>> G.nodes[0]['color'] = 'red'
>>> G.nodes[1]['color'] = 'red'
>>> G.nodes[2]['color'] = 'blue'
>>> rc = ['red', 'blue']
>>> nx.attr_matrix(G, node_attr='color', normalized=True, rc_order=rc)
matrix([[0.33333333, 0.66666667],
        [1. , 0. ]])
```

For example, the above tells us that for all edges \((u,v)\):

\[
\Pr(v \text{ is red} \mid u \text{ is red}) = 1/3 \quad \Pr(v \text{ is blue} \mid u \text{ is red}) = 2/3
\]

\[
\Pr(v \text{ is red} \mid u \text{ is blue}) = 1 \quad \Pr(v \text{ is blue} \mid u \text{ is blue}) = 0
\]

Finally, we can obtain the total weights listed by the node colors.

```python
>>> nx.attr_matrix(G, edge_attr='weight', node_attr='color', rc_order=rc)
matrix([[3., 2.],
        [2., 0.]])
```

Thus, the total weight over all edges \((u,v)\) with \(u\) and \(v\) having colors:

- (red, red) is 3 # the sole contribution is from edge (0,1) (red, blue) is 2 # contributions from edges (0,2) and (1,2) (blue, red) is 2 # same as (red, blue) since graph is undirected (blue, blue) is 0 # there are no edges with blue endpoints
6.5.2 networkx.linalg.attrmatrix.attr_sparse_matrix

**attr_sparse_matrix** *(G, edge_attr=None, node_attr=None, normalized=False, rc_order=None, dtype=None)*

Returns a SciPy sparse matrix using attributes from G.

If only G is passed in, then the adjacency matrix is constructed.

Let A be a discrete set of values for the node attribute *node_attr*. Then the elements of A represent the rows and columns of the constructed matrix. Now, iterate through every edge \( e=(u,v) \) in \( G \) and consider the value of the edge attribute *edge_attr*. If \( u_a \) and \( v_a \) are the values of the node attribute *node_attr* for \( u \) and \( v \), respectively, then the value of the edge attribute is added to the matrix element at \((u_a, v_a)\).

**Parameters**

- **G** *(graph)* – The NetworkX graph used to construct the NumPy matrix.
- **edge_attr** *(str, optional)* – Each element of the matrix represents a running total of the specified edge attribute for edges whose node attributes correspond to the rows/cols of the matrix. The attribute must be present for all edges in the graph. If no attribute is specified, then we just count the number of edges whose node attributes correspond to the matrix element.
- **node_attr** *(str, optional)* – Each row and column in the matrix represents a particular value of the node attribute. The attribute must be present for all nodes in the graph. Note, the values of this attribute should be reliably hashable. So, float values are not recommended. If no attribute is specified, then the rows and columns will be the nodes of the graph.
- **normalized** *(bool, optional)* – If True, then each row is normalized by the summation of its values.
- **rc_order** *(list, optional)* – A list of the node attribute values. This list specifies the ordering of rows and columns of the array. If no ordering is provided, then the ordering will be random (and also, a return value).

**Other Parameters**

- **dtype** *(NumPy data-type, optional)* – A valid NumPy dtype used to initialize the array. Keep in mind certain dtypes can yield unexpected results if the array is to be normalized. The parameter is passed to numpy.zeros(). If unspecified, the NumPy default is used.

**Returns**

- **M** *(SciPy sparse matrix)* – The attribute matrix.
- **ordering** *(list)* – If *rc_order* was specified, then only the matrix is returned. However, if *rc_order* was None, then the ordering used to construct the matrix is returned as well.

**Examples**

Construct an adjacency matrix:

```python
>>> G = nx.Graph()
>>> G.add_edge(0,1,thickness=1,weight=3)
>>> G.add_edge(0,2,thickness=2)
>>> G.add_edge(1,2,thickness=3)
>>> M = nx.attr_sparse_matrix(G, rc_order=[0,1,2])
>>> M.todense()
matrix([[0., 1., 1.],
        [1., 0., 1.],
        [1., 1., 0.]])
```
Alternatively, we can obtain the matrix describing edge thickness.

```python
>>> M = nx.attr_sparse_matrix(G, edge_attr='thickness', rc_order=[0,1,2])
>>> M.todense()
matrix([[0., 1., 2.],
        [1., 0., 3.],
        [2., 3., 0.]])
```

We can also color the nodes and ask for the probability distribution over all edges (u,v) describing:

\[ \text{Pr}(v \text{ has color } Y \mid u \text{ has color } X) \]

```python
>>> G.nodes[0]['color'] = 'red'
>>> G.nodes[1]['color'] = 'red'
>>> G.nodes[2]['color'] = 'blue'
>>> rc = ['red', 'blue']
>>> M = nx.attr_sparse_matrix(G, node_attr='color', normalized=True, rc_order=rc)
>>> M.todense()
matrix([[0.33333333, 0.66666667],
        [1. , 0. ]])
```

For example, the above tells us that for all edges (u,v):

\[ \text{Pr}(v \text{ is red } \mid u \text{ is red}) = 1/3 \]
\[ \text{Pr}(v \text{ is blue } \mid u \text{ is red}) = 2/3 \]
\[ \text{Pr}(v \text{ is red } \mid u \text{ is blue}) = 1 \]
\[ \text{Pr}(v \text{ is blue } \mid u \text{ is blue}) = 0 \]

Finally, we can obtain the total weights listed by the node colors.

```python
>>> M = nx.attr_sparse_matrix(G, edge_attr='weight', node_attr='color', rc_order=rc)
>>> M.todense()
matrix([[3., 2.],
        [2., 0.]])
```

Thus, the total weight over all edges (u,v) with u and v having colors:

- (red, red) is 3 # the sole contribution is from edge (0,1) (red, blue) is 2 # contributions from edges (0,2) and (1,2) (blue, red) is 2 # same as (red, blue) since graph is undirected (blue, blue) is 0 # there are no edges with blue endpoints

### 6.6 Modularity Matrices

Modularity matrix of graphs.

- **modularity_matrix**: \( G, \text{nodelist}, \text{weight} \)
  - Returns the modularity matrix of \( G \).

- **directed_modularity_matrix**: \( G, \text{nodelist}, \text{weight} \)
  - Returns the directed modularity matrix of \( G \).

#### 6.6.1 networkx.linalg.modularitymatrix.modularity_matrix

**modularity_matrix** \( (G, \text{nodelist}=\text{None}, \text{weight}=\text{None}) \)

- Returns the modularity matrix of \( G \).

The modularity matrix is the matrix \( B = A - \langle A \rangle \), where \( A \) is the adjacency matrix and \( \langle A \rangle \) is the average...
adjacency matrix, assuming that the graph is described by the configuration model.

More specifically, the element $B_{ij}$ of $B$ is defined as

$$A_{ij} - \frac{k_i k_j}{2m}$$

where $k_i$ is the degree of node $i$, and where $m$ is the number of edges in the graph. When weight is set to a name of an attribute edge, $A_{ij}$, $k_i$, $k_j$ and $m$ are computed using its value.

Parameters

- $G$ (Graph) – A NetworkX graph
- $nodelist$ (list, optional) – The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by $G$.nodes().
- $weight$ (string or None, optional (default=None)) – The edge attribute that holds the numerical value used for the edge weight. If None then all edge weights are 1.

Returns $B$ – The modularity matrix of $G$.

Return type Numpy matrix

Examples

```python
>>> import networkx as nx
>>> k = [3, 2, 2, 1, 0]
>>> G = nx.havel_hakimi_graph(k)
>>> B = nx.modularity_matrix(G)
```

See also:

to_numpy_matrix(), modularity_spectrum(), adjacency_matrix(),
directed_modularity_matrix()

References

6.6.2 networkx.linalg.modularitymatrix.directed_modularity_matrix

directed_modularity_matrix($G$, nodelist=None, weight=None)

Returns the directed modularity matrix of $G$.

The modularity matrix is the matrix $B = A - \langle A \rangle$, where $A$ is the adjacency matrix and $\langle A \rangle$ is the expected adjacency matrix, assuming that the graph is described by the configuration model.

More specifically, the element $B_{ij}$ of $B$ is defined as

$$B_{ij} = A_{ij} - \frac{k^\text{out}_i k^\text{in}_j}{m}$$

where $k^\text{in}_i$ is the in degree of node $i$, and $k^\text{out}_j$ is the out degree of node $j$, with $m$ the number of edges in the graph. When weight is set to a name of an attribute edge, $A_{ij}$, $k_i$, $k_j$ and $m$ are computed using its value.

Parameters

- $G$ (DiGraph) – A NetworkX DiGraph
- $nodelist$ (list, optional) – The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by $G$.nodes().
- **weight** *(string or None, optional (default=None))* – The edge attribute that holds the numerical value used for the edge weight. If None then all edge weights are 1.

**Returns**

- **B** – The modularity matrix of G.

**Return type** Numpy matrix

### Examples

```python
>>> import networkx as nx
>>> G = nx.DiGraph()
>>> G.add_edges_from(((1,2), (1,3), (3,1), (3,2), (3,5), (4,5), (4,6), ...
                      (5,4), (5,6), (6,4)))
>>> B = nx.directed_modularity_matrix(G)
```

### Notes

NetworkX defines the element $A_{ij}$ of the adjacency matrix as 1 if there is a link going from node i to node j. Leicht and Newman use the opposite definition. This explains the different expression for $B_{ij}$.

**See also:**

- `to_numpy_matrix()`, `modularity_spectrum()`, `adjacency_matrix()`, `modularity_matrix()`

### References

#### 6.7 Spectrum

Eigenvalue spectrum of graphs.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>adjacency_spectrum(G[, weight])</code></td>
<td>Returns eigenvalues of the adjacency matrix of G.</td>
</tr>
<tr>
<td><code>laplacian_spectrum(G[, weight])</code></td>
<td>Returns eigenvalues of the Laplacian of G</td>
</tr>
<tr>
<td><code>bethe_hessian_spectrum(G[, r])</code></td>
<td>Returns eigenvalues of the Bethe Hessian matrix of G.</td>
</tr>
<tr>
<td><code>normalized_laplacian_spectrum(G[, weight])</code></td>
<td>Return eigenvalues of the normalized Laplacian of G.</td>
</tr>
<tr>
<td><code>modularity_spectrum(G)</code></td>
<td>Returns eigenvalues of the modularity matrix of G.</td>
</tr>
</tbody>
</table>

#### 6.7.1 networkx.linalg.spectrum.adjacency_spectrum

**adjacency_spectrum** *(G, weight='weight')*

Returns eigenvalues of the adjacency matrix of G.

**Parameters**

- **G** *(graph)* – A NetworkX graph
- **weight** *(string or None, optional (default='weight'))* – The edge data key used to compute each value in the matrix. If None, then each edge has weight 1.

**Returns**

- **evals** – Eigenvalues

**Return type** NumPy array
Notes

For MultiGraph/MultiDiGraph, the edges weights are summed. See to_numpy_matrix for other options.

See also:

adjacency_matrix()

6.7.2 networkx.linalg.spectrum.laplacian_spectrum

```
laplacian_spectrum(G, weight='weight')
```

Returns eigenvalues of the Laplacian of G

Parameters

- G (graph) – A NetworkX graph
- weight (string or None, optional (default='weight')) – The edge data key used to compute each value in the matrix. If None, then each edge has weight 1.

Returns evals – Eigenvalues

Return type NumPy array

Notes

For MultiGraph/MultiDiGraph, the edges weights are summed. See to_numpy_matrix for other options.

See also:

laplacian_matrix()

6.7.3 networkx.linalg.spectrum.bethe_hessian_spectrum

```
bethe_hessian_spectrum(G, r=None)
```

Returns eigenvalues of the Bethe Hessian matrix of G.

Parameters

- G (Graph) – A NetworkX Graph or DiGraph
- r (float) – Regularizer parameter

Returns evals – Eigenvalues

Return type NumPy array

See also:

bethe_hessian_matrix()

References

6.7.4 networkx.linalg.spectrum.normalized_laplacian_spectrum

```
normalized_laplacian_spectrum(G, weight='weight')
```

Return eigenvalues of the normalized Laplacian of G

Parameters
- **G** *(graph)* – A NetworkX graph
- **weight** *(string or None, optional (default='weight'))* – The edge data key used to compute each value in the matrix. If None, then each edge has weight 1.

**Returns**
- **evals** – Eigenvalues

**Return type**
- NumPy array

**Notes**

For MultiGraph/MultiDiGraph, the edges weights are summed. See to_numpy_matrix for other options.

**See also:**
- normalized_laplacian_matrix()

### 6.7.5 networkx.linalg.spectrum.modularity_spectrum

**modularity_spectrum** *(G)*

Returns eigenvalues of the modularity matrix of G.

**Parameters**
- **G** *(Graph)* – A NetworkX Graph or DiGraph

**Returns**
- **evals** – Eigenvalues

**Return type**
- NumPy array

**See also:**
- modularity_matrix()

**References**
7.1 To NetworkX Graph

Functions to convert NetworkX graphs to and from other formats.

The preferred way of converting data to a NetworkX graph is through the graph constructor. The constructor calls the to_networkx_graph() function which attempts to guess the input type and convert it automatically.

Examples

Create a graph with a single edge from a dictionary of dictionaries

```python
>>> d = {0: {1: 1}}  # dict-of-dicts single edge (0,1)
>>> G = nx.Graph(d)
```

See also:
nx_agraph, nx_pydot

to_networkx_graph(data[, create_using, ...]) Make a NetworkX graph from a known data structure.

7.1.1 networkx.convert.to_networkx_graph

to_networkx_graph (data, create_using=None, multigraph_input=False) Make a NetworkX graph from a known data structure.

The preferred way to call this is automatically from the class constructor

```python
>>> d = {0: {1: {'weight':1}}} # dict-of-dicts single edge (0,1)
>>> G = nx.Graph(d)
```

instead of the equivalent

```python
>>> G = nx.from_dict_of_dicts(d)
```

Parameters

- data (object to be converted) –

Current known types are: any NetworkX graph dict-of-dicts dict-of-lists list of edges Pandas DataFrame (row per edge) numpy matrix numpy ndarray scipy sparse matrix pygraphviz agraph
• **create_using** (*NetworkX graph constructor, optional (default=nx.Graph]*) – Graph type to create. If graph instance, then cleared before populated.

• **multigraph_input** (*bool (default False]*) – If True and data is a dict_of_dicts, try to create a multigraph assuming dict_of_dict_of_lists. If data and create_using are both multigraphs then create a multigraph from a multigraph.

### 7.2 Dictionaries

**to_dict_of_dicts(G[, nodelist, edge_data])** Returns adjacency representation of graph as a dictionary of dictionaries.

**Parameters**

- **G** (*graph*) – A NetworkX graph
- **nodelist** (*list*) – Use only nodes specified in nodelist
- **edge_data** (*list, optional*) – If provided, the value of the dictionary will be set to edge_data for all edges. This is useful to make an adjacency matrix type representation with 1 as the edge data. If edgedata is None, the edgedata in G is used to fill the values. If G is a multigraph, the edgedata is a dict for each pair (u,v).

**from_dict_of_dicts(d[, create_using, ...])** Returns a graph from a dictionary of dictionaries.

**Parameters**

- **d** (*dictionary of dictionaries*) – A dictionary of dictionaries adjacency representation.
- **create_using** (*NetworkX graph constructor, optional (default=nx.Graph]*) – Graph type to create. If graph instance, then cleared before populated.
- **multigraph_input** (*bool (default False]*) – When True, the values of the inner dict are assumed to be containers of edge data for multiple edges. Otherwise this routine assumes the edge data are singletons.

#### 7.2.1 networkx.convert.to_dict_of_dicts

**to_dict_of_dicts** (*G, nodelist=None, edge_data=None*)

Returns adjacency representation of graph as a dictionary of dictionaries.

**Parameters**

- **G** (*graph*) – A NetworkX graph
- **nodelist** (*list*) – Use only nodes specified in nodelist
- **edge_data** (*list, optional*) – If provided, the value of the dictionary will be set to edge_data for all edges. This is useful to make an adjacency matrix type representation with 1 as the edge data. If edgedata is None, the edgedata in G is used to fill the values. If G is a multigraph, the edgedata is a dict for each pair (u,v).

#### 7.2.2 networkx.convert.from_dict_of_dicts

**from_dict_of_dicts** (*d, create_using=None, multigraph_input=False*)

Returns a graph from a dictionary of dictionaries.

**Parameters**

- **d** (*dictionary of dictionaries*) – A dictionary of dictionaries adjacency representation.
- **create_using** (*NetworkX graph constructor, optional (default=nx.Graph]*) – Graph type to create. If graph instance, then cleared before populated.
- **multigraph_input** (*bool (default False]*) – When True, the values of the inner dict are assumed to be containers of edge data for multiple edges. Otherwise this routine assumes the edge data are singletons.

#### Examples

```python
>>> dod = {0: {1: {'weight': 1}}}) # single edge (0,1)
>>> G = nx.from_dict_of_dicts(dod)
```

or

```python
>>> G = nx.Graph(dod) # use Graph constructor
```
7.3 Lists

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<tr>
<th>Function</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td><code>to_dict_of_lists(G[, nodelist])</code></td>
<td>Returns adjacency representation of graph as a dictionary of lists.</td>
</tr>
<tr>
<td><code>from_dict_of_lists(d[, create_using])</code></td>
<td>Returns a graph from a dictionary of lists.</td>
</tr>
<tr>
<td><code>to_edgelist(G[, nodelist])</code></td>
<td>Returns a list of edges in the graph.</td>
</tr>
<tr>
<td><code>from_edgelist(edgelist[, create_using])</code></td>
<td>Returns a graph from a list of edges.</td>
</tr>
</tbody>
</table>

### 7.3.1 networkx.convert.to_dict_of_lists

**to_dict_of_lists** *(G, nodelist=None)*

Returns adjacency representation of graph as a dictionary of lists.

**Parameters**

- **G** *(graph)* – A NetworkX graph
- **nodelist** *(list)* – Use only nodes specified in nodelist

**Notes**

Completely ignores edge data for MultiGraph and MultiDiGraph.

### 7.3.2 networkx.convert.from_dict_of_lists

**from_dict_of_lists** *(d, create_using=None)*

Returns a graph from a dictionary of lists.

**Parameters**

- **d** *(dictionary of lists)* – A dictionary of lists adjacency representation.
- **create_using** *(NetworkX graph constructor, optional (default=nx.Graph))* – Graph type to create. If graph instance, then cleared before populated.

**Examples**

```python
>>> dol = {0: [1]}  # single edge (0,1)
>>> G = nx.from_dict_of_lists(dol)
```

or

```python
>>> G = nx.Graph(dol)  # use Graph constructor
```

### 7.3.3 networkx.convert.to_edgelist

**to_edgelist** *(G, nodelist=None)*

Returns a list of edges in the graph.

**Parameters**

- **G** *(graph)* – A NetworkX graph
• **nodelist** (*list*) – Use only nodes specified in nodelist

### 7.3.4 networkx.convert.from_edgelist

**from_edgelist** (*edgelist*, *create_using=None*)

Returns a graph from a list of edges.

**Parameters**

- **edgelist** (*list or iterator*) – Edge tuples
- **create_using** (*NetworkX graph constructor*, *optional (default=nx.Graph]*) – Graph type to create. If graph instance, then cleared before populated.

**Examples**

```python
>>> edgelist = [(0, 1)] # single edge (0,1)
>>> G = nx.from_edgelist(edgelist)
```

or

```python
>>> G = nx.Graph(edgelist) # use Graph constructor
```

### 7.4 Numpy

Functions to convert NetworkX graphs to and from numpy/scipy matrices.

The preferred way of converting data to a NetworkX graph is through the graph constructor. The constructor calls the to_networkx_graph() function which attempts to guess the input type and convert it automatically.

**Examples**

Create a 10 node random graph from a numpy matrix

```python
>>> import numpy as np

>>> a = np.random.randint(0, 2, size=(10, 10))
>>> D = nx.DiGraph(a)
```

or equivalently

```python
>>> D = nx.to_networkx_graph(a, create_using=nx.DiGraph)
```

**See also:**

nx_agraph, nx_pydot

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>to_numpy_matrix</td>
<td>Returns the graph adjacency matrix as a NumPy matrix.</td>
</tr>
<tr>
<td>to_numpy_array</td>
<td>Returns the graph adjacency matrix as a NumPy array.</td>
</tr>
<tr>
<td>to_numpy_recarray</td>
<td>Returns the graph adjacency matrix as a NumPy recarray.</td>
</tr>
<tr>
<td>from_numpy_matrix</td>
<td>Returns a graph from numpy matrix.</td>
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<td>from_numpy_array</td>
<td>Returns a graph from NumPy array.</td>
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</tbody>
</table>
7.4.1 networkx.convert_matrix.to_numpy_matrix

to_numpy_matrix(G, nodelist=None, dtype=None, order=None, multigraph_weight=sum, weight='weight', nonedge=0.0)

Returns the graph adjacency matrix as a NumPy matrix.

Parameters

- G (graph) – The NetworkX graph used to construct the NumPy matrix.
- nodelist (list, optional) – The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- dtype (NumPy data type, optional) – A valid single NumPy data type used to initialize the array. This must be a simple type such as int or numpy.float64 and not a compound data type (see to_numpy_recarray). If None, then the NumPy default is used.
- order ({'C', 'F'}, optional) – Whether to store multidimensional data in C- or Fortran-contiguous (row- or column-wise) order in memory. If None, then the NumPy default is used.
- multigraph_weight ({sum, min, max}, optional) – An operator that determines how weights in multigraphs are handled. The default is to sum the weights of the multiple edges.
- weight (string or None optional (default = 'weight')) – The edge attribute that holds the numerical value used for the edge weight. If an edge does not have that attribute, then the value 1 is used instead.
- nonedge (float (default = 0.0)) – The matrix values corresponding to nonedges are typically set to zero. However, this could be undesirable if there are matrix values corresponding to actual edges that also have the value zero. If so, one might prefer nonedges to have some other value, such as nan.

Returns M – Graph adjacency matrix

Return type NumPy matrix

See also:

to_numpy_recarray(), from_numpy_matrix()

Notes

For directed graphs, entry i,j corresponds to an edge from i to j.

The matrix entries are assigned to the weight edge attribute. When an edge does not have a weight attribute, the value of the entry is set to the number 1. For multiple (parallel) edges, the values of the entries are determined by the multigraph_weight parameter. The default is to sum the weight attributes for each of the parallel edges.

When nodelist does not contain every node in G, the matrix is built from the subgraph of G that is induced by the nodes in nodelist.

The convention used for self-loop edges in graphs is to assign the diagonal matrix entry value to the weight attribute of the edge (or the number 1 if the edge has no weight attribute). If the alternate convention of doubling the edge weight is desired the resulting NumPy matrix can be modified as follows:

```python
>>> import numpy as np
>>> G = nx.Graph([(1, 1)])
>>> A = nx.to_numpy_matrix(G)
>>> A
```
Examples

```python
>>> G = nx.MultiDiGraph()
>>> G.add_edge(0, 1, weight=2)
0
>>> G.add_edge(1, 0)
0
>>> G.add_edge(2, 2, weight=3)
0
>>> G.add_edge(2, 2)
1
>>> nx.to_numpy_matrix(G, nodelist=[0, 1, 2])
matrix([[0., 2., 0.],
       [1., 0., 0.],
       [0., 0., 4.]])
```

### 7.4.2 networkx.convert_matrix.to_numpy_array

to_numpy_array(G, nodelist=None, dtype=None, order=None, multigraph_weight=<built-in function sum>, weight='weight', nonedge=0.0)

Returns the graph adjacency matrix as a NumPy array.

**Parameters**

- G (graph) – The NetworkX graph used to construct the NumPy array.
- nodelist (list, optional) – The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- dtype (NumPy data type, optional) – A valid single NumPy data type used to initialize the array. This must be a simple type such as int or numpy.float64 and not a compound data type (see to_numpy_recarray). If None, then the NumPy default is used.
- order (‘C’, ‘F’, optional) – Whether to store multidimensional data in C- or Fortran-contiguous (row- or column-wise) order in memory. If None, then the NumPy default is used.
- multigraph_weight (‘sum’, ‘min’, ‘max’, optional) – An operator that determines how weights in multigraphs are handled. The default is to sum the weights of the multiple edges.
- weight (string or None optional (default = ‘weight’)) – The edge attribute that holds the numerical value used for the edge weight. If an edge does not have that attribute, then the value 1 is used instead.
- nonedge (float (default = 0.0)) – The array values corresponding to nonedges are typically set to zero. However, this could be undesirable if there are array values corresponding to actual edges that also have the value zero. If so, one might prefer nonedges to have some other value, such as nan.

**Returns** A – Graph adjacency matrix
**Return type** NumPy ndarray

**See also:**

`from_numpy_array()`

**Notes**

For directed graphs, entry i,j corresponds to an edge from i to j.

Entries in the adjacency matrix are assigned to the weight edge attribute. When an edge does not have a weight attribute, the value of the entry is set to the number 1. For multiple (parallel) edges, the values of the entries are determined by the `multigraph_weight` parameter. The default is to sum the weight attributes for each of the parallel edges.

When `nodelist` does not contain every node in G, the adjacency matrix is built from the subgraph of G that is induced by the nodes in `nodelist`.

The convention used for self-loop edges in graphs is to assign the diagonal array entry value to the weight attribute of the edge (or the number 1 if the edge has no weight attribute). If the alternate convention of doubling the edge weight is desired the resulting NumPy array can be modified as follows:

```python
>>> import numpy as np
>>> G = nx.Graph([(1, 1)])
>>> A = nx.to_numpy_array(G)
>>> A
array([[1.]])
>>> A[np.diag_indices_from(A)] *= 2
>>> A
array([[2.]])
```

**Examples**

```python
>>> G = nx.MultiDiGraph()
>>> G.add_edge(0, 1, weight=2)
0
>>> G.add_edge(1, 0)
0
>>> G.add_edge(2, 2, weight=3)
0
>>> G.add_edge(2, 2)
1
>>> nx.to_numpy_array(G, nodelist=[0, 1, 2])
array([[0., 2., 0.],
       [1., 0., 0.],
       [0., 0., 4.]])
```

7.4.3 networkx.convert_matrix.to_numpy_recarray

`to_numpy_recarray` (G, nodelist=None, dtype=None, order=None) Returns the graph adjacency matrix as a NumPy recarray.

**Parameters**

- G (graph) – The NetworkX graph used to construct the NumPy matrix.
• **nodelist** (*list, optional*) – The rows and columns are ordered according to the nodes in `nodelist`. If `nodelist` is `None`, then the ordering is produced by `G.nodes()`.

• **dtype** (*NumPy data-type, optional*) – A valid NumPy named dtype used to initialize the NumPy recarray. The data type names are assumed to be keys in the graph edge attribute dictionary.

• **order** (*{'C', 'F'}, optional*) – Whether to store multidimensional data in C- or Fortran-contiguous (row- or column-wise) order in memory. If `None`, then the NumPy default is used.

**Returns**  
`M` – The graph with specified edge data as a Numpy recarray

**Return type**  
NumPy recarray

**Notes**

When `nodelist` does not contain every node in `G`, the matrix is built from the subgraph of `G` that is induced by the nodes in `nodelist`.

**Examples**

```python
>>> G = nx.Graph()
>>> G.add_edge(1, 2, weight=7.0, cost=5)
>>> A = nx.to_numpy_recarray(G, dtype=[('weight', float), ('cost', int)])
>>> print(A.weight)
[[0. 7.]
 [7. 0.]]
>>> print(A.cost)
[[0 5]
 [5 0]]
```

### 7.4.4 networkx.convert_matrix.from_numpy_matrix

**from_numpy_matrix** (*A, parallel_edges=False, create_using=None*)  
Returns a graph from numpy matrix.

The numpy matrix is interpreted as an adjacency matrix for the graph.

**Parameters**

• **A** (*nump matrix*) – An adjacency matrix representation of a graph

• **parallel_edges** (*Boolean*) – If True, `create_using` is a multigraph, and `A` is an integer matrix, then entry `(i, j)` in the matrix is interpreted as the number of parallel edges joining vertices `i` and `j` in the graph. If False, then the entries in the adjacency matrix are interpreted as the weight of a single edge joining the vertices.

• **create_using** (*NetworkX graph constructor, optional (default=nx.Graph)*) – Graph type to create. If graph instance, then cleared before populated.

**Notes**

For directed graphs, explicitly mention `create_using=nx.DiGraph`, and entry `i,j` of `A` corresponds to an edge from `i` to `j`.

---

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If `create_using` is `networkx.MultiGraph` or `networkx.MultiDiGraph`, `parallel_edges` is True, and the entries of `A` are of type `int`, then this function returns a multigraph (constructed from `create_using`) with parallel edges.

If `create_using` indicates an undirected multigraph, then only the edges indicated by the upper triangle of the matrix `A` will be added to the graph.

If the numpy matrix has a single data type for each matrix entry it will be converted to an appropriate Python data type.

If the numpy matrix has a user-specified compound data type the names of the data fields will be used as attribute keys in the resulting NetworkX graph.

See also:
`to_numpy_matrix()`, `to_numpy_recarray()`

**Examples**

Simple integer weights on edges:

```python
>>> import numpy as np
>>> A = np.array([[1, 1], [2, 1]])
>>> G = nx.from_numpy_matrix(A)
```

If `create_using` indicates a multigraph and the matrix has only integer entries and `parallel_edges` is False, then the entries will be treated as weights for edges joining the nodes (without creating parallel edges):

```python
>>> A = np.array([[1, 1], [1, 2]])
>>> G = nx.from_numpy_matrix(A, create_using=nx.MultiGraph)
>>> G[1][1]
AtlasView({0: {'weight': 2}})
```

If `create_using` indicates a multigraph and the matrix has only integer entries and `parallel_edges` is True, then the entries will be treated as the number of parallel edges joining those two vertices:

```python
>>> A = np.array([[1, 1], [1, 2]])
>>> temp = nx.MultiGraph()
>>> G = nx.from_numpy_matrix(A, parallel_edges=True, create_using=temp)
>>> G[1][1]
AtlasView({0: {'weight': 1}, 1: {'weight': 1}})
```

User defined compound data type on edges:

```python
>>> dt = [('weight', float), ('cost', int)]
>>> A = np.array([[1.0, 2]], dtype=dt)
>>> G = nx.from_numpy_matrix(A)
>>> list(G.edges())
[(0, 0)]
>>> G[0][0]['cost']
2
```

User defined compound data type on edges:

```python
>>> dt = [('weight', float), ('cost', int)]
>>> A = np.array([[1.0, 2]], dtype=dt)
>>> G = nx.from_numpy_matrix(A)
>>> list(G.edges())
[(0, 0)]
>>> G[0][0]['cost']
2
```
7.4.5 networkx.convert_matrix.from_numpy_array

from_numpy_array(A, parallel_edges=False, create_using=None)

Returns a graph from NumPy array.

The NumPy array is interpreted as an adjacency matrix for the graph.

Parameters

- A (NumPy ndarray) – An adjacency matrix representation of a graph
- parallel_edges (Boolean) – If this is True, create_using is a multigraph, and A is an integer array, then entry \((i, j)\) in the array is interpreted as the number of parallel edges joining vertices \(i\) and \(j\) in the graph. If it is False, then the entries in the array are interpreted as the weight of a single edge joining the vertices.
- create_using (NetworkX graph constructor, optional (default=nx.Graph)) – Graph type to create. If graph instance, then cleared before populated.

Notes

For directed graphs, explicitly mention create_using=nx.DiGraph, and entry \(i,j\) of A corresponds to an edge from \(i\) to \(j\).

If create_using is networkx.MultiGraph or networkx.MultiDiGraph, parallel_edges is True, and the entries of A are of type int, then this function returns a multigraph (of the same type as create_using) with parallel edges.

If create_using indicates an undirected multigraph, then only the edges indicated by the upper triangle of the array A will be added to the graph.

If the NumPy array has a single data type for each array entry it will be converted to an appropriate Python data type.

If the NumPy array has a user-specified compound data type the names of the data fields will be used as attribute keys in the resulting NetworkX graph.

See also:

to_numpy_array()

Examples

Simple integer weights on edges:

```python
>>> import numpy as np
>>> A = np.array([[1, 1], [2, 1]])
>>> G = nx.from_numpy_array(A)
>>> G.edges(data=True)
EdgeDataView([(0, 0, {'weight': 1}), (0, 1, {'weight': 2}), (1, 1, {'weight': 1})])
```

If create_using indicates a multigraph and the array has only integer entries and parallel_edges is False, then the entries will be treated as weights for edges joining the nodes (without creating parallel edges):

```python
>>> A = np.array([[1, 1], [1, 2]])
>>> G = nx.from_numpy_array(A, create_using=nx.MultiGraph)
>>> G[1][1]
AtlasView({0: {'weight': 2}})
```
If `create_using` indicates a multigraph and the array has only integer entries and `parallel_edges` is True, then the entries will be treated as the number of parallel edges joining those two vertices:

```python
>>> A = np.array([[1, 1], [1, 2]])
>>> temp = nx.MultiGraph()
>>> G = nx.from_numpy_array(A, parallel_edges=True, create_using=temp)
>>> G[1][1]
AtlasView({0: {'weight': 1}, 1: {'weight': 1}})
```

User defined compound data type on edges:

```python
>>> dt = [('weight', float), ('cost', int)]
>>> A = np.array([[(1.0, 2)]], dtype=dt)
>>> G = nx.from_numpy_array(A)
>>> G.edges()
EdgeView([(0, 0)])
>>> G[0][0]['cost']
2
>>> G[0][0]['weight']
1.0
```

### 7.5 Scipy

#### to_scipy_sparse_matrix

Returns the graph adjacency matrix as a SciPy sparse matrix.

**Parameters**

- **G (graph)** – The NetworkX graph used to construct the NumPy matrix.
- **nodelist (list, optional)** – The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- **dtype (NumPy data-type, optional)** – A valid NumPy dtype used to initialize the array. If None, then the NumPy default is used.
- **weight (string or None optional (default='weight')** – The edge attribute that holds the numerical value used for the edge weight. If None then all edge weights are 1.
- **format (str in {'bsr', 'csr', 'csc', 'coo', 'lil', 'dia', 'dok'})** – The type of the matrix to be returned (default ‘csr’). For some algorithms different implementations of sparse matrices can perform better. See\(^1\) for details.

**Returns** M – Graph adjacency matrix.

**Return type** SciPy sparse matrix

---

Notes

For directed graphs, matrix entry i,j corresponds to an edge from i to j.

The matrix entries are populated using the edge attribute held in parameter weight. When an edge does not have that attribute, the value of the entry is 1.

For multiple edges the matrix values are the sums of the edge weights.

When nodelist does not contain every node in G, the matrix is built from the subgraph of G that is induced by the nodes in nodelist.

Uses coo_matrix format. To convert to other formats specify the format= keyword.

The convention used for self-loop edges in graphs is to assign the diagonal matrix entry value to the weight attribute of the edge (or the number 1 if the edge has no weight attribute). If the alternate convention of doubling the edge weight is desired the resulting Scipy sparse matrix can be modified as follows:

```python
>>> import scipy as sp
>>> G = nx.Graph([(1, 1)])
>>> A = nx.to_scipy_sparse_matrix(G)
>>> print(A.todense())
[[1]]
>>> A.setdiag(A.diagonal() * 2)
>>> print(A.todense())
[[2]]
```

Examples

```python
>>> G = nx.MultiDiGraph()
>>> G.add_edge(0, 1, weight=2)
0
>>> G.add_edge(1, 0)
0
>>> G.add_edge(2, 2, weight=3)
0
>>> G.add_edge(2, 2)
1
>>> S = nx.to_scipy_sparse_matrix(G, nodelist=[0, 1, 2])
>>> print(S.todense())
[[0 2 0]
 [1 0 0]
 [0 0 4]]
```

References

7.5.2 networkx.convert_matrix.from_scipy_sparse_matrix

`from_scipy_sparse_matrix(A, parallel_edges=False, create_using=None, edge_attribute='weight')`

Creates a new graph from an adjacency matrix given as a SciPy sparse matrix.

Parameters

- A (scipy sparse matrix) – An adjacency matrix representation of a graph
- parallel_edges (Boolean) – If this is True, create_using is a multigraph, and A is an integer matrix, then entry (i, j) in the matrix is interpreted as the number of parallel edges
joining vertices $i$ and $j$ in the graph. If it is False, then the entries in the matrix are interpreted as the weight of a single edge joining the vertices.

- **create_using** (*NetworkX graph constructor, optional (default=nx.Graph)*) – Graph type to create. If graph instance, then cleared before populated.
- **edge_attribute** (*string*) – Name of edge attribute to store matrix numeric value. The data will have the same type as the matrix entry (int, float, (real,imag)).

**Notes**

For directed graphs, explicitly mention create_using=nx.DiGraph, and entry $i,j$ of $A$ corresponds to an edge from $i$ to $j$.

If create_using is `networkx.MultiGraph` or `networkx.MultiDiGraph`, parallel_edges is True, and the entries of $A$ are of type int, then this function returns a multigraph (constructed from create_using) with parallel edges. In this case, edge_attribute will be ignored.

If create_using indicates an undirected multigraph, then only the edges indicated by the upper triangle of the matrix $A$ will be added to the graph.

**Examples**

```python
>>> import scipy as sp
>>> A = sp.sparse.eye(2, 2, 1)
>>> G = nx.from_scipy_sparse_matrix(A)
```

If create_using indicates a multigraph and the matrix has only integer entries and parallel_edges is False, then the entries will be treated as weights for edges joining the nodes (without creating parallel edges):

```python
>>> A = sp.sparse.csr_matrix([[1, 1], [1, 2]])
>>> G = nx.from_scipy_sparse_matrix(A, create_using=nx.MultiGraph)
>>> G[1][1]
AtlasView({0: {'weight': 2}})
```

If create_using indicates a multigraph and the matrix has only integer entries and parallel_edges is True, then the entries will be treated as the number of parallel edges joining those two vertices:

```python
>>> A = sp.sparse.csr_matrix([[1, 1], [1, 2]])
>>> G = nx.from_scipy_sparse_matrix(A, parallel_edges=True,
...                                 create_using=nx.MultiGraph)
>>> G[1][1]
AtlasView({0: {'weight': 1}, 1: {'weight': 1}})
```

**7.6 Pandas**

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7.6.1 networkx.convert_matrix.to_pandas_adjacency

to_pandas_adjacency (G, nodelist=None, dtype=None, order=None, multigraph_weight=sum, weight='weight', nonedge=0.0)

Returns the graph adjacency matrix as a Pandas DataFrame.

Parameters

- G (graph) – The NetworkX graph used to construct the Pandas DataFrame.
- nodelist (list, optional) – The rows and columns are ordered according to the nodes in nodelist. If nodelist is None, then the ordering is produced by G.nodes().
- multigraph_weight (sum, min, max, optional) – An operator that determines how weights in multigraphs are handled. The default is to sum the weights of the multiple edges.
- weight (string or None, optional) – The edge attribute that holds the numerical value used for the edge weight. If an edge does not have that attribute, then the value 1 is used instead.
- nonedge (float, optional) – The matrix values corresponding to nonedges are typically set to zero. However, this could be undesirable if there are matrix values corresponding to actual edges that also have the value zero. If so, one might prefer nonedges to have some other value, such as nan.

Returns df – Graph adjacency matrix

Return type Pandas DataFrame

Notes

For directed graphs, entry i,j corresponds to an edge from i to j.

The DataFrame entries are assigned to the weight edge attribute. When an edge does not have a weight attribute, the value of the entry is set to the number 1. For multiple (parallel) edges, the values of the entries are determined by the ‘multigraph_weight’ parameter. The default is to sum the weight attributes for each of the parallel edges.

When nodelist does not contain every node in G, the matrix is built from the subgraph of G that is induced by the nodes in nodelist.

The convention used for self-loop edges in graphs is to assign the diagonal matrix entry value to the weight attribute of the edge (or the number 1 if the edge has no weight attribute). If the alternate convention of doubling the edge weight is desired the resulting Pandas DataFrame can be modified as follows:

```python
>>> import pandas as pd
>>> pd.options.display.max_columns = 20
>>> import numpy as np
>>> G = nx.Graph([(1, 1)])
>>> df = nx.to_pandas_adjacency(G, dtype=int)
>>> df
   1
1  1
>>> df.values[np.diag_indices_from(df)] *= 2
>>> df
   1
1  2
```
Examples

>>> G = nx.MultiDiGraph()
>>> G.add_edge(0, 1, weight=2)
0
>>> G.add_edge(1, 0)
0
>>> G.add_edge(2, 2, weight=3)
0
>>> G.add_edge(2, 2)
1
>>> nx.to_pandas_adjacency(G, nodelist=[0, 1, 2], dtype=int)
   0 1 2
0  0 2 0
1  1 0 0
2  0 0 4

7.6.2 networkx.convert_matrix.from_pandas_adjacency

from_pandas_adjacency (df, create_using=None)

Returns a graph from Pandas DataFrame.

The Pandas DataFrame is interpreted as an adjacency matrix for the graph.

Parameters

- df (Pandas DataFrame) – An adjacency matrix representation of a graph
- create_using (NetworkX graph constructor, optional (default=nx.Graph)) – Graph type to create. If graph instance, then cleared before populated.

Notes

For directed graphs, explicitly mention create_using=nx.DiGraph, and entry i,j of df corresponds to an edge from i to j.

If the numpy matrix has a single data type for each matrix entry it will be converted to an appropriate Python data type.

If the numpy matrix has a user-specified compound data type the names of the data fields will be used as attribute keys in the resulting NetworkX graph.

See also:

to_pandas_adjacency()

Examples

Simple integer weights on edges:

>>> import pandas as pd
>>> pd.options.display.max_columns = 20
>>> df = pd.DataFrame([[1, 1], [2, 1]])
>>> df
  0 1
0 1 1
1 1 1
(continues on next page)
1 2 1
>>> G = nx.from_pandas_adjacency(df)
>>> G.name = 'Graph from pandas adjacency matrix'
>>> print(nx.info(G))
Name: Graph from pandas adjacency matrix  
Type: Graph 
Number of nodes: 2 
Number of edges: 3 
Average degree: 3.0000

7.6.3 networkx.convert_matrix.to_pandas_edgelist

to_pandas_edgelist \( (G, \text{source='source'}, \text{target='target'}, \text{nodelist=None, dtype=None, order=None}) \)

Returns the graph edge list as a Pandas DataFrame.

Parameters

- \( G \) (graph) – The NetworkX graph used to construct the Pandas DataFrame.
- \( \text{source} \) (str or int, optional) – A valid column name (string or integer) for the source nodes (for the directed case).
- \( \text{target} \) (str or int, optional) – A valid column name (string or integer) for the target nodes (for the directed case).
- \( \text{nodelist} \) (list, optional) – Use only nodes specified in nodelist

Returns \( df \) – Graph edge list

Return type Pandas DataFrame

Examples

```python
>>> G = nx.Graph([('A', 'B', {'cost': 1, 'weight': 7}),
...    ('C', 'E', {'cost': 9, 'weight': 10})])
>>> df = nx.to_pandas_edgelist(G, nodelist=['A', 'C'])
>>> df[['source', 'target', 'cost', 'weight']]  
source target cost weight
0 A B 1 7
1 C E 9 10
```

7.6.4 networkx.convert_matrix.from_pandas_edgelist

from_pandas_edgelist \( (df, \text{source='source'}, \text{target='target'}, \text{edge_attr=None, create_using=None}) \)

Returns a graph from Pandas DataFrame containing an edge list.

The Pandas DataFrame should contain at least two columns of node names and zero or more columns of edge attributes. Each row will be processed as one edge instance.

Note: This function iterates over DataFrame.values, which is not guaranteed to retain the data type across columns in the row. This is only a problem if your row is entirely numeric and a mix of ints and floats. In that case, all values will be returned as floats. See the DataFrame.iterrows documentation for an example.

Parameters

- \( df \) (Pandas DataFrame) – An edge list representation of a graph
• **source** (*str or int*) – A valid column name (string or integer) for the source nodes (for the directed case).

• **target** (*str or int*) – A valid column name (string or integer) for the target nodes (for the directed case).

• **edge_attr** (*str or int, iterable, True, or None*) – A valid column name (*str or int*) or iterable of column names that are used to retrieve items and add them to the graph as edge attributes. If *True*, all of the remaining columns will be added. If *None*, no edge attributes are added to the graph.

• **create_using** (*NetworkX graph constructor, optional (default=nx.Graph]*) – Graph type to create. If graph instance, then cleared before populated.

**See also:**

to_pandas_edgelist()

**Examples**

Simple integer weights on edges:

```python
>>> import pandas as pd
>>> pd.options.display.max_columns = 20
>>> import numpy as np
>>> rng = np.random.RandomState(seed=5)
>>> ints = rng.randint(1, 11, size=(3,2))
>>> a = ['A', 'B', 'C']
>>> b = ['D', 'A', 'E']
>>> df = pd.DataFrame(ints, columns=['weight', 'cost'])
>>> df[0] = a
>>> df['b'] = b
>>> df[['weight', 'cost', 0, 'b']]
   weight  cost  0  b
0      4      7  A  D
1      7      1  B  A
2     10      9  C  E
>>> G = nx.from_pandas_edgelist(df, 0, 'b', ['weight', 'cost'])
>>> G['E']['C']['weight']
10
>>> G['E']['C']['cost']
9
>>> edges = pd.DataFrame({'source': [0, 1, 2],
...                        'target': [2, 2, 3],
...                        'weight': [3, 4, 5],
...                        'color': ['red', 'blue', 'blue']})
>>> G = nx.from_pandas_edgelist(edges, edge_attr=True)
>>> G[0][2]['color']
'red'
```
8.1 Relabeling

\[
\text{convert_node_labels_to_integers}(G, \ldots) \quad \text{Returns a copy of the graph } G \text{ with the nodes relabeled using consecutive integers.}
\]

\[
\text{relabel_nodes}(G, \text{mapping}, \text{copy}) \quad \text{Relabel the nodes of the graph } G.
\]

8.1.1 networkx.relabel.convert_node_labels_to_integers

\text{convert_node_labels_to_integers} (G, \text{first_label=0, ordering='default', label_attribute=None})

Returns a copy of the graph G with the nodes relabeled using consecutive integers.

Parameters

- \text{G (graph)} – A NetworkX graph
- \text{first_label (int, optional (default=0))} – An integer specifying the starting offset in numbering nodes. The new integer labels are numbered first_label, \ldots, n-1+first_label.
- \text{ordering (string)} – “default” : inherit node ordering from G.nodes() “sorted” : inherit node ordering from sorted(G.nodes()) “increasing degree” : nodes are sorted by increasing degree “decreasing degree” : nodes are sorted by decreasing degree
- \text{label_attribute (string, optional (default=None))} – Name of node attribute to store old label. If None no attribute is created.

Notes

Node and edge attribute data are copied to the new (reabeled) graph.

There is no guarantee that the relabeling of nodes to integers will give the same two integers for two (even identical graphs). Use the \text{ordering} argument to try to preserve the order.

See also:

- \text{relabel_nodes()}

8.1.2 networkx.relabel.relabel_nodes

\text{relabel_nodes} (G, \text{mapping, copy=True})

Relabel the nodes of the graph G.

Parameters
- **G (graph)** – A NetworkX graph
- **mapping (dictionary)** – A dictionary with the old labels as keys and new labels as values. A partial mapping is allowed.
- **copy (bool (optional, default=True))** – If True return a copy, or if False relabel the nodes in place.

### Examples

To create a new graph with nodes relabeled according to a given dictionary:

```python
>>> G = nx.path_graph(3)
>>> sorted(G)
[0, 1, 2]
>>> mapping = {0: 'a', 1: 'b', 2: 'c'}
>>> H = nx.relabel_nodes(G, mapping)
>>> sorted(H)
['a', 'b', 'c']
```

Nodes can be relabeled with any hashable object, including numbers and strings:

```python
>>> import string

>>> G = nx.path_graph(26)  # nodes are integers 0 through 25
>>> sorted(G)[:3]
[0, 1, 2]
>>> mapping = dict(zip(G, string.ascii_lowercase))
>>> G = nx.relabel_nodes(G, mapping)  # nodes are characters a through z
>>> sorted(G)[:3]
['a', 'b', 'c']
>>> mapping = dict(zip(G, range(1, 27)))
>>> G = nx.relabel_nodes(G, mapping)  # nodes are integers 1 through 26
>>> sorted(G)[:3]
[1, 2, 3]
```

To perform a partial in-place relabeling, provide a dictionary mapping only a subset of the nodes, and set the `copy` keyword argument to False:

```python
>>> G = nx.path_graph(3)  # nodes 0-1-2
>>> mapping = {0: 'a', 1: 'b'}  # 0->'a' and 1->'b'
>>> G = nx.relabel_nodes(G, mapping, copy=False)
>>> sorted(G, key=str)
[2, 'a', 'b']
```

A mapping can also be given as a function:

```python
>>> G = nx.path_graph(3)
>>> H = nx.relabel_nodes(G, lambda x: x ** 2)
>>> list(H)
[0, 1, 4]
```

### Notes

Only the nodes specified in the mapping will be relabeled.

The keyword setting `copy=False` modifies the graph in place. `Relabel_nodes` avoids naming collisions by building a directed graph from `mapping` which specifies the order of relabelings. Naming collisions, such as `a->b`,
b->c, are ordered such that “b” gets renamed to “c” before “a” gets renamed “b”. In cases of circular mappings (e.g. a->b, b->a), modifying the graph is not possible in-place and an exception is raised. In that case, use copy=True.

See also:

convert_node_labels_to_integers()
9.1 Adjacency List

9.1.1 Adjacency List

Read and write NetworkX graphs as adjacency lists.

Adjacency list format is useful for graphs without data associated with nodes or edges and for nodes that can be meaningfully represented as strings.

**Format**

The adjacency list format consists of lines with node labels. The first label in a line is the source node. Further labels
in the line are considered target nodes and are added to the graph along with an edge between the source node and
target node.

The graph with edges a-b, a-c, d-e can be represented as the following adjacency list (anything following the # in a
line is a comment):

```
a b c  # source target target
d e
```

<table>
<thead>
<tr>
<th>read_adjlist(path[, comments, delimiter, ...])</th>
<th>Read graph in adjacency list format from path.</th>
</tr>
</thead>
<tbody>
<tr>
<td>write_adjlist(G, path[, comments, ...])</td>
<td>Write graph G in single-line adjacency-list format to path.</td>
</tr>
<tr>
<td>parse_adjlist(lines[, comments, delimiter, ...])</td>
<td>Parse lines of a graph adjacency list representation.</td>
</tr>
<tr>
<td>generate_adjlist(G[, delimiter])</td>
<td>Generate a single line of the graph G in adjacency list format.</td>
</tr>
</tbody>
</table>

9.1.2 networkx.readwrite.adjlist.read_adjlist

read_adjlist (path, comments='#', delimiter=None, create_using=None, nodetype=None, encoding='utf-8')
Read graph in adjacency list format from path.

**Parameters**

- **path** *(string or file)* – Filename or file handle to read. Filenames ending in .gz or .bz2 will be un compressed.
- **create_using** *(NetworkX graph constructor, optional (default=nx.Graph))* – Graph type to
create. If graph instance, then cleared before populated.

- **nodetype** (Python type, optional) – Convert nodes to this type.
- **comments** (string, optional) – Marker for comment lines
- **delimiter** (string, optional) – Separator for node labels. The default is whitespace.

Returns  

**G** – The graph corresponding to the lines in adjacency list format.

**Return type**  

NetworkX graph

### Examples

```python
>>> G = nx.path_graph(4)
>>> nx.write_adjlist(G, "test.adjlist")
>>> G = nx.read_adjlist("test.adjlist")
```

The path can be a filehandle or a string with the name of the file. If a filehandle is provided, it has to be opened in 'rb' mode.

```python
>>> fh = open("test.adjlist", 'rb')
>>> G = nx.read_adjlist(fh)
```

Filenames ending in .gz or .bz2 will be compressed.

```python
>>> nx.write_adjlist(G,"test.adjlist.gz")
>>> G = nx.read_adjlist("test.adjlist.gz")
```

The optional nodetype is a function to convert node strings to nodetype.

For example

```python
>>> G = nx.read_adjlist("test.adjlist", nodetype=int)
```

will attempt to convert all nodes to integer type.

Since nodes must be hashable, the function nodetype must return hashable types (e.g. int, float, str, frozenset - or tuples of those, etc.)

The optional create_using parameter indicates the type of NetworkX graph created. The default is `nx.Graph`, an undirected graph. To read the data as a directed graph use

```python
>>> G = nx.read_adjlist("test.adjlist", create_using=nx.DiGraph)
```

### Notes

This format does not store graph or node data.

**See also:**

- `write_adjlist()`

### 9.1.3 networkx.readwrite.adjlist.write_adjlist

`write_adjlist(G, path, comments='#', delimiter=' ', encoding='utf-8')`

Write graph G in single-line adjacency-list format to path.
Parameters

- **G** (*NetworkX graph*)
- **path** (*string or file*) – Filename or file handle for data output. Filenames ending in .gz or .bz2 will be compressed.
- **comments** (*string, optional*) – Marker for comment lines
- **delimiter** (*string, optional*) – Separator for node labels
- **encoding** (*string, optional*) – Text encoding.

Examples

```python
>>> G = nx.path_graph(4)
>>> nx.write_adjlist(G, "test.adjlist")
```

The path can be a filehandle or a string with the name of the file. If a filehandle is provided, it has to be opened in `wb` mode.

```python
>>> fh = open("test.adjlist", 'wb')
>>> nx.write_adjlist(G, fh)
```

Notes

This format does not store graph, node, or edge data.

See also:

`read_adjlist()`, `generate_adjlist()`

9.1.4 networkx.readwrite.adjlist.parse_adjlist

`parse_adjlist` (*lines, comments=’#’, delimiter=None, create_using=None, nodetype=None*)

Parse lines of a graph adjacency list representation.

Parameters

- **lines** (*list or iterator of strings*) – Input data in adjlist format
- **create_using** (*NetworkX graph constructor, optional (default=nx.Graph]*) – Graph type to create. If graph instance, then cleared before populated.
- **nodetype** (*Python type, optional*) – Convert nodes to this type.
- **comments** (*string, optional*) – Marker for comment lines
- **delimiter** (*string, optional*) – Separator for node labels. The default is whitespace.

Returns **G** – The graph corresponding to the lines in adjacency list format.

Return type  *NetworkX graph*
NetworkX Reference, Release 2.5rc1.dev20191214022633

```python
>>> lines = ['1 2 5',
...         '2 3 4',
...         '3 5',
...         '4',
...         '5']
>>> G = nx.parse_adjlist(lines, nodetype=int)
>>> nodes = [1, 2, 3, 4, 5]
>>> all(node in G for node in nodes)
True
>>> edges = [(1, 2), (1, 5), (2, 3), (2, 4), (3, 5)]
>>> all((u, v) in G.edges() or (v, u) in G.edges() for (u, v) in edges)
True

See also:

read_adjlist()

9.1.5 networkx.readwrite.adjlist.generate_adjlist

generate_adjlist(G, delimiter=' ')
Generate a single line of the graph G in adjacency list format.

Parameters

• G (NetworkX graph)

• delimiter (string, optional) – Separator for node labels

Returns lines – Lines of data in adjlist format.

Return type string

Examples

```python
>>> G = nx.lollipop_graph(4, 3)
>>> for line in nx.generate_adjlist(G):
...     print(line)
0 1 2 3
1 2 3
2 3
3 4
4 5
5 6
6
```

See also:

write_adjlist(), read_adjlist()

9.2 Multiline Adjacency List

9.2.1 Multi-line Adjacency List

Read and write NetworkX graphs as multi-line adjacency lists.
The multi-line adjacency list format is useful for graphs with nodes that can be meaningfully represented as strings. With this format simple edge data can be stored but node or graph data is not.

**Format**

The first label in a line is the source node label followed by the node degree d. The next d lines are target node labels and optional edge data. That pattern repeats for all nodes in the graph.

The graph with edges a-b, a-c, d-e can be represented as the following adjacency list (anything following the # in a line is a comment):

```
# example.multiline-adjlist
a 2
b
c
d 1
e
```

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>read_multiline_adjlist</code></td>
<td>Read graph in multi-line adjacency list format from path.</td>
</tr>
<tr>
<td><code>write_multiline_adjlist</code></td>
<td>Write the graph G in multiline adjacency list format to path.</td>
</tr>
<tr>
<td><code>parse_multiline_adjlist</code></td>
<td>Parse lines of a multiline adjacency list representation of a graph.</td>
</tr>
<tr>
<td><code>generate_multiline_adjlist</code></td>
<td>Generate a single line of the graph G in multiline adjacency list format.</td>
</tr>
</tbody>
</table>

### 9.2.2 `networkx.readwrite.multiline_adjlist.read_multiline_adjlist`

**read_multiline_adjlist** *(path[, comments, delimiter=None, create_using=None, nodetype=None, edgetype=None, encoding='utf-8'])*

Read graph in multi-line adjacency list format from path.

**Parameters**

- **path** *(string or file)* – Filename or file handle to read. Filenames ending in .gz or .bz2 will be uncompressed.
- **create_using** *(NetworkX graph constructor, optional (default=nx.Graph))* – Graph type to create. If graph instance, then cleared before populated.
- **nodetype** *(Python type, optional)* – Convert nodes to this type.
- **edgetype** *(Python type, optional)* – Convert edge data to this type.
- **comments** *(string, optional)* – Marker for comment lines
- **delimiter** *(string, optional)* – Separator for node labels. The default is whitespace.

**Returns**

- **G** *(NetworkX graph)*

**Return type** NetworkX graph
Examples

```python
>>> G=nx.path_graph(4)
>>> nx.write_multiline_adjlist(G,"test.adjlist")
>>> G=nx.read_multiline_adjlist("test.adjlist")
```

The path can be a file or a string with the name of the file. If a file is provided, it has to be opened in ‘rb’ mode.

```python
>>> fh=open("test.adjlist", 'rb')
>>> G=nx.read_multiline_adjlist(fh)
```

Filenames ending in .gz or .bz2 will be compressed.

```python
>>> nx.write_multiline_adjlist(G,"test.adjlist.gz")
>>> G=nx.read_multiline_adjlist("test.adjlist.gz")
```

The optional nodetype is a function to convert node strings to nodetype.

For example

```python
>>> G=nx.read_multiline_adjlist("test.adjlist", nodetype=int)
```

will attempt to convert all nodes to integer type.

The optional edgetype is a function to convert edge data strings to edgetype.

```python
>>> G=nx.read_multiline_adjlist("test.adjlist")
```

The optional create_using parameter is a NetworkX graph container. The default is Graph(), an undirected graph. To read the data as a directed graph use

```python
>>> G=nx.read_multiline_adjlist("test.adjlist", create_using=nx.DiGraph)
```

Notes

This format does not store graph, node, or edge data.

See also:

write_multiline_adjlist()

9.2.3 networkx.readwrite.multiline_adjlist

write_multiline_adjlist (G, path, delimiter=' ', comments='#', encoding='utf-8')

Write the graph G in multiline adjacency list format to path

Parameters

- G (NetworkX graph)
- comments (string, optional) – Marker for comment lines
- delimiter (string, optional) – Separator for node labels
- encoding (string, optional) – Text encoding.
Examples

```python
>>> G=nx.path_graph(4)
>>> nx.write_multiline_adjlist(G,"test.adjlist")
```

The path can be a file handle or a string with the name of the file. If a file handle is provided, it has to be opened in 'wb' mode.

```python
>>> fh=open("test.adjlist","wb")
>>> nx.write_multiline_adjlist(G,fh)
```

Filenames ending in .gz or .bz2 will be compressed.

```python
>>> nx.write_multiline_adjlist(G,"test.adjlist.gz")
```

See also:

`read_multiline_adjlist()`

### 9.2.4 networkx.readwrite.multiline_adjlist.parse_multiline_adjlist

`parse_multiline_adjlist` *(lines, comments='#', delimiter=None, create_using=None, nodetype=None, edgetype=None)*

Parse lines of a multiline adjacency list representation of a graph.

**Parameters**

- `lines` *(list or iterator of strings)* – Input data in multiline adjlist format
- `create_using` *(NetworkX graph constructor, optional (default=nx.Graph))* – Graph type to create. If graph instance, then cleared before populated.
- `nodetype` *(Python type, optional)* – Convert nodes to this type.
- `comments` *(string, optional)* – Marker for comment lines
- `delimiter` *(string, optional)* – Separator for node labels. The default is whitespace.

**Returns**

- `G` – The graph corresponding to the lines in multiline adjacency list format.

**Return type**

NetworkX graph

**Examples**

```python
>>> lines = ['1 2',
...         '2 (\"weight\":3, \"name\": \"Frodo\")',
...         '3 (\"name\": \"Gandalf\")',
...         '2 1',
...         '5 (\"weight\":6, \"name\": \"Saruman\")']
>>> G = nx.parse_multiline_adjlist(iter(lines), nodetype=int)
>>> list(G)
[1, 2, 3, 5]
```

### 9.2.5 networkx.readwrite.multiline_adjlist.generate_multiline_adjlist

`generate_multiline_adjlist` *(G, delimiter=\'\')*

Generate a single line of the graph G in multiline adjacency list format.
Parameters

- \( G \) (NetworkX graph)
- delimiter (string, optional) – Separator for node labels

Returns

- lines – Lines of data in multiline adjlist format.

Return type

- string

Examples

```python
>>> G = nx.lollipop_graph(4, 3)
>>> for line in nx.generate_multiline_adjlist(G):
...     print(line)
0 3
1 {}
2 {}
3 {}
1 2
2 {}
3 {}
2 1
3 {}
3 1
4 {}
4 1
5 {}
5 1
6 {}
6 0
```

See also:

`write_multiline_adjlist()`, `read_multiline_adjlist()`

9.3 Edge List

9.3.1 Edge Lists

Read and write NetworkX graphs as edge lists.

The multi-line adjacency list format is useful for graphs with nodes that can be meaningfully represented as strings. With the edgelist format simple edge data can be stored but node or graph data is not. There is no way of representing isolated nodes unless the node has a self-loop edge.

Format

You can read or write three formats of edge lists with these functions.

Node pairs with no data:

```
1 2
```

Python dictionary as data:
Arbitrary data:

```
1 2 7 green
```

### read_edgelist(path[, comments, delimiter, ...])
Read a graph from a list of edges.

#### Parameters
- `path` *(file or string)* – File or filename to read. If a file is provided, it must be opened in ‘rb’ mode. Filenames ending in .gz or .bz2 will be uncompressed.
- `comments` *(string, optional)* – The character used to indicate the start of a comment.
- `delimiter` *(string, optional)* – The string used to separate values. The default is whitespace.
- `create_using` *(NetworkX graph constructor, optional (default=nx.Graph))* – Graph type to create. If graph instance, then cleared before populated.
- `nodetype` *(int, float, str, Python type, optional)* – Convert node data from strings to specified type
- `data` *(bool or list of (label,type) tuples)* – Tuples specifying dictionary key names and types for edge data
- `edgetype` *(int, float, str, Python type, optional OBSOLETE)* – Convert edge data from strings to specified type and use as ‘weight’
- `encoding` *(string, optional)* – Specify which encoding to use when reading file.

#### Returns
- `G` – A networkx Graph or other type specified with create_using

#### Example
```
>>> nx.write_edgelist(nx.path_graph(4), "test.edgelist")
>>> G=nx.read_edgelist("test.edgelist")
```

```
>>> fh=open("test.edgelist", 'rb')
>>> G=nx.read_edgelist(fh)
>>> fh.close()
```
Edgelist with data in a list:

```python
>>> fh = open('test.edgelist', 'w')
>>> d = fh.write('1 2 3')
>>> fh.close()
```

```
G = nx.read_edgelist('test.edgelist', nodetype=int, data=((('weight', float),)))
list(G)
[1, 2]
list(G.edges(data=True))
[(1, 2, {'weight': 3.0})]
```

See `read_write.edgelist.parse_edgelist()` for more examples of formatting.

See also:

`parse_edgelist()`, `write_edgelist()`

Notes

Since nodes must be hashable, the function `nodetype` must return hashable types (e.g. int, float, str, frozenset - or tuples of those, etc.)

### 9.3.3 `networkx.readwrite.edgelist.write_edgelist`

**write_edgelist** *(G, path, comments='#', delimiter=' ', data=True, encoding='utf-8')*

Write graph as a list of edges.

**Parameters**

- **G** *(graph)* – A NetworkX graph
- **path** *(file or string)* – File or filename to write. If a file is provided, it must be opened in ‘wb’ mode. Filenames ending in .gz or .bz2 will be compressed.
- **comments** *(string, optional)* – The character used to indicate the start of a comment
- **delimiter** *(string, optional)* – The string used to separate values. The default is whitespace.
- **data** *(bool or list, optional)* – If False write no edge data. If True write a string representation of the edge data dictionary... If a list (or other iterable) is provided, write the keys specified in the list.
- **encoding** *(string, optional)* – Specify which encoding to use when writing file.

**Examples**

```python
>>> G = nx.path_graph(4)
>>> nx.write_edgelist(G, "test.edgelist")
>>> G = nx.path_graph(4)
>>> fh = open("test.edgelist", 'wb')
>>> nx.write_edgelist(G, fh)
>>> nx.write_edgelist(G, "test.edgelist.gz")
>>> nx.write_edgelist(G, "test.edgelist.gz", data=False)
```
>>> G=nx.Graph()
>>> G.add_edge(1,2,weight=7,color='red')
>>> nx.write_edgelist(G,'test.edgelist',data=False)
>>> nx.write_edgelist(G,'test.edgelist',data=['color'])
>>> nx.write_edgelist(G,'test.edgelist',data=['color','weight'])

See also:

read_edgelist(), write_weighted_edgelist()

9.3.4 networkx.readwrite.edgelist.read_weighted_edgelist

read_weighted_edgelist (path, comments='#', delimiter=None, create_using=None, nodetype=None, encoding='utf-8')
Read a graph as list of edges with numeric weights.

Parameters

- **path** (file or string) – File or filename to read. If a file is provided, it must be opened in 'rb' mode. Filenames ending in .gz or .bz2 will be uncompressed.
- **comments** (string, optional) – The character used to indicate the start of a comment.
- **delimiter** (string, optional) – The string used to separate values. The default is whitespace.
- **create_using** (NetworkX graph constructor, optional (default=nx.Graph)) – Graph type to create. If graph instance, then cleared before populated.
- **nodetype** (int, float, str, Python type, optional) – Convert node data from strings to specified type
- **encoding** (string, optional) – Specify which encoding to use when reading file.

Returns  
G – A networkx Graph or other type specified with create_using

Return type  
graph

Notes

Since nodes must be hashable, the function nodetype must return hashable types (e.g. int, float, str, frozenset - or tuples of those, etc.)

Example edgelist file format.

With numeric edge data:

```bash
# >>> G=nx.read_weighted_edgelist(fh)
# source target data
a b 1
a c 3.14159
d e 42
```

See also:

write_weighted_edgelist()
9.3.5 networkx.readwrite.edgelist.write_weighted_edgelist

**write_weighted_edgelist** *(G, path, comments='#', delimiter=' ', encoding='utf-8')*

Write graph G as a list of edges with numeric weights.

**Parameters**

- **G** *(graph)* – A NetworkX graph
- **path** *(file or string)* – File or filename to write. If a file is provided, it must be opened in ‘wb’ mode. Filenames ending in .gz or .bz2 will be compressed.
- **comments** *(string, optional)* – The character used to indicate the start of a comment
- **delimiter** *(string, optional)* – The string used to separate values. The default is whitespace.
- **encoding** *(string, optional)* – Specify which encoding to use when writing file.

**Examples**

```python
>>> G = nx.Graph()
>>> G.add_edge(1, 2, weight=7)
>>> nx.write_weighted_edgelist(G, 'test.weighted.edgelist')
```

See also:

*read_edgelist(), write_edgelist(), read_weighted_edgelist()*

9.3.6 networkx.readwrite.edgelist.generate_edgelist

**generate_edgelist** *(G, delimiter=' ', data=True)*

Generate a single line of the graph G in edge list format.

**Parameters**

- **G** *(NetworkX graph)*
- **delimiter** *(string, optional)* – Separator for node labels
- **data** *(bool or list of keys)* – If False generate no edge data. If True use a dictionary representation of edge data. If a list of keys use a list of data values corresponding to the keys.

**Returns** lines – Lines of data in adjlist format.

**Return type** string

**Examples**

```python
>>> G = nx.lollipop_graph(4, 3)
>>> G[1][2]['weight'] = 3
>>> G[3][4]['capacity'] = 12
>>> for line in nx.generate_edgelist(G, data=False):
...       print(line)
0 1
0 2
0 3
1 2
1 3
```
>>> for line in nx.generate_edgelist(G):
...    print(line)
0 1 {}
0 2 {}
0 3 {}
1 2 {'weight': 3}
1 3 {}
2 3 {}
3 4 {'capacity': 12}
4 5 {}
5 6 {}

>>> for line in nx.generate_edgelist(G, data=['weight']):
...    print(line)
0 1
0 2
0 3
1 2 3
1 3
2 3
3 4
4 5
5 6

See also:
write_adjlist(), read_adjlist()

9.3.7 networkx.readwrite.edgelist.parse_edgelist

parse_edgelist (lines, comments='#', delimiter=None, create_using=None, nodetype=None, data=True)
Parse lines of an edge list representation of a graph.

Parameters

- **lines** *(list or iterator of strings)* – Input data in edgelist format
- **comments** *(string, optional)* – Marker for comment lines
- **delimiter** *(string, optional)* – Separator for node labels
- **create_using** *(NetworkX graph constructor, optional (default=nx.Graph))* – Graph type to create. If graph instance, then cleared before populated.
- **nodetype** *(Python type, optional)* – Convert nodes to this type.
- **data** *(bool or list of (label, type) tuples)* – If False generate no edge data or if True use a dictionary representation of edge data or a list tuples specifying dictionary key names and types for edge data.

Returns

- **G** – The graph corresponding to lines

Return type

NetworkX Graph

Examples

Edgelist with no data:

```python
>>> lines = ['1 2',
          ... '2 3',
          ... '3 4']
>>> G = nx.parse_edgelist(lines, nodetype = int)
>>> list(G)
[1, 2, 3, 4]
>>> list(G.edges())
[(1, 2), (2, 3), (3, 4)]
```

Edgelist with data in Python dictionary representation:

```python
>>> lines = ['1 2 {'weight':3}',
          ... '2 3 {'weight':27}',
          ... '3 4 {'weight':3.0}']
>>> G = nx.parse_edgelist(lines, nodetype = int)
>>> list(G)
[1, 2, 3, 4]
>>> list(G.edges(data=True))
[(1, 2, {'weight': 3}), (2, 3, {'weight': 27}), (3, 4, {'weight': 3.0})]
```

Edgelist with data in a list:

```python
>>> lines = ['1 2 3',
          ... '2 3 27',
          ... '3 4 3.0']
>>> G = nx.parse_edgelist(lines, nodetype = int, data=((('weight',float),)))
>>> list(G)
[1, 2, 3, 4]
>>> list(G.edges(data=True))
[(1, 2, {'weight': 3.0}), (2, 3, {'weight': 27.0}), (3, 4, {'weight': 3.0})]
```

See also:

`read_weighted_edgelist()`

9.4 GEXF

Read and write graphs in GEXF format.

GEXF (Graph Exchange XML Format) is a language for describing complex network structures, their associated data and dynamics.

This implementation does not support mixed graphs (directed and undirected edges together).

9.4.1 Format

GEXF is an XML format. See https://gephi.org/gexf/format/schema.html for the specification and https://gephi.org/gexf/format/basic.html for examples.

```python
read_gexf(path[, node_type, relabel, version]) Read graph in GEXF format from path.```

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<td>Write G in GEXF format to path.</td>
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<tr>
<td><code>generate_gexf(G[, encoding, prettyprint, ...])</code></td>
<td>Generate lines of GEXF format representation of G.</td>
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<tr>
<td><code>relabel_gexf_graph(G)</code></td>
<td>Relabel graph using “label” node keyword for node label.</td>
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9.4.2 networkx.readwrite.gexf.read_gexf

**read_gexf** *(path, node_type=None, relabel=False, version='1.2draft')*

Read graph in GEXF format from path.

“GEXF (Graph Exchange XML Format) is a language for describing complex networks structures, their associated data and dynamics”\(^1\).

**Parameters**

- **path** *(file or string)* – File or file name to write. File names ending in .gz or .bz2 will be compressed.
- **node_type** *(Python type (default: None))* – Convert node ids to this type if not None.
- **relabel** *(bool (default: False))* – If True relabel the nodes to use the GEXF node “label” attribute instead of the node “id” attribute as the NetworkX node label.
- **version** *(string (default: 1.2draft))* – Version of GEXF File Format (see https://gephi.org/gexf/format/schema.html) – Supported values: “1.1draft”, “1.2draft”

**Returns** *graph* – If no parallel edges are found a Graph or DiGraph is returned. Otherwise a MultiGraph or MultiDiGraph is returned.

**Return type** NetworkX graph

**Notes**

This implementation does not support mixed graphs (directed and undirected edges together).

**References**

9.4.3 networkx.readwrite.gexf.write_gexf

**write_gexf** *(G, path, encoding='utf-8', prettyprint=True, version='1.2draft')*

Write G in GEXF format to path.

“GEXF (Graph Exchange XML Format) is a language for describing complex networks structures, their associated data and dynamics”\(^1\).

Node attributes are checked according to the version of the GEXF schemas used for parameters which are not user defined, e.g. visualization ‘viz’\(^2\). See example for usage.

**Parameters**

- **G** *(graph)* – A NetworkX graph

---

\(^1\) GEXF File Format, https://gephi.org/gexf/format/
\(^1\) GEXF File Format, https://gephi.org/gexf/format/
\(^2\) GEXF viz schema 1.1, https://gephi.org/gexf/1.1draft/viz
• **path** (*file or string*) – File or file name to write. File names ending in .gz or .bz2 will be compressed.
• **encoding** (*string (optional, default: ‘utf-8’)*) – Encoding for text data.
• **prettyprint** (*bool (optional, default: True)*) – If True use line breaks and indenting in output XML.

### Examples

```python
g = nx.path_graph(4)
nx.write_gexf(g, "test.gexf")
```

# visualization data
```python
G.nodes[0]["viz"] = {'size': 54}
G.nodes[0]["viz"]['position'] = {'x': 0, 'y': 1}
G.nodes[0]["viz"]['color'] = {'r': 0, 'g': 0, 'b': 256}
```

### Notes

This implementation does not support mixed graphs (directed and undirected edges together).

The node id attribute is set to be the string of the node label. If you want to specify an id use set it as node data, e.g. `node[‘a’][‘id’]=1` to set the id of node ‘a’ to 1.

### References

#### 9.4.4 networkx.readwrite.gexf.generate_gexf

**generate_gexf** *(G, encoding='utf-8', prettyprint=True, version='1.2draft')*

Generate lines of GEXF format representation of G.

“GEXF (Graph Exchange XML Format) is a language for describing complex networks structures, their associated data and dynamics”¹.

**Parameters**

- **G** (*graph*)
  - A NetworkX graph
- **encoding** (*string (optional, default: ‘utf-8’)*)
  - Encoding for text data.
- **prettyprint** (*bool (optional, default: True)*)
  - If True use line breaks and indenting in output XML.
- **version** (*string (default: 1.2draft)*)
  - Version of GEFX File Format (see [https://gephi.org/gexf/format/schema.html](https://gephi.org/gexf/format/schema.html))

---

¹ GEXF File Format, [https://gephi.org/gexf/format/](https://gephi.org/gexf/format/)
Examples

```python
>>> G = nx.path_graph(4)
>>> linefeed = chr(10) # linefeed=

>>> s = linefeed.join(nx.generate_gexf(G))
>>> for line in nx.generate_gexf(G):
...     print(line)
```

Notes

This implementation does not support mixed graphs (directed and undirected edges together).

The node id attribute is set to be the string of the node label. If you want to specify an id use set it as node data, e.g. `node[‘a’][‘id’]=1` to set the id of node ‘a’ to 1.

References

**9.4.5 networkx.readwrite.gexf.relabel_gexf_graph**

`relabel_gexf_graph(G)`

Relabel graph using “label” node keyword for node label.

- **Parameters** `G (graph)` – A NetworkX graph read from GEXF data
- **Returns** `H` – A NetworkX graph with relabeled nodes
- **Return type** `graph`
- **Raises** `NetworkXError` – If node labels are missing or not unique while relabel=True.

Notes

This function relabels the nodes in a NetworkX graph with the “label” attribute. It also handles relabeling the specific GEXF node attributes “parents”, and “pid”.

**9.5 GML**

Read graphs in GML format.

“GML, the Graph Modelling Language, is our proposal for a portable file format for graphs. GML’s key features are portability, simple syntax, extensibility and flexibility. A GML file consists of a hierarchical key-value lists. Graphs can be annotated with arbitrary data structures. The idea for a common file format was born at the GD’95; this proposal is the outcome of many discussions. GML is the standard file format in the Graphlet graph editor system. It has been overtaken and adapted by several other systems for drawing graphs.”

GML files are stored using a 7-bit ASCII encoding with any extended ASCII characters (iso8859-1) appearing as HTML character entities. You will need to give some thought into how the exported data should interact with different languages and even different Python versions. Re-importing from gml is also a concern.

Without specifying a `stringizer/destringizer`, the code is capable of handling `int/float/str/dict/list` data as required by the GML specification. For other data types, you need to explicitly supply a `stringizer/destringizer`. 

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For additional documentation on the GML file format, please see the GML website. Several example graphs in GML format may be found on Mark Newman’s Network data page.

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<td><code>literal_destringizer(rep)</code></td>
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<td>Convert a value to a Python literal in GML representation.</td>
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### 9.5.1 networkx.readwrite.gml.read_gml

**read_gml** *(path, label='label', destringizer=None)*  
Read graph in GML format from path.

**Parameters**

- **path** *(filename or filehandle)* – The filename or filehandle to read from.
- **label** *(string, optional)* – If not None, the parsed nodes will be renamed according to node attributes indicated by label. Default value: ‘label’.
- **destringizer** *(callable, optional)* – A destringizer that recovers values stored as strings in GML. If it cannot convert a string to a value, a `ValueError` is raised. Default value: None.

**Returns** G – The parsed graph.

**Return type** NetworkX graph

**Raises** `NetworkXError` – If the input cannot be parsed.

**See also:**

`write_gml()`, `parse_gml()`

**Notes**

GML files are stored using a 7-bit ASCII encoding with any extended ASCII characters (iso8859-1) appearing as HTML character entities. Without specifying a stringizer/destringizer, the code is capable of handling `int/float/str/dict/list` data as required by the GML specification. For other data types, you need to explicitly supply a stringizer/destringizer.

For additional documentation on the GML file format, please see the GML url.

See the module docstring `networkx.readwrite.gml` for more details.

**Examples**

```python
>>> G = nx.path_graph(4)
>>> nx.write_gml(G, 'test.gml')
>>> H = nx.read_gml('test.gml')
```
9.5.2 networkx.readwrite.gml.write_gml

write_gml (G, path, stringizer=None)
Write a graph G in GML format to the file or file handle path.

Parameters
- G (NetworkX graph) – The graph to be converted to GML.
- path (filename or filehandle) – The filename or filehandle to write. Files whose names end with .gz or .bz2 will be compressed.
- stringizer (callable, optional) – A stringizer which converts non-int/non-float/non-dict values into strings. If it cannot convert a value into a string, it should raise a ValueError to indicate that. Default value: None.

Raises NetworkXError – If stringizer cannot convert a value into a string, or the value to convert is not a string while stringizer is None.

See also: read_gml(), generate_gml()

Notes
Graph attributes named ‘directed’, ‘multigraph’, ‘node’ or ‘edge’, node attributes named ‘id’ or ‘label’, edge attributes named ‘source’ or ‘target’ (or ‘key’ if G is a multigraph) are ignored because these attribute names are used to encode the graph structure.

GML files are stored using a 7-bit ASCII encoding with any extended ASCII characters (iso8859-1) appearing as HTML character entities. Without specifying a stringizer/destringizer, the code is capable of handling int/float/str/dict/list data as required by the GML specification. For other data types, you need to explicitly supply a stringizer/destringizer.

Note that while we allow non-standard GML to be read from a file, we make sure to write GML format. In particular, underscores are not allowed in attribute names. For additional documentation on the GML file format, please see the GML url.

See the module docstring networkx.readwrite.gml for more details.

Examples

```python
>>> G = nx.path_graph(4)
>>> nx.write_gml(G, "test.gml")
```

Filenames ending in .gz or .bz2 will be compressed.

```python
>>> nx.write_gml(G, "test.gml.gz")
```

9.5.3 networkx.readwrite.gml.parse_gml

parse_gml (lines, label='label', destringizer=None)
Parse GML graph from a string or iterable.

Parameters
- lines (string or iterable of strings) – Data in GML format.
• **label** *(string, optional)* – If not None, the parsed nodes will be renamed according to node attributes indicated by `label`. Default value: ‘label’.

• **destringizer** *(callable, optional)* – A destringizer that recovers values stored as strings in GML. If it cannot convert a string to a value, a `ValueError` is raised. Default value: None.

**Returns**  
`G` – The parsed graph.

**Return type**  
`NetworkX graph`

**Raises**  
`NetworkXError` – If the input cannot be parsed.

**See also:**

`write_gml()`, `read_gml()`

**Notes**

This stores nested GML attributes as dictionaries in the NetworkX graph, node, and edge attribute structures.

GML files are stored using a 7-bit ASCII encoding with any extended ASCII characters (iso8859-1) appearing as HTML character entities. Without specifying a stringizer/destringizer, the code is capable of handling `int/float/str/dict/list` data as required by the GML specification. For other data types, you need to explicitly supply a stringizer/destringizer.

For additional documentation on the GML file format, please see the GML url.

See the module docstring `networkx.readwrite.gml` for more details.

### 9.5.4 networkx.readwrite.gml.generate_gml

**generate_gml** *(G, stringizer=None)*

Generate a single entry of the graph `G` in GML format.

**Parameters**

• **G** *(NetworkX graph)* – The graph to be converted to GML.

• **stringizer** *(callable, optional)* – A stringizer which converts non-int/non-float/non-dict values into strings. If it cannot convert a value into a string, it should raise a `ValueError` to indicate that. Default value: None.

**Returns**  
`lines` – Lines of GML data. Newlines are not appended.

**Return type**  
generator of strings

**Raises**  
`NetworkXError` – If `stringizer` cannot convert a value into a string, or the value to convert is not a string while `stringizer` is None.

**Notes**

Graph attributes named ‘directed’, ‘multigraph’, ‘node’ or ‘edge’, node attributes named ‘id’ or ‘label’, edge attributes named ‘source’ or ‘target’ (or ‘key’ if `G` is a multigraph) are ignored because these attribute names are used to encode the graph structure.

GML files are stored using a 7-bit ASCII encoding with any extended ASCII characters (iso8859-1) appearing as HTML character entities. Without specifying a stringizer/destringizer, the code is capable of handling `int/float/str/dict/list` data as required by the GML specification. For other data types, you need to explicitly supply a stringizer/destringizer.
For additional documentation on the GML file format, please see the GML url.
See the module docstring networkx.readwrite.gml for more details.

Examples

```python
>>> G = nx.Graph()
>>> G.add_node("1")
>>> print("\n".join(nx.generate_gml(G))

graph [  
    node [  
        id 0  
        label "1"
    ]
]

>>> G = nx.OrderedMultiGraph([("a", "b"), ("a", "b")])
>>> print("\n".join(nx.generate_gml(G)))

graph [  
    multigraph 1  
    node [  
        id 0  
        label "a"
    ]  
    node [  
        id 1  
        label "b"
    ]  
    edge [  
        source 0  
        target 1  
        key 0
    ]  
    edge [  
        source 0  
        target 1  
        key 1
    ]
]
```

9.5.5 networkx.readwrite.gml.literal_destringizer

`literal_destringizer(rep)`

Convert a Python literal to the value it represents.

Parameters

- `rep (string)` – A Python literal.

Returns

- `value` – The value of the Python literal.

Return type

- `object`

Raises `ValueError` – If `rep` is not a Python literal.

9.5.6 networkx.readwrite.gml.literal_stringizer

`literal_stringizer(value)`

Convert a value to a Python literal in GML representation.
Parameters value (object) – The value to be converted to GML representation.

Returns rep – A double-quoted Python literal representing value. Unprintable characters are replaced by XML character references.

Return type string

Raises ValueError – If value cannot be converted to GML.

Notes

literal_stringizer is largely the same as repr in terms of functionality but attempts prefix unicode and bytes literals with u and b to provide better interoperability of data generated by Python 2 and Python 3.

The original value can be recovered using the networkx.readwrite.gml.literal_destringizer() function.

9.6 Pickle

9.6.1 Pickled Graphs

Read and write NetworkX graphs as Python pickles.

“The pickle module implements a fundamental, but powerful algorithm for serializing and de-serializing a Python object structure. “Pickling” is the process whereby a Python object hierarchy is converted into a byte stream, and “unpickling” is the inverse operation, whereby a byte stream is converted back into an object hierarchy.”

Note that NetworkX graphs can contain any hashable Python object as node (not just integers and strings). For arbitrary data types it may be difficult to represent the data as text. In that case using Python pickles to store the graph data can be used.

Format

See https://docs.python.org/2/library/pickle.html

read_gpickle(path) Read graph object in Python pickle format.
write_gpickle(G, path[, protocol]) Write graph in Python pickle format.

9.6.2 networkx.readwrite.gpickle.read_gpickle

read_gpickle(path)

Read graph object in Python pickle format.

Pickles are a serialized byte stream of a Python object\(^1\). This format will preserve Python objects used as nodes or edges.

Parameters path (file or string) – File or filename to write. Filenames ending in .gz or .bz2 will be uncompressed.

Returns G – A NetworkX graph

Return type graph

\(^1\) https://docs.python.org/2/library/pickle.html
Examples

```python
>>> G = nx.path_graph(4)
>>> nx.write_gpickle(G, "test.gpickle")
>>> G = nx.read_gpickle("test.gpickle")
```

References

9.6.3 networkx.readwrite.gpickle.write_gpickle

write_gpickle \((G, path, protocol=4)\)
Write graph in Python pickle format.

Pickles are a serialized byte stream of a Python object\(^1\). This format will preserve Python objects used as nodes or edges.

Parameters

- **G** *(graph)* – A NetworkX graph
- **path** *(file or string)* – File or filename to write. Filenames ending in .gz or .bz2 will be compressed.
- **protocol** *(integer)* – Pickling protocol to use. Default value: pickle.HIGHEST_PROTOCOL.

Examples

```python
>>> G = nx.path_graph(4)
>>> nx.write_gpickle(G, "test.gpickle")
```

References

9.7 GraphML

9.7.1 GraphML

Read and write graphs in GraphML format.

This implementation does not support mixed graphs (directed and undirected edges together), hyperedges, nested graphs, or ports.

“GraphML is a comprehensive and easy-to-use file format for graphs. It consists of a language core to describe the structural properties of a graph and a flexible extension mechanism to add application-specific data. Its main features include support of

- directed, undirected, and mixed graphs,
- hypergraphs,
- hierarchical graphs,
- graphical representations.

\(^1\) [https://docs.python.org/2/library/pickle.html](https://docs.python.org/2/library/pickle.html)
• references to external data,
• application-specific attribute data, and
• light-weight parsers.

Unlike many other file formats for graphs, GraphML does not use a custom syntax. Instead, it is based on XML and hence ideally suited as a common denominator for all kinds of services generating, archiving, or processing graphs.”

http://graphml.graphdrawing.org/

Format

GraphML is an XML format. See http://graphml.graphdrawing.org/specification.html for the specification and http://graphml.graphdrawing.org/primer/graphml-primer.html for examples.

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<tr>
<td>write_graphml(G, path[, encoding, ...])</td>
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<td>generate_graphml(G[, encoding, prettyprint])</td>
<td>Generate GraphML lines for G</td>
</tr>
<tr>
<td>parse_graphml(graphml_string[, node_type])</td>
<td>Read graph in GraphML format from string.</td>
</tr>
</tbody>
</table>

9.7.2 networkx.readwrite.graphml.read_graphml

read_graphml (path, node_type=<class 'str'>, edge_key_type=<class 'int'>)

Read graph in GraphML format from path.

Parameters

- **path** (file or string) – File or filename to write. Filenames ending in .gz or .bz2 will be compressed.
- **node_type** (Python type (default: str)) – Convert node ids to this type
- **edge_key_type** (Python type (default: int)) – Convert graphml edge ids to this type as key of multi-edges

Returns graph – If no parallel edges are found a Graph or DiGraph is returned. Otherwise a MultiGraph or MultiDiGraph is returned.

Return type NetworkX graph

Notes

Default node and edge attributes are not propagated to each node and edge. They can be obtained from G.graph and applied to node and edge attributes if desired using something like this:

```python
>>> default_color = G.graph['node_default']['color']
>>> for node, data in G.nodes(data=True):
...   if 'color' not in data:
...     data['color'] = default_color
>>> default_color = G.graph['edge_default']['color']
>>> for u, v, data in G.edges(data=True):
...   if 'color' not in data:
...     data['color'] = default_color
```

This implementation does not support mixed graphs (directed and undirected edges together), hypergraphs, nested graphs, or ports.
For multigraphs the GraphML edge “id” will be used as the edge key. If not specified then they “key” attribute will be used. If there is no “key” attribute a default NetworkX multigraph edge key will be provided.

Files with the yEd “yfiles” extension will can be read but the graphics information is discarded.
yEd compressed files (“file.graphmlz” extension) can be read by renaming the file to “file.graphml.gz”.

### 9.7.3 networkx.readwrite.graphml.write_graphml

**write_graphml** *(G, path, encoding='utf-8', prettyprint=True, infer_numeric_types=False)*

Write G in GraphML XML format to path

This function uses the LXML framework and should be faster than the version using the xml library.

**Parameters**

- **G (graph)** – A networkx graph
- **path (file or string)** – File or filename to write. Filenames ending in .gz or .bz2 will be compressed.
- **encoding (string (optional))** – Encoding for text data.
- **prettyprint (bool (optional))** – If True use line breaks and indenting in output XML.
- **infer_numeric_types (bool) –** Determine if numeric types should be generalized. For example, if edges have both int and float ‘weight’ attributes, we infer in GraphML that both are floats.

**Examples**

```python
>>> G = nx.path_graph(4)
>>> nx.write_graphml_lxml(G, "fourpath.graphml")
```

**Notes**

This implementation does not support mixed graphs (directed and undirected edges together) hyperedges, nested graphs, or ports.

### 9.7.4 networkx.readwrite.graphml.generate_graphml

**generate_graphml** *(G, encoding='utf-8', prettyprint=True)*

Generate GraphML lines for G

**Parameters**

- **G (graph)** – A networkx graph
- **encoding (string (optional))** – Encoding for text data.
- **prettyprint (bool (optional))** – If True use line breaks and indenting in output XML.
Examples

```python
>>> G = nx.path_graph(4)
>>> linefeed = chr(10)  # linefeed =

>>> s = linefeed.join(nx.generate_graphml(G))
>>> for line in nx.generate_graphml(G):
...    print(line)
```

Notes

This implementation does not support mixed graphs (directed and undirected edges together) hyperedges, nested graphs, or ports.

9.7.5 networkx.readwrite.graphml.parse_graphml

`parse_graphml(graphml_string, node_type=<class 'str'>)`

Read graph in GraphML format from string.

Parameters

- `graphml_string (string)` – String containing graphml information (e.g., contents of a graphml file).
- `node_type (Python type (default: str))` – Convert node ids to this type

Returns `graph` – If no parallel edges are found a Graph or DiGraph is returned. Otherwise a MultiGraph or MultiDiGraph is returned.

Return type NetworkX graph

Examples

```python
>>> G = nx.path_graph(4)
>>> linefeed = chr(10)  # linefeed =

>>> s = linefeed.join(nx.generate_graphml(G))
>>> H = nx.parse_graphml(s)
```

Notes

Default node and edge attributes are not propagated to each node and edge. They can be obtained from `G.graph` and applied to node and edge attributes if desired using something like this:

```python
>>> default_color = G.graph['node_default']['color']
>>> for node, data in G.nodes(data=True):
...    if 'color' not in data:
...       data['color']=default_color
>>> default_color = G.graph['edge_default']['color']
>>> for u, v, data in G.edges(data=True):
...    if 'color' not in data:
...       data['color']=default_color
```
This implementation does not support mixed graphs (directed and undirected edges together), hypergraphs, nested graphs, or ports.

For multigraphs the GraphML edge “id” will be used as the edge key. If not specified then they “key” attribute will be used. If there is no “key” attribute a default NetworkX multigraph edge key will be provided.

9.8 JSON

9.8.1 JSON data

Generate and parse JSON serializable data for NetworkX graphs.

These formats are suitable for use with the d3.js examples https://d3js.org/

The three formats that you can generate with NetworkX are:

- node-link like in the d3.js example https://bl.ocks.org/mbostock/4062045
- tree like in the d3.js example https://bl.ocks.org/mbostock/4063550
- adjacency like in the d3.js example https://bost.ocks.org/mike/miserables/

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<td>Returns data in node-link format that is suitable for JSON serialization and use in Javascript documents.</td>
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<td>Returns graph from node-link data format.</td>
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<td>adjacency_data(G[, attrs])</td>
<td>Returns data in adjacency format that is suitable for JSON serialization and use in Javascript documents.</td>
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<tr>
<td>adjacency_graph(data[, directed, ...])</td>
<td>Returns graph from adjacency data format.</td>
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<tr>
<td>cytoscape_data(G[, attrs])</td>
<td>Returns data in Cytoscape JSON format (cyjs).</td>
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<td>Returns data in tree format that is suitable for JSON serialization and use in Javascript documents.</td>
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9.8.2 networkx.readwrite.json_graph.node_link_data

node_link_data(G, attrs=None)

Returns data in node-link format that is suitable for JSON serialization and use in Javascript documents.

Parameters

- **G** (NetworkX graph)
- **attrs** (dict) – A dictionary that contains five keys ‘source’, ‘target’, ‘name’, ‘key’ and ‘link’. The corresponding values provide the attribute names for storing NetworkX-internal graph data. The values should be unique. Default value:

\[
\text{dict}(source='source', target='target', name='id', key='key', link='links')
\]

If some user-defined graph data use these attribute names as data keys, they may be silently dropped.
Returns data – A dictionary with node-link formatted data.

Return type dict

Raises NetworkXError – If values in attrs are not unique.

Examples

```python
>>> from networkx.readwrite import json_graph
>>> G = nx.Graph([('A', 'B')])
>>> data1 = json_graph.node_link_data(G)
>>> H = nx.gn_graph(2)
>>> data2 = json_graph.node_link_data(H, {'link': 'edges', 'source': 'from',
→'target': 'to'})
```

To serialize with json

```python
>>> import json
>>> s1 = json.dumps(data1)
>>> s2 = json.dumps(data2, default={'link': 'edges', 'source': 'from', 'target':
→'to'})
```

Notes

Graph, node, and link attributes are stored in this format. Note that attribute keys will be converted to strings in order to comply with JSON.

Attribute ‘key’ is only used for multigraphs.

See also:

node_link_graph(), adjacency_data(), tree_data()

9.8.3 networkx.readwrite.json_graph.node_link_graph

node_link_graph (data, directed=False, multigraph=True, attrs=None)

Returns graph from node-link data format.

Parameters

- data (dict) – node-link formatted graph data
- directed (bool) – If True, and direction not specified in data, return a directed graph.
- multigraph (bool) – If True, and multigraph not specified in data, return a multigraph.
- attrs (dict) – A dictionary that contains five keys ‘source’, ‘target’, ‘name’, ‘key’ and ‘link’.
  The corresponding values provide the attribute names for storing NetworkX-internal graph data. Default value:
  ```python
dict(source='source', target='target', name='id', key='key', link='links')
```

Returns G – A NetworkX graph object

Return type NetworkX graph
Examples

```python
>>> from networkx.readwrite import json_graph
>>> G = nx.Graph([('A', 'B')])
>>> data = json_graph.node_link_data(G)
>>> H = json_graph.node_link_graph(data)
```

Notes

Attribute ‘key’ is only used for multigraphs.

See also:

`node_link_data()`, `adjacency_data()`, `tree_data()`

9.8.4 networkx.readwrite.json_graph.adjacency_data

adjacency_data(G, attrs={'id': 'id', 'key': 'key'})

Returns data in adjacency format that is suitable for JSON serialization and use in Javascript documents.

Parameters

- G (NetworkX graph)
- attrs (dict) – A dictionary that contains two keys ‘id’ and ‘key’. The corresponding values provide the attribute names for storing NetworkX-internal graph data. The values should be unique. Default value: `dict(id='id', key='key')`.

If some user-defined graph data use these attribute names as data keys, they may be silently dropped.

Returns data – A dictionary with adjacency formatted data.

Return type dict

Raises NetworkXError – If values in attrs are not unique.

Examples

```python
>>> from networkx.readwrite import json_graph
>>> G = nx.Graph([(1,2)])
>>> data = json_graph.adjacency_data(G)
```

To serialize with json

```python
>>> import json
>>> s = json.dumps(data)
```

Notes

Graph, node, and link attributes will be written when using this format but attribute keys must be strings if you want to serialize the resulting data with JSON.

The default value of attrs will be changed in a future release of NetworkX.

See also:
adjacency_graph(), node_link_data(), tree_data()

9.8.5 networkx.readwrite.json_graph.adjacency_graph

adjacency_graph(data, directed=False, multigraph=True, attrs={'id': 'id', 'key': 'key'})

Returns graph from adjacency data format.

Parameters data (dict) – Adjacency list formatted graph data

Returns

• G (NetworkX graph) – A NetworkX graph object
• directed (bool) – If True, and direction not specified in data, return a directed graph.
• multigraph (bool) – If True, and multigraph not specified in data, return a multigraph.
• attrs (dict) – A dictionary that contains two keys ‘id’ and ‘key’. The corresponding values provide the attribute names for storing NetworkX-internal graph data. The values should be unique. Default value: dict(id='id', key='key').

Examples

```python
>>> from networkx.readwrite import json_graph
>>> G = nx.Graph([(1,2)])
>>> data = json_graph.adjacency_data(G)
>>> H = json_graph.adjacency_graph(data)
```

Notes

The default value of attrs will be changed in a future release of NetworkX.

See also:

adjacency_graph(), node_link_data(), tree_data()

9.8.6 networkx.readwrite.json_graph.cytoscape_data

cytoscape_data(G, attrs=None)

Returns data in Cytoscape JSON format (cyjs).

Parameters G (NetworkX Graph)

Returns data – A dictionary with cyjs formatted data.

Return type dict

Raises NetworkXError – If values in attrs are not unique.

9.8.7 networkx.readwrite.json_graph.cytoscape_graph

cytoscape_graph(data, attrs=None)
9.8.8 networkx.readwrite.json_graph.tree_data

**tree_data** *(G, root, attrs= dict('children': 'children', 'id': 'id'))*

Returns data in tree format that is suitable for JSON serialization and use in Javascript documents.

**Parameters**

- **G** *(NetworkX graph)* – G must be an oriented tree
- **root** *(node)* – The root of the tree
- **attrs** *(dict)* – A dictionary that contains two keys ‘id’ and ‘children’. The corresponding values provide the attribute names for storing NetworkX-internal graph data. The values should be unique. Default value: `dict(id='id', children='children')`.

If some user-defined graph data use these attribute names as data keys, they may be silently dropped.

**Returns** data – A dictionary with node-link formatted data.

**Return type** **dict**

**Raises** **NetworkXError** – If values in attrs are not unique.

**Examples**

```python
>>> from networkx.readwrite import json_graph
>>> G = nx.DiGraph([(1,2)])
>>> data = json_graph.tree_data(G, root=1)
```

To serialize with json

```python
>>> import json
>>> s = json.dumps(data)
```

**Notes**

Node attributes are stored in this format but keys for attributes must be strings if you want to serialize with JSON.

Graph and edge attributes are not stored.

The default value of attrs will be changed in a future release of NetworkX.

**See also:**

*tree_graph(), node_link_data(), node_link_data()*

9.8.9 networkx.readwrite.json_graph.tree_graph

**tree_graph** *(data, attrs= dict('children': 'children', 'id': 'id'))*

Returns graph from tree data format.

**Parameters** **data** *(dict)* – Tree formatted graph data

**Returns**

- **G** *(NetworkX DiGraph)*
• **attrs** (dict) – A dictionary that contains two keys ‘id’ and ‘children’. The corresponding values provide the attribute names for storing NetworkX-internal graph data. The values should be unique. Default value: `dict(id='id', children='children')`.

**Examples**

```python
>>> from networkx.readwrite import json_graph
>>> G = nx.DiGraph([(1,2)])
>>> data = json_graph.tree_data(G, root=1)
>>> H = json_graph.tree_graph(data)
```

**Notes**

The default value of attrs will be changed in a future release of NetworkX.

See also:

`tree_graph()`, `node_link_data()`, `adjacency_data()`

### 9.8.10 `networkx.readwrite.json_graph.jit_data`

**jit_data** *(G, indent=None)*

Returns data in JIT JSON format.

**Parameters**

- **G** *(NetworkX Graph)*
- **indent** *(optional, default=None)* – If indent is a non-negative integer, then JSON array elements and object members will be pretty-printed with that indent level. An indent level of 0, or negative, will only insert newlines. None (the default) selects the most compact representation.

**Returns** data

**Return type** JIT JSON string

### 9.8.11 `networkx.readwrite.json_graph.jit_graph`

**jit_graph** *(data, create_using=None)*

Read a graph from JIT JSON.

**Parameters**

- **data** *(JSON Graph Object)*
- **create_using** *(Networkx Graph, optional (default: Graph))* – Return graph of this type. The provided instance will be cleared.

**Returns** G

**Return type** NetworkX Graph built from create_using if provided.
9.9 LEDA

Read graphs in LEDA format.
LEDA is a C++ class library for efficient data types and algorithms.

9.9.1 Format

See http://www.algorithmic-solutions.info/leda_guide/graphs/leda_native_graph_fileformat.html

| **read_leda** (path[, encoding]) | Read graph in LEDA format from path. |
| **parse_leda** (lines)         | Read graph in LEDA format from string or iterable. |

9.9.2 networkx.readwrite.leda.read_leda

**read_leda** *(path, encoding='UTF-8')*
Read graph in LEDA format from path.

- **Parameters**:
  - *path* *(file or string)* – File or filename to read. Filenames ending in .gz or .bz2 will be uncompressed.

- **Returns**:
  - *G*  

- **Return type**:
  - NetworkX graph

**Examples**

G=nx.read_leda('file.leda')

**References**

9.9.3 networkx.readwrite.leda.parse_leda

**parse_leda** *(lines)*
Read graph in LEDA format from string or iterable.

- **Parameters**:
  - *lines* *(string or iterable)* – Data in LEDA format.

- **Returns**:
  - *G*  

- **Return type**:
  - NetworkX graph

**Examples**

G=nx.parse_leda(string)
References

9.10 YAML

9.10.1 YAML

Read and write NetworkX graphs in YAML format.

“YAML is a data serialization format designed for human readability and interaction with scripting languages.” See http://www.yaml.org for documentation.

Format

http://pyyaml.org/wiki/PyYAML

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<td>Read graph in YAML format from path.</td>
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<td>Write graph G in YAML format to path.</td>
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9.10.2 networkx.readwrite.nx_yaml.read_yaml

read_yaml(path)

Read graph in YAML format from path.

YAML is a data serialization format designed for human readability and interaction with scripting languages\(^1\).

Parameters

- **path (file or string)** – File or filename to read. Filenames ending in .gz or .bz2 will be uncompressed.

Returns

- **G**

Return type

- NetworkX graph

Examples

```python
>>> G=nx.path_graph(4)
>>> nx.write_yaml(G,'test.yaml')
>>> G=nx.read_yaml('test.yaml')
```

9.10.3 networkx.readwrite.nx_yaml.write_yaml

write_yaml(G_to_be_yaml, path_for_yaml_output, **kwds)

Write graph G in YAML format to path.

YAML is a data serialization format designed for human readability and interaction with scripting languages\(^1\).

Parameters

- **G (graph)** – A NetworkX graph

---

\(^1\) http://www.yaml.org

---
• **path** *(file or string)* – File or filename to write. Filenames ending in .gz or .bz2 will be compressed.

**Notes**

To use encoding on the output file include e.g. `encoding='utf-8'` in the keyword arguments.

**Examples**

```python
>>> G=nx.path_graph(4)
>>> nx.write_yaml(G,'test.yaml')
```

**References**

### 9.11 SparseGraph6

Functions for reading and writing graphs in the `graph6` or `sparse6` file formats.

According to the author of these formats,

*graph6* and *sparse6* are formats for storing undirected graphs in a compact manner, using only printable ASCII characters. Files in these formats have text type and contain one line per graph.

*graph6* is suitable for small graphs, or large dense graphs. *sparse6* is more space-efficient for large sparse graphs.

—graph6 and sparse6 homepage

#### 9.11.1 Graph6

Functions for reading and writing graphs in the `graph6` format.

The `graph6` file format is suitable for small graphs or large dense graphs. For large sparse graphs, use the `sparse6` format.

For more information, see the [graph6 homepage](http://www.graph6.org/).

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<td>Write a simple undirected graph to a path in <code>graph6</code> format.</td>
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**networkx.readwrite.graph6.from_graph6_bytes**

`from_graph6_bytes` *(string)*

Read a simple undirected graph in `graph6` format from string.
Parameters  

string \((string)\) – Data in graph6 format, without a trailing newline.

Returns  

\(G\)

Return type  

Graph

Raises

- **NetworkXError** – If the string is unable to be parsed in graph6 format
- **ValueError** – If any character \(c\) in the input string does not satisfy \(63 \leq \text{ord}(c) < 127\).

Examples

```python
>>> G = nx.from_graph6_bytes(b'A_')
>>> sorted(G.edges())
[\{(0, 1)\}]
```

See also:

read_graph6(), write_graph6()

References

networkx.readwrite.graph6.read_graph6

read_graph6 \((path)\)
Read simple undirected graphs in graph6 format from path.

Parameters  

path \((file \ or \ string)\) – File or filename to write.

Returns  

\(G\) – If the file contains multiple lines then a list of graphs is returned

Return type  

Graph or list of Graphs

Raises  

NetworkXError – If the string is unable to be parsed in graph6 format

Examples

You can read a graph6 file by giving the path to the file:

```python
>>> import tempfile
>>> with tempfile.NamedTemporaryFile() as f:
...     _ = f.write(b'>>graph6<<A_
...
...     _ = f.seek(0)
...     G = nx.read_graph6(f.name)
>>> list(G.edges())
[\{(0, 1)\}]
```

You can also read a graph6 file by giving an open file-like object:

```python
>>> import tempfile
>>> with tempfile.NamedTemporaryFile() as f:
...     _ = f.write(b'>>graph6<<A_
...
...     _ = f.seek(0)
...     G = nx.read_graph6(f)
```
>>> list(G.edges())
[(0, 1)]

See also:

from_graph6_bytes(), write_graph6()

References

networkx.readwrite.graph6.to_graph6_bytes
to_graph6_bytes(G, nodes=None, header=True)
Convert a simple undirected graph to bytes in graph6 format.

Parameters

• G (Graph (undirected))
• nodes (list or iterable) – Nodes are labeled 0...n-1 in the order provided. If None the ordering given by G.nodes() is used.
• header (bool) – If True add ‘>>graph6<<’ bytes to head of data.

Raises

• NetworkXNotImplemented – If the graph is directed or is a multigraph.
• ValueError – If the graph has at least \(2^{36}\) nodes; the graph6 format is only defined for graphs of order less than \(2^{36}\).

Examples

>>> nx.to_graph6_bytes(nx.path_graph(2))
b'>>graph6<<A\n'

See also:

from_graph6_bytes(), read_graph6(), write_graph6_bytes()

Notes

The returned bytes end with a newline character.

The format does not support edge or node labels, parallel edges or self loops. If self loops are present they are silently ignored.

References

networkx.readwrite.graph6.write_graph6

write_graph6(G, path, nodes=None, header=True)
Write a simple undirected graph to a path in graph6 format.

Parameters

• G (Graph (undirected))
• **path** (*str*) – The path naming the file to which to write the graph.

• **nodes** (*list or iterable*) – Nodes are labeled 0…n-1 in the order provided. If None the ordering given by `G.nodes()` is used.

• **header** (*bool*) – If True add ‘>>graph6<<’ string to head of data

**Raises**

• **NetworkXNotImplemented** – If the graph is directed or is a multigraph.

• **ValueError** – If the graph has at least \(2 \times 36\) nodes; the graph6 format is only defined for graphs of order less than \(2 \times 36\).

**Examples**

You can write a graph6 file by giving the path to a file:

```python
>>> import tempfile
>>> with tempfile.NamedTemporaryFile() as f:
...    nx.write_graph6(nx.path_graph(2), f.name)
...    _ = f.seek(0)
...    print(f.read())

b'>>graph6<<A_\n'
```

**See also:**

`from_graph6_bytes()`, `read_graph6()`

**Notes**

The function writes a newline character after writing the encoding of the graph.

The format does not support edge or node labels, parallel edges or self loops. If self loops are present they are silently ignored.

**References**

**9.11.2 Sparse6**

Functions for reading and writing graphs in the *sparse6* format.

The *sparse6* file format is a space-efficient format for large sparse graphs. For small graphs or large dense graphs, use the *graph6* file format.

For more information, see the *sparse6* homepage.

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<td><code>write_sparse6(G, path[, nodes, header])</code></td>
<td>Write graph G to given path in sparse6 format.</td>
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</tbody>
</table>
networkx.readwrite.sparse6.from_sparse6_bytes

**from_sparse6_bytes**(string)  
Read an undirected graph in sparse6 format from string.

**Parameters**  
string *(string)* – Data in sparse6 format

**Returns**  
G

**Return type**  
Graph

**Raises** NetworkXError – If the string is unable to be parsed in sparse6 format

**Examples**

```python
>>> G = nx.from_sparse6_bytes(b':A_')
>>> sorted(G.edges())
[(0, 1), (0, 1), (0, 1)]
```

See also:  
read_sparse6(), write_sparse6()

**References**

networkx.readwrite.sparse6.read_sparse6

**read_sparse6**(path)  
Read an undirected graph in sparse6 format from path.

**Parameters**  
path *(file or string)* – File or filename to write.

**Returns**  
G – If the file contains multiple lines then a list of graphs is returned

**Return type**  
Graph/Multigraph or list of Graphs/MultiGraphs

**Raises** NetworkXError – If the string is unable to be parsed in sparse6 format

**Examples**

You can read a sparse6 file by giving the path to the file:

```python
>>> import tempfile
>>> with tempfile.NamedTemporaryFile() as f:
...    _ = f.write(b'>>sparse6<<:An
...
...
...
G = nx.read_sparse6(f.name)
>>> list(G.edges())
[(0, 1)]
```

You can also read a sparse6 file by giving an open file-like object:

```python
>>> import tempfile
>>> with tempfile.NamedTemporaryFile() as f:
...    _ = f.write(b'>>sparse6<<:An
...
...
...
G = nx.read_sparse6(f)
```
See also:

```
read_sparse6(), from_sparse6_bytes()
```

References

networkx.readwrite.sparse6.to_sparse6_bytes

to_sparse6_bytes(G, nodes=None, header=True)

Convert an undirected graph to bytes in sparse6 format.

- **G** (*Graph (undirected)*)
- **nodes** (*list or iterable*) – Nodes are labeled 0...n-1 in the order provided. If None the ordering given by `G.nodes()` is used.
- **header** (*bool*) – If True add ‘>>>sparse6<<’ bytes to head of data.

Raises

- **NetworkXNotImplemented** – If the graph is directed.
- **ValueError** – If the graph has at least $2^{36}$ nodes; the sparse6 format is only defined for graphs of order less than $2^{36}$.

Examples

```
>>> nx.to_sparse6_bytes(nx.path_graph(2))
b'>>>sparse6<<:An\n'
```

See also:

```
to_sparse6_bytes(), read_sparse6(), write_sparse6_bytes()
```

Notes

The returned bytes end with a newline character.

The format does not support edge or node labels.

References

networkx.readwrite.sparse6.write_sparse6

write_sparse6(G, path, nodes=None, header=True)

Write graph G to given path in sparse6 format.

- **G** (*Graph (undirected)*)
• **path** (*file or string*) – File or filename to write

• **nodes** (*list or iterable*) – Nodes are labeled 0…n-1 in the order provided. If None the ordering given by G.nodes() is used.

• **header** (*bool*) – If True add ‘>>sparse6<<’ string to head of data

Raises **NetworkXError** – If the graph is directed

### Examples

You can write a sparse6 file by giving the path to the file:

```python
>>> import tempfile
>>> with tempfile.NamedTemporaryFile() as f:
...     nx.write_sparse6(nx.path_graph(2), f.name)
...     print(f.read())
b'>>sparse6<<:An
'
```

You can also write a sparse6 file by giving an open file-like object:

```python
>>> with tempfile.NamedTemporaryFile() as f:
...     nx.write_sparse6(nx.path_graph(2), f)
...     _ = f.seek(0)
...     print(f.read())
b'>>sparse6<<:An\n'
```

See also:

`read_sparse6()`, `from_sparse6_bytes()`

### Notes

The format does not support edge or node labels.

### References

9.12 Pajek

9.12.1 Pajek

Read graphs in Pajek format.

This implementation handles directed and undirected graphs including those with self loops and parallel edges.

### Format


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</table>
```
9.12.2 networkx.readwrite.pajek.read_pajek

`read_pajek(path, encoding='UTF-8')`
Read graph in Pajek format from path.

Parameters  
path (file or string) – File or filename to write. Filenames ending in .gz or .bz2 will be uncompressed.

Returns  
G

Return type  
NetworkX MultiGraph or MultiDiGraph.

Examples

```python
>>> G = nx.path_graph(4)
>>> nx.write_pajek(G, "test.net")
>>> G = nx.read_pajek("test.net")
```

To create a Graph instead of a MultiGraph use

```python
>>> G1 = nx.Graph(G)
```

References


9.12.3 networkx.readwrite.pajek.write_pajek

`write_pajek(G, path, encoding='UTF-8')`
Write graph in Pajek format to path.

Parameters  
- G (graph) – A Networkx graph  
- path (file or string) – File or filename to write. Filenames ending in .gz or .bz2 will be compressed.

Examples

```python
>>> G = nx.path_graph(4)
>>> nx.write_pajek(G, "test.net")
```

**Warning:** Optional node attributes and edge attributes must be non-empty strings. Otherwise it will not be written into the file. You will need to convert those attributes to strings if you want to keep them.

References

9.12.4 networkx.readwrite.pajek.parse_pajek

parse_pajek (lines)
Parse Pajek format graph from string or iterable.

Parameters
lines (string or iterable) – Data in Pajek format.

Returns
G

Return type
NetworkX graph

See also:
read_pajek()

9.12.5 networkx.readwrite.pajek.generate_pajek

generate_pajek (G)
Generate lines in Pajek graph format.

Parameters
G (graph) – A Networkx graph

References

9.13 GIS Shapefile

9.13.1 Shapefile

Generates a networkx.DiGraph from point and line shapefiles.

“The Esri Shapefile or simply a shapefile is a popular geospatial vector data format for geographic information systems software. It is developed and regulated by Esri as a (mostly) open specification for data interoperability among Esri and other software products.” See https://en.wikipedia.org/wiki/Shapefile for additional information.

| read_shp (path[, simplify, geom_attrs, strict]) | Generates a networkx.DiGraph from shapefiles. |
| write_shp (G, outdir) | Writes a networkx.DiGraph to two shapefiles, edges and nodes. |

9.13.2 networkx.readwrite.nx_shp.read_shp

read_shp (path, simplify=True, geom_attrs=True, strict=True)
Generates a networkx.DiGraph from shapefiles. Point geometries are translated into nodes, lines into edges. Coordinate tuples are used as keys. Attributes are preserved, line geometries are simplified into start and end coordinates. Accepts a single shapefile or directory of many shapefiles.

“The Esri Shapefile or simply a shapefile is a popular geospatial vector data format for geographic information systems software.”

Parameters

1 https://en.wikipedia.org/wiki/Shapefile
• **path (file or string)** – File, directory, or filename to read.

• **simplify (bool)** – If True, simplify line geometries to start and end coordinates. If False, and line feature geometry has multiple segments, the non-geometric attributes for that feature will be repeated for each edge comprising that feature.

• **geom_attrs (bool)** – If True, include the Wkb, Wkt and Json geometry attributes with each edge.

  NOTE: if these attributes are available, write_shp will use them to write the geometry. If nodes store the underlying coordinates for the edge geometry as well (as they do when they are read via this method) and they change, your geometry will be out of sync.

• **strict (bool)** – If True, raise NetworkXError when feature geometry is missing or GeometryType is not supported. If False, silently ignore missing or unsupported geometry in features.

**Returns**  
\[ G \]

**Return type**  
NetworkX graph

**Raises**

• **ImportError** – If ogr module is not available.

• **RuntimeError** – If file cannot be open or read.

• **NetworkXError** – If strict=True and feature is missing geometry or GeometryType is not supported.

**Examples**

```python
>>> G = nx.read_shp('test.shp')
```

**References**

9.13.3 networkx.readwrite.nx_shp.write_shp

**write_shp** \((G, outdir)\)

Writes a networkx.DiGraph to two shapefiles, edges and nodes. Nodes and edges are expected to have a Well Known Binary (Wkb) or Well Known Text (Wkt) key in order to generate geometries. Also acceptable are nodes with a numeric tuple key \((x,y)\).

“The Esri Shapefile or simply a shapefile is a popular geospatial vector data format for geographic information systems software.”[1]

**Parameters**  
outdir (directory path) – Output directory for the two shapefiles.

**Returns**

**Return type**  
None

**Examples**

```python
nx.write_shp(digraph, '/shapefiles') # doctest +SKIP
```

---

References
NetworkX provides basic functionality for visualizing graphs, but its main goal is to enable graph analysis rather than perform graph visualization. In the future, graph visualization functionality may be removed from NetworkX or only available as an add-on package.

Proper graph visualization is hard, and we highly recommend that people visualize their graphs with tools dedicated to that task. Notable examples of dedicated and fully-featured graph visualization tools are Cytoscape, Gephi, Graphviz and, for LaTeX typesetting, PGF/TikZ. To use these and other such tools, you should export your NetworkX graph into a format that can be read by those tools. For example, Cytoscape can read the GraphML format, and so, `networkx.write_graphml(G, path)` might be an appropriate choice.

### 10.1 Matplotlib

#### 10.1.1 Matplotlib

Draw networks with matplotlib.

See also:

- [matplotlib](http://matplotlib.org/)
- [pygraphviz](http://pygraphviz.github.io/)

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10.1.2 networkx.drawing.nx_pylab.draw

draw \((G, \text{pos}=\text{None}, ax=\text{None}, **\text{kwds})\)

Draw the graph \(G\) with Matplotlib.

Draw the graph as a simple representation with no node labels or edge labels and using the full Matplotlib figure area and no axis labels by default. See draw_networkx() for more full-featured drawing that allows title, axis labels etc.

**Parameters**

- **\(G\) (graph)** – A networkx graph
- **\text{pos} (dictionary, optional)** – A dictionary with nodes as keys and positions as values. If not specified a spring layout positioning will be computed. See \textit{networkx.drawing.layout} for functions that compute node positions.
- **\text{ax} (Matplotlib Axes object, optional)** – Draw the graph in specified Matplotlib axes.
- **\text{kwds} (optional keywords)** – See \textit{networkx.draw_networkx()} for a description of optional keywords.

**Examples**

```python
>>> G = nx.dodecahedral_graph()
>>> nx.draw(G)
>>> nx.draw(G, pos=nx.spring_layout(G))  # use spring layout
```

See also:

- \textit{draw_networkx()}
- \textit{draw_networkx_nodes()}
- \textit{draw_networkx_edges()}
- \textit{draw_networkx_labels()}
- \textit{draw_networkx_edge_labels()}

**Notes**

This function has the same name as \textit{pylab.draw} and \textit{pyplot.draw} so beware when using \textit{from networkx import *}

since you might overwrite the \textit{pylab.draw} function.

With \textit{pyplot} use

```python
>>> import matplotlib.pyplot as plt
>>> import networkx as nx
>>> G = nx.dodecahedral_graph()
>>> nx.draw(G)
>>> plt.draw()  # pyplot draw()
```

Also see the NetworkX drawing examples at https://networkx.github.io/documentation/latest/auto_examples/index.html

10.1.3 networkx.drawing.nx_pylab.draw_networkx

draw_networkx \((G, \text{pos}=\text{None}, \text{arrows}=\text{True}, \text{with}_\text{labels}=\text{True}, **\text{kwds})\)

Draw the graph \(G\) using Matplotlib.
Parameters

- **G** *(graph)* – A networkx graph
- **pos** *(dictionary, optional)* – A dictionary with nodes as keys and positions as values. If not specified a spring layout positioning will be computed. See `networkx.drawing.layout` for functions that compute node positions.
- **arrows** *(bool, optional (default=True))* – For directed graphs, if True draw arrowheads. Note: Arrows will be the same color as edges.
- **arrowstyle** *(str, optional (default='^>|'))* – For directed graphs, choose the style of the arrowheads. See :py:class:`matplotlib.patches.ArrowStyle` for more options.
- **arrowsize** *(int, optional (default=10))* – For directed graphs, choose the size of the arrow head head’s length and width. See :py:class:`matplotlib.patches.FancyArrowPatch` for attribute `mutation_scale` for more info.
- **with_labels** *(bool, optional (default=True))* – Set to True to draw labels on the nodes.
- **ax** *(Matplotlib Axes object, optional)* – Draw the graph in the specified Matplotlib axes.
- **nodelist** *(list, optional (default G.nodes()))* – Draw only specified nodes
- **edgelist** *(list, optional (default=G.edges()))* – Draw only specified edges
- **node_size** *(scalar or array, optional (default=300))* – Size of nodes. If an array is specified it must be the same length as nodelist.
- **node_color** *(color or array of colors (default='#1f78b4'))* – Node color. Can be a single color or a sequence of colors with the same length as nodelist. Color can be string, or rgb (or rgba) tuple of floats from 0-1. If numeric values are specified they will be mapped to colors using the cmap and vmin,vmax parameters. See matplotlib.scatter for more details.
- **node_shape** *(string, optional (default='o'))* – The shape of the node. Specification is as matplotlib.scatter marker, one of ‘so^>v<dph8’.
- **alpha** *(float, optional (default=None))* – The node and edge transparency
- **cmap** *(Matplotlib colormap, optional (default=None))* – Colormap for mapping intensities of nodes
- **vmin,vmax** *(float, optional (default=None))* – Minimum and maximum for node colormap scaling
- **linewidths** *(None | scalar | sequence)* – Line width of symbol border (default =1.0)
- **width** *(float, optional (default=1.0))* – Line width of edges
- **edge_color** *(color or array of colors (default='k'))* – Edge color. Can be a single color or a sequence of colors with the same length as edgelist. Color can be string, or rgb (or rgba) tuple of floats from 0-1. If numeric values are specified they will be mapped to colors using the edge_cmap and edge_vmin,edge_vmax parameters.
- **edge_cmap** *(Matplotlib colormap, optional (default=None))* – Colormap for mapping intensities of edges
- **edge_vmin,edge_vmax** *(floats, optional (default=None))* – Minimum and maximum for edge colormap scaling
- **style** *(string, optional (default='solid'))* – Edge line style (solid,dashed,dotted,dashdot)
• **labels** *(dictionary, optional (default=None)) –* Node labels in a dictionary keyed by node of text labels

• **font_size** *(int, optional (default=12)) –* Font size for text labels

• **font_color** *(string, optional (default='k' black)) –* Font color string

• **font_weight** *(string, optional (default='normal')) –* Font weight

• **font_family** *(string, optional (default='sans-serif')) –* Font family

• **label** *(string, optional) –* Label for graph legend

**Notes**

For directed graphs, arrows are drawn at the head end. Arrows can be turned off with keyword arrows=False.

**Examples**

```python
>>> G = nx.dodecahedral_graph()
>>> nx.draw(G)
>>> nx.draw(G, pos=nx.spring_layout(G))  # use spring layout

>>> import matplotlib.pyplot as plt
>>> limits = plt.axis('off')  # turn of axis
```

Also see the NetworkX drawing examples at https://networkx.github.io/documentation/latest/auto_examples/index.html

See also:

draw(), draw_networkx_nodes(), draw_networkx_edges(), draw_networkx_labels(), draw_networkx_edge_labels()

10.1.4 networkx.drawing.nx_pylab.draw_networkx_nodes

draw_networkx_nodes *(G, pos, nodelist=None, node_size=300, node_color='#1f78b4', node_shape='o', alpha=None, cmap=None, vmin=None, vmax=None, ax=None, linewidths=None, edgecolors=None, label=None, **kwds)*

Draw the nodes of the graph G.

This draws only the nodes of the graph G.

**Parameters**

• **G (graph)** – A networkx graph

• **pos (dictionary)** – A dictionary with nodes as keys and positions as values. Positions should be sequences of length 2.

• **ax (Matplotlib Axes object, optional)** – Draw the graph in the specified Matplotlib axes.

• **nodelist (list, optional)** – Draw only specified nodes (default G.nodes())

• **node_size (scalar or array)** – Size of nodes (default=300). If an array is specified it must be the same length as nodelist.
• **node_color** *(color or array of colors (default=='#1f78b4'))* – Node color. Can be a single color or a sequence of colors with the same length as nodelist. Color can be string, or rgb (or rgba) tuple of floats from 0-1. If numeric values are specified they will be mapped to colors using the cmap and vmin,vmax parameters. See matplotlib.scatter for more details.

• **node_shape** *(string)* – The shape of the node. Specification is as matplotlib.scatter marker, one of ‘so^>v<dph8’ (default=’o’).

• **alpha** *(float or array of floats)* – The node transparency. This can be a single alpha value (default=None), in which case it will be applied to all the nodes of color. Otherwise, if it is an array, the elements of alpha will be applied to the colors in order (cycling through alpha multiple times if necessary).

• **cmap** *(Matplotlib colormap)* – Colormap for mapping intensities of nodes (default=None)

• **vmin,vmax** *(floats)* – Minimum and maximum for node colormap scaling (default=None)

• **linewidths** *(None | scalar | sequence)* – Line width of symbol border (default =1.0)

• **edgecolors** *(None | scalar | sequence)* – Colors of node borders (default = node_color)

• **label** *(None| string)* – Label for legend

• **min_source_margin** *(int, optional (default=0))* – The minimum margin (gap) at the beginning of the edge at the source.

• **min_target_margin** *(int, optional (default=0))* – The minimum margin (gap) at the end of the edge at the target.

**Returns** *PathCollection* of the nodes.

**Return type** *matplotlib.collections.PathCollection*

**Examples**

```python
>>> G = nx.dodecahedral_graph()
>>> nodes = nx.draw_networkx_nodes(G, pos=nx.spring_layout(G))
```

Also see the NetworkX drawing examples at https://networkx.github.io/documentation/latest/auto_examples/index.html

**See also:**

draw(), draw_networkx(), draw_networkx_edges(), draw_networkx_labels(), draw_networkx_edge_labels()
- **pos** (*dictionary*) – A dictionary with nodes as keys and positions as values. Positions should be sequences of length 2.

- **edgelist** (*collection of edge tuples*) – Draw only specified edges (default=G.edges())

- **width** (*float, or array of floats*) – Line width of edges (default=1.0)

- **edge_color** (*color or array of colors (default='k')*) – Edge color. Can be a single color or a sequence of colors with the same length as edgelist. Color can be string, or rgb (or rgba) tuple of floats from 0-1. If numeric values are specified they will be mapped to colors using the edge_cmap and edge_vmin,edge_vmax parameters.

- **style** (*string*) – Edge line style (default=’solid’) (solid|dashed|dotted,dashdot)

- **alpha** (*float*) – The edge transparency (default=None)

- **edge_cmap** (*Matplotlib colormap*) – Colormap for mapping intensities of edges (default=None)

- **edge_vmin,edge_vmax** (*floats*) – Minimum and maximum for edge colormap scaling (default=None)

- **ax** (*Matplotlib Axes object, optional*) – Draw the graph in the specified Matplotlib axes.

- **arrows** (*bool, optional (default=True)*) – For directed graphs, if True draw arrowheads. Note: Arrows will be the same color as edges.

- **arrowstyle** (*str, optional (default='->'))* – For directed graphs, choose the style of the arrow heads. See :py:class: matplotlib.patches.ArrowStyle for more options.

- **arrowsize** (*int, optional (default=10)*) – For directed graphs, choose the size of the arrow head’s length and width. See :py:class: matplotlib.patches.FancyArrowPatch for attribute mutation_scale for more info.

- **connectionstyle** (*str, optional (default=None)*) – Pass the connectionstyle parameter to create curved arc of rounding radius rad. For example, connectionstyle=’arc3,rad=0.2’. See :py:class: matplotlib.patches.ConnectionStyle and :py:class: matplotlib.patches.FancyArrowPatch for more info.

- **label** (*[None| string]*) – Label for legend

- **min_source_margin** (*int, optional (default=0)*) – The minimum margin (gap) at the beginning of the edge at the source.

- **min_target_margin** (*int, optional (default=0)*) – The minimum margin (gap) at the end of the edge at the target.

**Returns**

- **matplotlib.collection.LineCollection** – LineCollection of the edges

- **list of matplotlib.patches.FancyArrowPatch** – FancyArrowPatch instances of the directed edges

- Depending whether the drawing includes arrows or not.

**Notes**

For directed graphs, arrows are drawn at the head end. Arrows can be turned off with keyword arrows=False. Be sure to include node_size as a keyword argument; arrows are drawn considering the size of nodes.
Examples

```python
>>> G = nx.dodecahedral_graph()
>>> edges = nx.draw_networkx_edges(G, pos=nx.spring_layout(G))

>>> G = nx.DiGraph()
>>> G.add_edges_from([(1, 2), (1, 3), (2, 3)])
>>> arcs = nx.draw_networkx_edges(G, pos=nx.spring_layout(G))
>>> alphas = [0.3, 0.4, 0.5]
>>> for i, arc in enumerate(arcs):
# change alpha values of arcs
...    arc.set_alpha(alphas[i])
```

Also see the NetworkX drawing examples at https://networkx.github.io/documentation/latest/auto_examples/index.html

See also:

draw(), draw_networkx(), draw_networkx_nodes(), draw_networkx_labels(),
draw_networkx_edge_labels()

10.1.6 networkx.drawing.nx_pylab.draw_networkx_labels

draw_networkx_labels(G, pos, labels=None, font_size=12, font_color='k', font_family='sans-serif', font_weight='normal', alpha=None, bbox=None, ax=None, **kwds)

Draw node labels on the graph G.

Parameters

- **G** *(graph) – A networkx graph*
- **pos** *(dictionary) – A dictionary with nodes as keys and positions as values. Positions should be sequences of length 2.*
- **labels** *(dictionary, optional (default=None)) – Node labels in a dictionary keyed by node of text labels Node-keys in labels should appear as keys in pos. If needed use: {n:lab for n,lab in labels.items() if n in pos}*
- **font_size** *(int) – Font size for text labels (default=12)*
- **font_color** *(string) – Font color string (default='k' black)*
- **font_family** *(string) – Font family (default='sans-serif')*
- **font_weight** *(string) – Font weight (default='normal')*
- **alpha** *(float or None) – The text transparency (default=None)*
- **ax** *(Matplotlib Axes object, optional) – Draw the graph in the specified Matplotlib axes.*

Returns **dict** of labels keyed on the nodes

Return type **dict**

Examples

```python
>>> G = nx.dodecahedral_graph()
>>> labels = nx.draw_networkx_labels(G, pos=nx.spring_layout(G))
```
10.1.7 networkx.drawing.nx_pylab.draw_networkx_edge_labels

draw_networkx_edge_labels(G, pos, edge_labels=None, label_pos=0.5, font_size=10, 
font_color='k', font_family='sans-serif', font_weight='normal', 
alpha=None, bbox=None, ax=None, rotate=True, **kwds)

Draw edge labels.

Parameters

- **G (graph)** – A networkx graph
- **pos (dictionary)** – A dictionary with nodes as keys and positions as values. Positions should be sequences of length 2.
- **ax (Matplotlib Axes object, optional)** – Draw the graph in the specified Matplotlib axes.
- **alpha (float or None)** – The text transparency (default=None)
- **edge_labels (dictionary)** – Edge labels in a dictionary keyed by edge two-tuple of text labels (default=None). Only labels for the keys in the dictionary are drawn.
- **label_pos (float)** – Position of edge label along edge (0=head, 0.5=center, 1=tail)
- **font_size (int)** – Font size for text labels (default=12)
- **font_color (string)** – Font color string (default='k’ black)
- **font_weight (string)** – Font weight (default='normal’)
- **font_family (string)** – Font family (default='sans-serif’)
- **bbox (Matplotlib bbox)** – Specify text box shape and colors.
- **clip_on (bool)** – Turn on clipping at axis boundaries (default=True)

Returns dict of labels keyed on the edges

Return type dict

Examples

```python
g = nx.dodecahedral_graph()
edge_labels = nx.draw_networkx_edge_labels(g, pos=nx.spring_layout(G))
```

Also see the NetworkX drawing examples at https://networkx.github.io/documentation/latest/auto_examples/index.html

See also:

draw(), draw_networkx(), draw_networkx_nodes(), draw_networkx_edges(), draw_networkx_labels()
10.1.8 networkx.drawing.nx_pylab.draw_circular

draw_circular(G, **kwargs)
Draw the graph G with a circular layout.

Parameters
• G (graph) – A networkx graph
• kwargs (optional keywords) – See networkx.draw_networkx() for a description of optional keywords, with the exception of the pos parameter which is not used by this function.

10.1.9 networkx.drawing.nx_pylab.draw_kamada_kawai

draw_kamada_kawai(G, **kwargs)
Draw the graph G with a Kamada-Kawai force-directed layout.

Parameters
• G (graph) – A networkx graph
• kwargs (optional keywords) – See networkx.draw_networkx() for a description of optional keywords, with the exception of the pos parameter which is not used by this function.

10.1.10 networkx.drawing.nx_pylab.draw_planar

draw_planar(G, **kwargs)
Draw a planar networkx graph with planar layout.

Parameters
• G (graph) – A planar networkx graph
• kwargs (optional keywords) – See networkx.draw_networkx() for a description of optional keywords, with the exception of the pos parameter which is not used by this function.

10.1.11 networkx.drawing.nx_pylab.draw_random

draw_random(G, **kwargs)
Draw the graph G with a random layout.

Parameters
• G (graph) – A networkx graph
• kwargs (optional keywords) – See networkx.draw_networkx() for a description of optional keywords, with the exception of the pos parameter which is not used by this function.

10.1.12 networkx.drawing.nx_pylab.draw_spectral

draw_spectral(G, **kwargs)
Draw the graph G with a spectral 2D layout.

Using the unnormalized Laplacian, the layout shows possible clusters of nodes which are an approximation of the ratio cut. The positions are the entries of the second and third eigenvectors corresponding to the ascending eigenvalues starting from the second one.
Parameters

- **G (graph)** – A networkx graph
- **kwargs (optional keywords)** – See networkx.draw_networkx() for a description of optional keywords, with the exception of the pos parameter which is not used by this function.

### 10.1.13 networkx.drawing.nx_pylab.draw_spring

**draw_spring** *(G, **kwargs)*

Draw the graph G with a spring layout.

Parameters

- **G (graph)** – A networkx graph
- **kwargs (optional keywords)** – See networkx.draw_networkx() for a description of optional keywords, with the exception of the pos parameter which is not used by this function.

### 10.1.14 networkx.drawing.nx_pylab.draw_shell

**draw_shell** *(G, **kwargs)*

Draw networkx graph with shell layout.

Parameters

- **G (graph)** – A networkx graph
- **kwargs (optional keywords)** – See networkx.draw_networkx() for a description of optional keywords, with the exception of the pos parameter which is not used by this function.

### 10.2 Graphviz AGraph (dot)

#### 10.2.1 Graphviz AGraph

Interface to pygraphviz AGraph class.

**Examples**

```python
>>> G = nx.complete_graph(5)
>>> A = nx.nx_agraph.to_agraph(G)
>>> H = nx.nx_agraph.from_agraph(A)
```

See also:

Pygraphviz [http://pygraphviz.github.io/](http://pygraphviz.github.io/)

- **from_agraph** *(A[, create_using])* – Returns a NetworkX Graph or DiGraph from a PyGraphviz graph.
- **to_agraph** *(N)* – Returns a pygraphviz graph from a NetworkX graph N.
- **write_dot** *(G, path)* – Write NetworkX graph G to Graphviz dot format on path.
- **read_dot** *(path)* – Returns a NetworkX graph from a dot file on path.

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<td><code>pygraphviz_layout(G[, prog, root, args])</code></td>
<td>Create node positions for G using Graphviz.</td>
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10.2.2 networkx.drawing.nx_agraph.from_agraph

`from_agraph(A, create_using=None)`

Returns a NetworkX Graph or DiGraph from a PyGraphviz graph.

**Parameters**

- `A` (*PyGraphviz AGraph*) – A graph created with PyGraphviz
- `create_using` (*NetworkX graph constructor, optional (default=nx.Graph)*) – Graph type to create. If graph instance, then cleared before populated.

**Examples**

```python
>>> K5 = nx.complete_graph(5)
>>> A = nx.nx_agraph.to_agraph(K5)
>>> G = nx.nx_agraph.from_agraph(A)
```

**Notes**

The Graph G will have a dictionary `G.graph_attr` containing the default graphviz attributes for graphs, nodes and edges.

Default node attributes will be in the dictionary `G.node_attr` which is keyed by node.

Edge attributes will be returned as edge data in G. With `edge_attr=False` the edge data will be the Graphviz edge weight attribute or the value 1 if no edge weight attribute is found.

10.2.3 networkx.drawing.nx_agraph.to_agraph

`to_agraph(N)`

Returns a pygraphviz graph from a NetworkX graph N.

**Parameters**

- `N` (*NetworkX graph*) – A graph created with NetworkX

**Examples**

```python
>>> K5 = nx.complete_graph(5)
>>> A = nx.nx_agraph.to_agraph(K5)
```

**Notes**

If N has an dict `N.graph_attr` an attempt will be made first to copy properties attached to the graph (see `from_agraph`) and then updated with the calling arguments if any.
10.2.4 networkx.drawing.nx_agraph.write_dot

write_dot (G, path)
Write NetworkX graph G to Graphviz dot format on path.

Parameters
• G (graph) – A networkx graph
• path (filename) – Filename or file handle to write

10.2.5 networkx.drawing.nx_agraph.read_dot

read_dot (path)
Returns a NetworkX graph from a dot file on path.

Parameters path (file or string) – File name or file handle to read.

10.2.6 networkx.drawing.nx_agraph.graphviz_layout

graphviz_layout (G, prog='neato', root=None, args='')
Create node positions for G using Graphviz.

Parameters
• G (NetworkX graph) – A graph created with NetworkX
• prog (string) – Name of Graphviz layout program
• root (string, optional) – Root node for twopi layout
• args (string, optional) – Extra arguments to Graphviz layout program

Returns

Return type Dictionary of x, y, positions keyed by node.

Examples

>>> G = nx.petersen_graph()
>>> pos = nx.nx_agraph.graphviz_layout(G)
>>> pos = nx.nx_agraph.graphviz_layout(G, prog='dot')

Notes

This is a wrapper for pygraphviz_layout.

10.2.7 networkx.drawing.nx_agraph.pygraphviz_layout

pygraphviz_layout (G, prog='neato', root=None, args='')
Create node positions for G using Graphviz.

Parameters
• G (NetworkX graph) – A graph created with NetworkX
• **prog** (*string*) – Name of Graphviz layout program
• **root** (*string, optional*) – Root node for twopi layout
• **args** (*string, optional*) – Extra arguments to Graphviz layout program
• **Returns** (*dictionary*) – Dictionary of x, y, positions keyed by node.

**Examples**

```python
>>> G = nx.petersen_graph()
>>> pos = nx.nx_agraph.graphviz_layout(G)
>>> pos = nx.nx_agraph.graphviz_layout(G, prog='dot')
```

**Notes**

If you use complex node objects, they may have the same string representation and GraphViz could treat them as the same node. The layout may assign both nodes a single location. See Issue #1568 If this occurs in your case, consider relabeling the nodes just for the layout computation using something similar to:

```python
H = nx.convert_node_labels_to_integers(G, label_attribute='node_label')
H_layout = nx.nx_agraph.pygraphviz_layout(G, prog='dot')
G_layout = {H.nodes[n]['node_label']: p for n, p in H_layout.items()}
```

### 10.3 Graphviz with pydot

#### 10.3.1 Pydot

Import and export NetworkX graphs in Graphviz dot format using pydot. Either this module or nx_agraph can be used to interface with graphviz.

**See also:**
- [pydot](https://github.com/erocarrera/pydot)
- [Graphviz](https://www.graphviz.org)
- [DOT](https://www.graphviz.org)

**Functions**

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<td><code>pydot_layout(G[, prog, root])</code></td>
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10.3.2 networkx.drawing.nx_pydot.from_pydot

from_pydot \( P \)  
Returns a NetworkX graph from a Pydot graph.

**Parameters**  
\( P \) (*Pydot graph*) – A graph created with Pydot

**Returns**  
\( G \) – A MultiGraph or MultiDiGraph.

**Return type**  
NetworkX multigraph

**Examples**

```python
>>> K5 = nx.complete_graph(5)
>>> A = nx.nx_pydot.to_pydot(K5)
>>> G = nx.nx_pydot.from_pydot(A)  # return MultiGraph

# make a Graph instead of MultiGraph >>> G = nx.Graph(nx.nx_pydot.from_pydot(A))
```

10.3.3 networkx.drawing.nx_pydot.to_pydot

to_pydot \( N \)  
Returns a pydot graph from a NetworkX graph \( N \).

**Parameters**  
\( N \) (*NetworkX graph*) – A graph created with NetworkX

**Examples**

```python
>>> K5 = nx.complete_graph(5)
>>> P = nx.nx_pydot.to_pydot(K5)
```

**Notes**

10.3.4 networkx.drawing.nx_pydot.write_dot

write_dot \( G, path \)  
Write NetworkX graph \( G \) to Graphviz dot format on path.  
Path can be a string or a file handle.

10.3.5 networkx.drawing.nx_pydot.read_dot

read_dot \( path \)  
Returns a NetworkX MultiGraph or MultiDiGraph from the dot file with the passed path.  
If this file contains multiple graphs, only the first such graph is returned. All graphs _except_ the first are silently ignored.

**Parameters**  
\( path \) (*str or file*) – Filename or file handle.

**Returns**  
\( G \) – A MultiGraph or MultiDiGraph.

**Return type**  
*MultiGraph* or *MultiDiGraph*
Notes

Use \( G = \text{nx.Graph(read_dot(path))} \) to return a Graph instead of a MultiGraph.

10.3.6 networkx.drawing.nx_pydot.graphviz_layout

digraphviz_layout \((G, \text{prog='neato', root=None})\)

Create node positions using Pydot and Graphviz.

Returns a dictionary of positions keyed by node.

Parameters

- \( G \) (NetworkX Graph) – The graph for which the layout is computed.
- \( \text{prog} \) (string (default: ‘neato’)) – The name of the GraphViz program to use for layout.
  Options depend on GraphViz version but may include: ‘dot’, ‘twopi’, ‘fdp’, ‘sfdp’, ‘circo’
- \( \text{root} \) (Node from G or None (default: None)) – The node of G from which to start some layout algorithms.

Returns

Return type Dictionary of (x, y) positions keyed by node.

Examples

```python
>>> G = nx.complete_graph(4)
>>> pos = nx.nx_pydot.graphviz_layout(G)
>>> pos = nx.nx_pydot.graphviz_layout(G, prog='dot')
```

Notes

This is a wrapper for pydot_layout.

10.3.7 networkx.drawing.nx_pydot.pydot_layout

pydot_layout \((G, \text{prog='neato', root=None})\)

Create node positions using pydot and Graphviz.

Parameters

- \( G \) (Graph) – NetworkX graph to be laid out.
- \( \text{prog} \) (string (default: ‘neato’)) – Name of the GraphViz command to use for layout.
  Options depend on GraphViz version but may include: ‘dot’, ‘twopi’, ‘fdp’, ‘sfdp’, ‘circo’
- \( \text{root} \) (Node from G or None (default: None)) – The node of G from which to start some layout algorithms.

Returns

Return type dict
Examples

```python
>>> G = nx.complete_graph(4)
>>> pos = nx.nx_pydot.pydot_layout(G)
>>> pos = nx.nx_pydot.pydot_layout(G, prog='dot')
```

Notes

If you use complex node objects, they may have the same string representation and GraphViz could treat them as the same node. The layout may assign both nodes a single location. See Issue #1568 If this occurs in your case, consider relabeling the nodes just for the layout computation using something similar to:

```python
H = nx.convert_node_labels_to_integers(G, label_attribute='node_label')
H_layout = nx.nx_pydot.pydot_layout(G, prog='dot')
G_layout = {H.nodes[n]['node_label']: p for n, p in H_layout.items()}
```

10.4 Graph Layout

10.4.1 Layout

Node positioning algorithms for graph drawing.

For `random_layout()` the possible resulting shape is a square of side [0, scale] (default: [0, 1]) Changing center shifts the layout by that amount.

For the other layout routines, the extent is [center - scale, center + scale] (default: [-1, 1]).

Warning: Most layout routines have only been tested in 2-dimensions.

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10.4.2 `networkx.drawing.layout.bipartite_layout`

`bipartite_layout(G, nodes[, align, scale, ...])` Position nodes in two straight lines.

Parameters
• **G** *(NetworkX graph or list of nodes)* – A position will be assigned to every node in G.

• **nodes** *(list or container)* – Nodes in one node set of the bipartite graph. This set will be placed on left or top.

• **align** *(string (default='vertical'))* – The alignment of nodes. Vertical or horizontal.

• **scale** *(number (default: 1))* – Scale factor for positions.

• **center** *(array-like or None)* – Coordinate pair around which to center the layout.

• **aspect_ratio** *(number (default=4/3):)* – The ratio of the width to the height of the layout.

Returns **pos** – A dictionary of positions keyed by node.

Return type **dict**

**Examples**

```python
>>> G = nx.bipartite.gnmk_random_graph(3, 5, 10, seed=123)
>>> top = nx.bipartite.sets(G)[0]
>>> pos = nx.bipartite_layout(G, top)
```

**Notes**

This algorithm currently only works in two dimensions and does not try to minimize edge crossings.

### 10.4.3 networkx.drawing.layout.circular_layout

circular_layout *(G, scale=1, center=None, dim=2)*

Position nodes on a circle.

**Parameters**

• **G** *(NetworkX graph or list of nodes)* – A position will be assigned to every node in G.

• **scale** *(number (default: 1))* – Scale factor for positions.

• **center** *(array-like or None)* – Coordinate pair around which to center the layout.

• **dim** *(int)* – Dimension of layout. If dim>2, the remaining dimensions are set to zero in the returned positions. If dim<2, a ValueError is raised.

Returns **pos** – A dictionary of positions keyed by node

Return type **dict**

**Raises** **ValueError** – If dim < 2

**Examples**

```python
>>> G = nx.path_graph(4)
>>> pos = nx.circular_layout(G)
```

**Notes**

This algorithm currently only works in two dimensions and does not try to minimize edge crossings.
10.4.4 networkx.drawing.layout.kamada_kawai_layout

kamada_kawai_layout \((G, \text{dist}=\text{None}, \text{pos}=\text{None}, \text{weight}=\text{weight'}, \text{scale}=1, \text{center}=\text{None}, \text{dim}=2)\)

Position nodes using Kamada-Kawai path-length cost-function.

Parameters

- **G** (*NetworkX graph or list of nodes*) – A position will be assigned to every node in G.
- **dist** (*float (default=None]*) – A two-level dictionary of optimal distances between nodes, indexed by source and destination node. If None, the distance is computed using shortest_path_length().
- **pos** (*dict or None optional (default=None]*) – Initial positions for nodes as a dictionary with node as keys and values as a coordinate list or tuple. If None, then use circular_layout() for dim >= 2 and a linear layout for dim == 1.
- **weight** (*string or None optional (default='weight')*) – The edge attribute that holds the numerical value used for the edge weight. If None, then all edge weights are 1.
- **scale** (*number (default: 1)*) – Scale factor for positions.
- **center** (*array-like or None*) – Coordinate pair around which to center the layout.
- **dim** (*int*) – Dimension of layout.

Returns **pos** – A dictionary of positions keyed by node

Return type **dict**

Examples

```python
>>> G = nx.path_graph(4)
>>> pos = nx.kamada_kawai_layout(G)
```

10.4.5 networkx.drawing.layout.planar_layout

planar_layout \((G, \text{scale}=1, \text{center}=\text{None}, \text{dim}=2)\)

Position nodes without edge intersections.

Parameters **G** (*NetworkX graph or list of nodes*) – A position will be assigned to every node in G. If G is of type PlanarEmbedding, the positions are selected accordingly.

Parameters

- **G** (*NetworkX graph or list of nodes*) – A position will be assigned to every node in G. If G is of type nx.PlanarEmbedding, the positions are selected accordingly.
- **scale** (*number (default: 1)*) – Scale factor for positions.
- **center** (*array-like or None*) – Coordinate pair around which to center the layout.
- **dim** (*int*) – Dimension of layout.

Returns **pos** – A dictionary of positions keyed by node

Return type **dict**

Raises **NetworkXException** – If G is not planar
Examples

```python
>>> G = nx.path_graph(4)
>>> pos = nx.planar_layout(G)
```

10.4.6 networkx.drawing.layout.random_layout

`random_layout(G, center=None, dim=2, seed=None)`

Position nodes uniformly at random in the unit square.

For every node, a position is generated by choosing each of \( \text{dim} \) coordinates uniformly at random on the interval [0.0, 1.0).

NumPy (http://scipy.org) is required for this function.

Parameters

- \( G \) (NetworkX graph or list of nodes) – A position will be assigned to every node in \( G \).
- \( \text{center} \) (array-like or None) – Coordinate pair around which to center the layout.
- \( \text{dim} \) (int) – Dimension of layout.
- \( \text{seed} \) (int, RandomState instance or None optional (default=None)) – Set the random state for deterministic node layouts. If int, \( \text{seed} \) is the seed used by the random number generator, if numpy.random.RandomState instance, \( \text{seed} \) is the random number generator, if None, the random number generator is the RandomState instance used by numpy.random.

Returns \( \text{pos} \) – A dictionary of positions keyed by node

Return type \( \text{dict} \)

Examples

```python
>>> G = nx.lollipop_graph(4, 3)
>>> pos = nx.random_layout(G)
```

10.4.7 networkx.drawing.layout.rescale_layout

`rescale_layout(pos, scale=1)`

Returns scaled position array to (-scale, scale) in all axes.

The function acts on NumPy arrays which hold position information. Each position is one row of the array. The dimension of the space equals the number of columns. Each coordinate in one column.

To rescale, the mean (center) is subtracted from each axis separately. Then all values are scaled so that the largest magnitude value from all axes equals \( \text{scale} \) (thus, the aspect ratio is preserved). The resulting NumPy Array is returned (order of rows unchanged).

Parameters

- \( \text{pos} \) (numpy array) – positions to be scaled. Each row is a position.
- \( \text{scale} \) (number (default: 1)) – The size of the resulting extent in all directions.

Returns \( \text{pos} \) – scaled positions. Each row is a position.

Return type \( \text{numpy array} \)
10.4.8 networkx.drawing.layout.shell_layout

**shell_layout** *(G, nlist=None, scale=1, center=None, dim=2)*

Position nodes in concentric circles.

**Parameters**

- **G** *(NetworkX graph or list of nodes)* – A position will be assigned to every node in G.
- **nlist** *(list of lists)* – List of node lists for each shell.
- **scale** *(number (default: 1))* – Scale factor for positions.
- **center** *(array-like or None)* – Coordinate pair around which to center the layout.
- **dim** *(int)* – Dimension of layout, currently only dim=2 is supported. Other dimension values result in a ValueError.

**Returns**

**pos** – A dictionary of positions keyed by node

**Return type**

*dict*

**Raises**

*ValueError* – If dim != 2

**Examples**

```python
>>> G = nx.path_graph(4)
>>> shells = [[[0], [1, 2, 3]]
>>> pos = nx.shell_layout(G, shells)
```

**Notes**

This algorithm currently only works in two dimensions and does not try to minimize edge crossings.

10.4.9 networkx.drawing.layout.spring_layout

**spring_layout** *(G, k=None, pos=None, fixed=None, iterations=50, threshold=0.0001, weight='weight', scale=1, center=None, dim=2, seed=None)*

Position nodes using Fruchterman-Reingold force-directed algorithm.

The algorithm simulates a force-directed representation of the network treating edges as springs holding nodes close, while treating nodes as repelling objects, sometimes called an anti-gravity force. Simulation continues until the positions are close to an equilibrium.

There are some hard-coded values: minimal distance between nodes (0.01) and “temperature” of 0.1 to ensure nodes don’t fly away. During the simulation, **k** helps determine the distance between nodes, though **scale** and **center** determine the size and place after rescaling occurs at the end of the simulation.

Fixing some nodes doesn’t allow them to move in the simulation. It also turns off the rescaling feature at the simulation’s end. In addition, setting **scale** to **None** turns off rescaling.

**Parameters**

- **G** *(NetworkX graph or list of nodes)* – A position will be assigned to every node in G.
- **k** *(float (default=None))* – Optimal distance between nodes. If None the distance is set to 1/sqrt(n) where n is the number of nodes. Increase this value to move nodes farther apart.
- **pos** *(dict or None optional (default=None)) –* Initial positions for nodes as a dictionary with node as keys and values as a coordinate list or tuple. If None, then use random initial positions.

- **fixed** *(list or None optional (default=None)) –* Nodes to keep fixed at initial position. ValueError raised if `fixed` specified and `pos` not.

- **iterations** *(int optional (default=50)) –* Maximum number of iterations taken

- **threshold** *(float optional (default = 1e-4)) –* Threshold for relative error in node position changes. The iteration stops if the error is below this threshold.

- **weight** *(string or None optional (default=`weight`) –* The edge attribute that holds the numerical value used for the edge weight. If None, then all edge weights are 1.

- **scale** *(number or None (default: 1)) –* Scale factor for positions. Not used unless `fixed` is None. If scale is None, no rescaling is performed.

- **center** *(array-like or None) –* Coordinate pair around which to center the layout. Not used unless `fixed` is None.

- **dim** *(int) –* Dimension of layout.

- **seed** *(int, RandomState instance or None optional (default=None)) –* Set the random state for deterministic node layouts. If int, `seed` is the seed used by the random number generator, if `numpy.random.RandomState` instance, `seed` is the random number generator, if None, the random number generator is the RandomState instance used by `numpy.random`.

Returns **pos** – A dictionary of positions keyed by node

Return type **dict**

Examples

```python
>>> G = nx.path_graph(4)
>>> pos = nx.spring_layout(G)
# The same using longer but equivalent function name >>> pos = nx.fruchterman_reingold_layout(G)
```

10.4.10 networkx.drawing.layout.spectral_layout

**spectral_layout** *(G, weight=`weight`, scale=1, center=None, dim=2)*

Position nodes using the eigenvectors of the graph Laplacian.

Using the unnormalized Laplacian, the layout shows possible clusters of nodes which are an approximation of the ratio cut. If dim is the number of dimensions then the positions are the entries of the dim eigenvectors corresponding to the ascending eigenvalues starting from the second one.

Parameters

- **G** *(NetworkX graph or list of nodes)* – A position will be assigned to every node in G.

- **weight** *(string or None optional (default=`weight`) –* The edge attribute that holds the numerical value used for the edge weight. If None, then all edge weights are 1.

- **scale** *(number (default: 1)) –* Scale factor for positions.

- **center** *(array-like or None) –* Coordinate pair around which to center the layout.

- **dim** *(int) –* Dimension of layout.
Returns pos – A dictionary of positions keyed by node

Return type dict

Examples

```python
>>> G = nx.path_graph(4)
>>> pos = nx.spectral_layout(G)
```

Notes

Directed graphs will be considered as undirected graphs when positioning the nodes. For larger graphs (>500 nodes) this will use the SciPy sparse eigenvalue solver (ARPACK).

10.4.11 networkx.drawing.layout.spiral_layout

spiral_layout(G, scale=1, center=None, dim=2, resolution=0.35, equidistant=False)

Position nodes in a spiral layout.

Parameters

- G (NetworkX graph or list of nodes) – A position will be assigned to every node in G.
- scale (number (default: 1)) – Scale factor for positions.
- center (array-like or None) – Coordinate pair around which to center the layout.
- dim (int) – Dimension of layout, currently only dim=2 is supported. Other dimension values result in a ValueError.
- resolution (float) – The compactness of the spiral layout returned. Lower values result in more compressed spiral layouts.
- equidistant (bool) – If True, nodes will be plotted equidistant from each other.

Returns pos – A dictionary of positions keyed by node

Return type dict

Raises ValueError – If dim != 2

Examples

```python
>>> G = nx.path_graph(4)
>>> pos = nx.spiral_layout(G)
```

Notes

This algorithm currently only works in two dimensions.
Random Number Generators (RNGs) are often used when generating, drawing and computing properties or manipulating networks. NetworkX provides functions which use one of two standard RNGs: NumPy’s package `numpy.random` or Python’s built-in package `random`. They each provide the same algorithm for generating numbers (Mersenne Twister). Their interfaces are similar (dangerously similar) and yet distinct. They each provide a global default instance of their generator that is shared by all programs in a single session. For the most part you can use the RNGs as NetworkX has them set up and you’ll get reasonable pseudorandom results (results that are statistically random, but created in a deterministic manner).

Sometimes you want more control over how the numbers are generated. In particular, you need to set the `seed` of the generator to make your results reproducible – either for scientific publication or for debugging. Both RNG packages have easy functions to set the seed to any integer, thus determining the subsequent generated values. Since this package (and many others) use both RNGs you may need to set the `seed` of both RNGs. Even if we strictly only used one of the RNGs, you may find yourself using another package that uses the other. Setting the state of the two global RNGs is as simple setting the seed of each RNG to an arbitrary integer:

```python
>>> import random
>>> random.seed(246)  # or any integer
>>> import numpy
>>> numpy.random.seed(4812)
```

Many users will be satisfied with this level of control.

For people who want even more control, we include an optional argument to functions that use an RNG. This argument is called `seed`, but determines more than the seed of the RNG. It tells the function which RNG package to use, and whether to use a global or local RNG.

```python
>>> from networkx import path_graph, random_layout
>>> G = path_graph(9)
>>> pos = random_layout(G, seed=None)  # use (either) global default RNG
>>> pos = random_layout(G, seed=42)  # local RNG just for this call
>>> pos = random_layout(G, seed=numpy.random)  # use numpy global RNG
>>> random_state = numpy.random.RandomState(42)
>>> pos = random_layout(G, seed=random_state)  # use/reuse your own RNG
```

Each NetworkX function that uses an RNG was written with one RNG package in mind. It either uses `random` or `numpy.random` by default. But some users want to only use a single RNG for all their code. This `seed` argument provides a mechanism so that any function can use a `numpy.random` RNG even if the function is written for `random`. It works as follows.

The default behavior (when `seed=None`) is to use the global RNG for the function’s preferred package. If `seed` is set to an integer value, a local RNG is created with the indicated seed value and is used for the duration of that function (including any calls to other functions) and then discarded. Alternatively, you can specify `seed=numpy.random` to ensure that the global numpy RNG is used whether the function expects it or not. Finally, you can provide a numpy
RNG to be used by the function. The RNG is then available to use in other functions or even other package like
sklearn. In this way you can use a single RNG for all random numbers in your project.

While it is possible to assign `seed` a random-style RNG for NetworkX functions written for the `random` package
API, the numpy RNG interface has too many nice features for us to ensure a random-style RNG will work in all
functions. In practice, you can do most things using only `random` RNGs (useful if numpy is not available). But your
experience will be richer if numpy is available.

To summarize, you can easily ignore the `seed` argument and use the global RNGs. You can specify to use only the
numpy global RNG with `seed=numpy.random`. You can use a local RNG by providing an integer `seed` value. And
you can provide your own numpy RNG, reusing it for all functions. It is easier to use numpy RNGs if you want a
single RNG for your computations.
12.1 Exceptions

Base exceptions and errors for NetworkX.

**class NetworkXException**
Base class for exceptions in NetworkX.

**class NetworkXError**
Exception for a serious error in NetworkX

**class NetworkXPointlessConcept**
Raised when a null graph is provided as input to an algorithm that cannot use it.

The null graph is sometimes considered a pointless concept \(^1\), thus the name of the exception.

**References**

**class NetworkXAlgorithmError**
Exception for unexpected termination of algorithms.

**class NetworkXUnfeasible**
Exception raised by algorithms trying to solve a problem instance that has no feasible solution.

**class NetworkXNoPath**
Exception for algorithms that should return a path when running on graphs where such a path does not exist.

**class NetworkXNoCycle**
Exception for algorithms that should return a cycle when running on graphs where such a cycle does not exist.

**class NodeNotFound**
Exception raised if requested node is not present in the graph

**class HasACycle**
Raised if a graph has a cycle when an algorithm expects that it will have no cycles.

**class NetworkXUnbounded**
Exception raised by algorithms trying to solve a maximization or a minimization problem instance that is unbounded.

**class NetworkXNotImplemented**
Exception raised by algorithms not implemented for a type of graph.

---

class AmbiguousSolution
    Raised if more than one valid solution exists for an intermediary step of an algorithm.
    In the face of ambiguity, refuse the temptation to guess. This may occur, for example, when trying to determine
    the bipartite node sets in a disconnected bipartite graph when computing bipartite matchings.

class ExceededMaxIterations
    Raised if a loop iterates too many times without breaking.
    This may occur, for example, in an algorithm that computes progressively better approximations to a value but
    exceeds an iteration bound specified by the user.

class PowerIterationFailedConvergence (num_iterations, *args, **kw)
    Raised when the power iteration method fails to converge within a specified iteration limit.
    num_iterations is the number of iterations that have been completed when this exception was raised.
13.1 Helper Functions

Miscellaneous Helpers for NetworkX.

These are not imported into the base networkx namespace but can be accessed, for example, as

```python
>>> import networkx

>>> networkx.utils.is_list_of_ints([1, 2, 3])
True

>>> networkx.utils.is_list_of_ints([1, 2, "spam"])
False
```

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>is_string_like(obj)</code></td>
<td>Check if obj is string.</td>
</tr>
<tr>
<td><code>flatten(obj[, result])</code></td>
<td>Return flattened version of (possibly nested) iterable object.</td>
</tr>
<tr>
<td><code>iterable(obj)</code></td>
<td>Return True if obj is iterable with a well-defined <code>len()</code>.</td>
</tr>
<tr>
<td><code>is_list_of_ints(intlist)</code></td>
<td>Return True if list is a list of ints.</td>
</tr>
<tr>
<td><code>make_list_of_ints(sequence)</code></td>
<td>Return list of ints from sequence of integral numbers.</td>
</tr>
<tr>
<td><code>make_str(x)</code></td>
<td>Returns the string representation of x.</td>
</tr>
<tr>
<td><code>generate_unique_node()</code></td>
<td>Generate a unique node label.</td>
</tr>
<tr>
<td><code>default_opener(filename)</code></td>
<td>Opens <code>filename</code> using system’s default program.</td>
</tr>
<tr>
<td><code>pairwise(iterable[, cyclic])</code></td>
<td><code>s -&gt; (s0, s1), (s1, s2), (s2, s3), ...</code></td>
</tr>
<tr>
<td><code>groups(many_to_one)</code></td>
<td>Converts a many-to-one mapping into a one-to-many mapping.</td>
</tr>
<tr>
<td><code>create_random_state([random_state])</code></td>
<td>Returns a numpy.random.RandomState instance depending on input.</td>
</tr>
</tbody>
</table>

13.1.1 networkx.utils.misc.is_string_like

`is_string_like(obj)`
Check if obj is string.

13.1.2 networkx.utils.misc.flatten

`flatten(obj, result=None)`
Return flattened version of (possibly nested) iterable object.
13.1.3 networkx.utils.misc.iterable

iterable(obj)
Return True if obj is iterable with a well-defined len().

13.1.4 networkx.utils.misc.is_list_of_ints

is_list_of_ints(intlist)
Return True if list is a list of ints.

13.1.5 networkx.utils.misc.make_list_of_ints

make_list_of_ints(sequence)
Return list of ints from sequence of integral numbers.
All elements of the sequence must satisfy int(element) == element or a ValueError is raised. Sequence is iterated through once.
If sequence is a list, the non-int values are replaced with ints. So, no new list is created.

13.1.6 networkx.utils.misc.make_str

make_str(x)
Returns the string representation of t.

13.1.7 networkx.utils.misc.generate_unique_node

generate_unique_node()
Generate a unique node label.

13.1.8 networkx.utils.misc.default_opener

default_opener(filename)
Opens filename using system’s default program.
Parameters filename (str) – The path of the file to be opened.

13.1.9 networkx.utils.misc.pairwise

pairwise(iterable, cyclic=False)
s -> (s0, s1), (s1, s2), (s2, s3), . . .

13.1.10 networkx.utils.misc.groups

groups(many_to_one)
Converts a many-to-one mapping into a one-to-many mapping.
many_to_one must be a dictionary whose keys and values are all hashable.
The return value is a dictionary mapping values from `many_to_one` to sets of keys from `many_to_one` that have that value.

For example:

```python
>>> from networkx.utils import groups
>>> many_to_one = {'a': 1, 'b': 1, 'c': 2, 'd': 3, 'e': 3}
>>> groups(many_to_one)
{1: {'a', 'b'}, 2: {'c'}, 3: {'d', 'e'}}
```

### 13.1.11 networkx.utils.misc.create_random_state

#### create_random_state(random_state=None)

Returns a `numpy.random.RandomState` instance depending on input.

**Parameters**

- `random_state` *(int or RandomState instance or None optional (default=None))*  
  If int, return a `numpy.random.RandomState` instance set with seed=int. If `numpy.random.RandomState` instance, return it. If None or `numpy.random`, return the global random number generator used by `numpy.random`.

### 13.2 Data Structures and Algorithms

Union-find data structure.

**UnionFind.union(*objects)**  
Find the sets containing the objects and merge them all.

### 13.2.1 networkx.utils.union_find.UnionFind.union

**UnionFind.union(*objects)**  
Find the sets containing the objects and merge them all.

### 13.3 Random Sequence Generators

Utilities for generating random numbers, random sequences, and random selections.

**powerlaw_sequence(n[, exponent, seed])**  
Return sample sequence of length n from a power law distribution.

**cumulative_distribution(distribution)**  
Returns normalized cumulative distribution from discrete distribution.

**discrete_sequence(n[, distribution, ...])**  
Return sample sequence of length n from a given discrete distribution or discrete cumulative distribution.

**zipf_rv(alpha[, xmin, seed])**  
Returns a random value chosen from the Zipf distribution.

**random_weighted_sample(mapping, k[, seed])**  
Returns k items without replacement from a weighted sample.

**weighted_choice(mapping[, seed])**  
Returns a single element from a weighted sample.
13.3.1 networkx.utils.random_sequence.powerlaw_sequence

powerlaw_sequence(n, exponent=2.0, seed=None)

Return sample sequence of length n from a power law distribution.

13.3.2 networkx.utils.random_sequence.cumulative_distribution

cumulative_distribution(distribution)

Returns normalized cumulative distribution from discrete distribution.

13.3.3 networkx.utils.random_sequence.discrete_sequence

discrete_sequence(n, distribution=None, cdistribution=None, seed=None)

Return sample sequence of length n from a given discrete distribution or discrete cumulative distribution.

One of the following must be specified.

distribution = histogram of values, will be normalized

cdistribution = normalized discrete cumulative distribution

13.3.4 networkx.utils.random_sequence.zipf_rv

zipf_rv(alpha, xmin=1, seed=None)

Returns a random value chosen from the Zipf distribution.

The return value is an integer drawn from the probability distribution

\[ p(x) = \frac{x^{-\alpha}}{\zeta(\alpha, x_{\min})} \]

where \( \zeta(\alpha, x_{\min}) \) is the Hurwitz zeta function.

Parameters

- alpha (float) – Exponent value of the distribution
- xmin (int) – Minimum value
- seed (integer, random_state, or None (default)) – Indicator of random number generation state. See Randomness.

Returns x – Random value from Zipf distribution

Return type int

Raises ValueError: – If xmin < 1 or If alpha <= 1

Notes

The rejection algorithm generates random values for a the power-law distribution in uniformly bounded expected time dependent on parameters. See\(^1\) for details on its operation.

\(^1\) Luc Devroye, Non-Uniform Random Variate Generation, Springer-Verlag, New York, 1986.
Examples

```python
>>> nx.utils.zipf_rv(alpha=2, xmin=3, seed=42)
8
```

References

13.3.5 networkx.utils.random_sequence.random_weighted_sample

`random_weighted_sample(mapping, k, seed=None)`

Returns k items without replacement from a weighted sample.

The input is a dictionary of items with weights as values.

13.3.6 networkx.utils.random_sequence.weighted_choice

`weighted_choice(mapping, seed=None)`

Returns a single element from a weighted sample.

The input is a dictionary of items with weights as values.

13.4 Decorators

<table>
<thead>
<tr>
<th>Decorator</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td><code>open_file(path_arg[, mode])</code></td>
<td>Decorator to ensure clean opening and closing of files.</td>
</tr>
<tr>
<td><code>not_implemented_for(*graph_types)</code></td>
<td>Decorator to mark algorithms as not implemented</td>
</tr>
<tr>
<td><code>nodes_or_number(which_args)</code></td>
<td>Decorator to allow number of nodes or container of nodes.</td>
</tr>
<tr>
<td><code>preserve_random_state(func)</code></td>
<td>Decorator to preserve the numpy.random state during a function.</td>
</tr>
<tr>
<td><code>random_state(random_state_index)</code></td>
<td>Decorator to generate a numpy.random.RandomState instance.</td>
</tr>
</tbody>
</table>

13.4.1 networkx.utils.decorators.open_file

`open_file(path_arg, mode='r')`

Decorator to ensure clean opening and closing of files.

**Parameters**

- `path_arg (int)` – Location of the path argument in args. Even if the argument is a named positional argument (with a default value), you must specify its index as a positional argument.

- `mode (str)` – String for opening mode.

**Returns**

- `_open_file` – Function which cleanly executes the io.

**Return type**

`function`
Examples

Decorate functions like this:

```python
@open_file(0,'r')
def read_function(pathname):
    pass

@open_file(1,'w')
def write_function(G,pathname):
    pass

@open_file(1,'w')
def write_function(G, pathname='graph.dot')
    pass

@open_file('path', 'w+')
def another_function(arg, **kwargs):
    path = kwargs['path']
    pass
```

13.4.2 networkx.utils.decorators.not_implemented_for

`not_implemented_for(*graph_types)`

Decorator to mark algorithms as not implemented

Parameters `graph_types` *(container of strings)* – Entries must be one of ‘directed’, ‘undirected’, ‘multigraph’, ‘graph’.

Returns `_require` – The decorated function.

Return type `function`

Raises

* `NetworkXNotImplemented` –
* If any of the packages cannot be imported –

Notes

Multiple types are joined logically with “and”. For “or” use multiple `@not_implemented_for()` lines.

Examples

Decorate functions like this:

```python
@not_implemented_for('directed')
def sp_function(G):
    pass

@not_implemented_for('directed','multigraph')
def sp_np_function(G):
    pass
```
13.4.3 networkx.utils.decorators.nodes_or_number

`nodes_or_number`*(which_args)*
Decorator to allow number of nodes or container of nodes.

**Parameters** `which_args` *(int or sequence of ints)* – Location of the node arguments in args. Even if the argument is a named positional argument (with a default value), you must specify its index as a positional argument. If more than one node argument is allowed, can be a list of locations.

**Returns** `_nodes_or_numbers` – Function which replaces int args with ranges.

**Return type** function

**Examples**

Decorate functions like this:

```python
@nodes_or_number(0)
def empty_graph(nodes):
    pass

@nodes_or_number([0,1])
def grid_2d_graph(m1, m2, periodic=False):
    pass

@nodes_or_number(1)
def full_rary_tree(r, n):
    # r is a number. n can be a number of a list of nodes
    pass
```

13.4.4 networkx.utils.decorators.preserve_random_state

`preserve_random_state`*(func)*
Decorator to preserve the numpy.random state during a function.

**Parameters** `func` *(function)* – function around which to preserve the random state.

**Returns** `wrapper` – Function which wraps the input function by saving the state before calling the function and restoring the function afterward.

**Return type** function

**Examples**

Decorate functions like this:

```python
@preserve_random_state
def do_random_stuff(x, y):
    return x + y * numpy.random.random()
```

**Notes**

If numpy.random is not importable, the state is not saved or restored.
13.4.5 `networkx.utils.decorators.random_state`

`random_state(random_state_index)`

Decorator to generate a numpy.random.RandomState instance.

Argument position `random_state_index` is processed by `create_random_state`. The result is a numpy.random.RandomState instance.

**Parameters** `random_state_index` (*int*) – Location of the random_state argument in args that is to be used to generate the numpy.random.RandomState instance. Even if the argument is a named positional argument (with a default value), you must specify its index as a positional argument.

**Returns** `random_state` – Function whose random_state keyword argument is a RandomState instance.

**Return type** function

### Examples

Decorate functions like this:

```python
@np_random_state(0)
def random_float(random_state=None):
    return random_state.rand()

@np_random_state(1)
def random_array(dims, random_state=1):
    return random_state.rand(*dims)
```

See also:

`py_random_state()`

13.5 Cuthill-Mckee Ordering

Cuthill-McKee ordering of graph nodes to produce sparse matrices

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
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<tbody>
<tr>
<td><code>cuthill_mckee_ordering(G[, heuristic])</code></td>
<td>Generate an ordering (permutation) of the graph nodes to make a sparse matrix.</td>
</tr>
<tr>
<td><code>reverse_cuthill_mckee_ordering(G[, heuristic])</code></td>
<td>Generate an ordering (permutation) of the graph nodes to make a sparse matrix.</td>
</tr>
</tbody>
</table>

13.5.1 `networkx.utils.rcm.cuthill_mckee_ordering`

`cuthill_mckee_ordering(G, heuristic=None)`

Generate an ordering (permutation) of the graph nodes to make a sparse matrix.

Uses the Cuthill-McKee heuristic (based on breadth-first search)\(^1\).

**Parameters**

- **G (graph)** – A NetworkX graph

---

• **heuristic** *(function, optional)* – Function to choose starting node for RCM algorithm. If None a node from a pseudo-peripheral pair is used. A user-defined function can be supplied that takes a graph object and returns a single node.

**Returns** nodes – Generator of nodes in Cuthill-McKee ordering.

**Return type** generator

**Examples**

```python
>>> from networkx.utils import cuthill_mckee_ordering
>>> G = nx.path_graph(4)
>>> rcm = list(cuthill_mckee_ordering(G))
>>> A = nx.adjacency_matrix(G, nodelist=rcm)
```

Smallest degree node as heuristic function:

```python
>>> def smallest_degree(G):
...     return min(G, key=G.degree)
>>> rcm = list(cuthill_mckee_ordering(G, heuristic=smallest_degree))
```

**See also:**

reverse_cuthill_mckee_ordering()

**Notes**

The optimal solution the the bandwidth reduction is NP-complete\(^2\).

**References**

13.5.2 networkx.utils.rcm.reverse_cuthill_mckee_ordering

**reverse_cuthill_mckee_ordering** *(G, heuristic=None)*

Generate an ordering (permutation) of the graph nodes to make a sparse matrix. Uses the reverse Cuthill-McKee heuristic (based on breadth-first search)\(^1\).

**Parameters**

• **G** *(graph)* – A NetworkX graph

• **heuristic** *(function, optional)* – Function to choose starting node for RCM algorithm. If None a node from a pseudo-peripheral pair is used. A user-defined function can be supplied that takes a graph object and returns a single node.

**Returns** nodes – Generator of nodes in reverse Cuthill-McKee ordering.

**Return type** generator


13.5. Cuthill-McKee Ordering 727
**Examples**

```python
>>> from networkx.utils import reverse_cuthill_mckee_ordering
>>> G = nx.path_graph(4)
>>> rcm = list(reverse_cuthill_mckee_ordering(G))
>>> A = nx.adjacency_matrix(G, nodelist=rcm)
```

Smallest degree node as heuristic function:

```python
>>> def smallest_degree(G):
...    return min(G, key=G.degree)
>>> rcm = list(reverse_cuthill_mckee_ordering(G, heuristic=smallest_degree))
```

**See also:**

cuthill_mckee_ordering()

**Notes**

The optimal solution of the bandwidth reduction is NP-complete\(^2\).

**References**

13.6 Context Managers

---

**reversed(G)**

A context manager for temporarily reversing a directed graph in place.

### 13.6.1 networkx.utils.contextmanagers.reversed

**reversed(G)**

A context manager for temporarily reversing a directed graph in place.

This is a no-op for undirected graphs.

**Parameters**

\( G (\text{graph}) \) – A NetworkX graph.

---

dictionary  A Python dictionary maps keys to values. Also known as “hashes”, or “associative arrays” in other programming languages. See https://docs.python.org/2/tutorial/datastructures.html#dictionaries

dege  Edges are either two-tuples of nodes \((u, v)\) or three tuples of nodes with an edge attribute dictionary \((u,v,\text{dict})\).

ebunch  An iterable container of edge tuples like a list, iterator, or file.

dge attribute  Edges can have arbitrary Python objects assigned as attributes by using keyword/value pairs when adding an edge assigning to the \(G\text{.edges}[u][v]\) attribute dictionary for the specified edge \(u-v\).

hashable  An object is hashable if it has a hash value which never changes during its lifetime (it needs a \(__hash__()\) method), and can be compared to other objects (it needs an \(__eq__()\) or \(__cmp__()\) method). Hashable objects which compare equal must have the same hash value.

Hashability makes an object usable as a dictionary key and a set member, because these data structures use the hash value internally.

All of Python’s immutable built-in objects are hashable, while no mutable containers (such as lists or dictionaries) are. Objects which are instances of user-defined classes are hashable by default; they all compare unequal, and their hash value is their \(\text{id()}\).

Definition from https://docs.python.org/2/glossary.html

nbunch  An nbunch is a single node, container of nodes or None (representing all nodes). It can be a list, set, graph, etc. To filter an nbunch so that only nodes actually in \(G\) appear, use \(G\text{.nbunch_iter(nbunch)}\).

node  A node can be any hashable Python object except None.

node attribute  Nodes can have arbitrary Python objects assigned as attributes by using keyword/value pairs when adding a node or assigning to the \(G\text{.nodes}[n]\) attribute dictionary for the specified node \(n\).
This guide can help you start working with NetworkX.

**A.1 Creating a graph**

Create an empty graph with no nodes and no edges.

```python
>>> import networkx as nx
>>> G = nx.Graph()
```

By definition, a *Graph* is a collection of nodes (vertices) along with identified pairs of nodes (called edges, links, etc). In NetworkX, nodes can be any hashable object e.g., a text string, an image, an XML object, another Graph, a customized node object, etc.

**Note**: Python’s *None* object should not be used as a node as it determines whether optional function arguments have been assigned in many functions.

**A.2 Nodes**

The graph $G$ can be grown in several ways. NetworkX includes many graph generator functions and facilities to read and write graphs in many formats. To get started though we’ll look at simple manipulations. You can add one node at a time,

```python
>>> G.add_node(1)
```

add a list of nodes,

```python
>>> G.add_nodes_from([2, 3])
```

or add any iterable container of nodes. You can also add nodes along with node attributes if your container yields 2-tuples (node, node_attribute_dict). Node attributes are discussed further below.

```python
>>> H = nx.path_graph(10)
>>> G.add_nodes_from(H)
```

Note that $G$ now contains the nodes of $H$ as nodes of $G$. In contrast, you could use the graph $H$ as a node in $G$.

```python
>>> G.add_node(H)
```
The graph $G$ now contains $H$ as a node. This flexibility is very powerful as it allows graphs of graphs, graphs of files, graphs of functions and much more. It is worth thinking about how to structure your application so that the nodes are useful entities. Of course you can always use a unique identifier in $G$ and have a separate dictionary keyed by identifier to the node information if you prefer.

**Note:** You should not change the node object if the hash depends on its contents.

### A.3 Edges

$G$ can also be grown by adding one edge at a time,

```python
>>> G.add_edge(1, 2)
>>> e = (2, 3)
>>> G.add_edge(*e)  # unpack edge tuple
```

by adding a list of edges,

```python
>>> G.add_edges_from([(1, 2), (1, 3)])
```

or by adding any *ebunch* of edges. An *ebunch* is any iterable container of edge-tuples. An edge-tuple can be a 2-tuple of nodes or a 3-tuple with 2 nodes followed by an edge attribute dictionary, e.g., (2, 3, {'weight': 3.1415}). Edge attributes are discussed further below

```python
>>> G.add_edges_from(H.edges)
```

There are no complaints when adding existing nodes or edges. For example, after removing all nodes and edges,

```python
>>> G.clear()
```

we add new nodes/edges and NetworkX quietly ignores any that are already present.

```python
>>> G.add_edges_from([(1, 2), (1, 3)])
>>> G.add_node(1)
>>> G.add_edge(1, 2)
>>> G.add_node("spam")  # adds node "spam"
>>> G.add_nodes_from("spam")  # adds 4 nodes: 's', 'p', 'a', 'm'
>>> G.add_edge(3, 'm')
```

At this stage the graph $G$ consists of 8 nodes and 3 edges, as can be seen by:

```python
>>> G.number_of_nodes()
8
>>> G.number_of_edges()
3
```

We can examine the nodes and edges. Four basic graph properties facilitate reporting: $G$.nodes, $G$.edges, $G$.adj and $G$.degree. These are set-like views of the nodes, edges, neighbors (adjacencies), and degrees of nodes in a graph. They offer a continually updated read-only view into the graph structure. They are also dict-like in that you can look up node and edge data attributes via the views and iterate with data attributes using methods .items(), .data('span'). If you want a specific container type instead of a view, you can specify one. Here we use lists, though sets, dicts, tuples and other containers may be better in other contexts.
One can specify to report the edges and degree from a subset of all nodes using an `nbunch`. An `nbunch` is any of: None (meaning all nodes), a node, or an iterable container of nodes that is not itself a node in the graph.

```python
>>> G.edges([2, 'm'])
EdgeDataView([(2, 1), ('m', 3)])
>>> G.degree([2, 3])
DegreeView({2: 1, 3: 2})
```

One can remove nodes and edges from the graph in a similar fashion to adding. Use methods `Graph.remove_node()`, `Graph.remove_nodes_from()`, `Graph.remove_edge()` and `Graph.remove_edges_from()`, e.g.

```python
>>> G.remove_node(2)
>>> G.remove_nodes_from("spam")
>>> list(G.nodes)
[1, 3, 'spam']
>>> G.remove_edge(1, 3)
```

When creating a graph structure by instantiating one of the graph classes you can specify data in several formats.

```python
>>> G.add_edge(1, 2)
>>> H = nx.DiGraph(G)  # create a DiGraph using the connections from G
>>> list(H.edges())
[(1, 2), (2, 1)]
>>> edgelist = [(0, 1), (1, 2), (2, 3)]
>>> H = nx.Graph(edgelist)
```

## A.4 What to use as nodes and edges

You might notice that nodes and edges are not specified as NetworkX objects. This leaves you free to use meaningful items as nodes and edges. The most common choices are numbers or strings, but a node can be any hashable object (except `None`), and an edge can be associated with any object `x` using `G.add_edge(n1, n2, object=x)`.

As an example, `n1` and `n2` could be protein objects from the RCSB Protein Data Bank, and `x` could refer to an XML record of publications detailing experimental observations of their interaction.

We have found this power quite useful, but its abuse can lead to unexpected surprises unless one is familiar with Python. If in doubt, consider using `convert_node_labels_to_integers()` to obtain a more traditional graph with integer labels.

## A.5 Accessing edges and neighbors

In addition to the views `Graph.edges()`, and `Graph.adj()`, access to edges and neighbors is possible using subscript notation.
You can get/set the attributes of an edge using subscript notation if the edge already exists.

```python
>>> G.add_edge(1, 3)
>>> G[1][3]['color'] = 'blue'
>>> G.edges[1, 2]['color'] = 'red'
```

Fast examination of all (node, adjacency) pairs is achieved using `G.adjacency()` or `G.adj.items()`. Note that for undirected graphs, adjacency iteration sees each edge twice.

```python
>>> FG = nx.Graph()
>>> FG.add_weighted_edges_from([(1, 2, 0.125), (1, 3, 0.75), (2, 4, 1.2), (3, 4, 0.375)])
>>> for n, nbrs in FG.adj.items():
...     for nbr, eattr in nbrs.items():
...         wt = eattr['weight']
...         if wt < 0.5: print('(%d, %d, %.3f)' % (n, nbr, wt))
(1, 2, 0.125)
(2, 1, 0.125)
(3, 4, 0.375)
(4, 3, 0.375)
```

Convenient access to all edges is achieved with the `edges` property.

```python
>>> for (u, v, wt) in FG.edges.data('weight'):
...     if wt < 0.5: print('(%d, %d, %.3f)' % (u, v, wt))
(1, 2, 0.125)
(3, 4, 0.375)
```

## A.6 Adding attributes to graphs, nodes, and edges

Attributes such as weights, labels, colors, or whatever Python object you like, can be attached to graphs, nodes, or edges.

Each graph, node, and edge can hold key/value attribute pairs in an associated attribute dictionary (the keys must be hashable). By default these are empty, but attributes can be added or changed using `add_edge`, `add_node` or direct manipulation of the attribute dictionaries named `G.graph`, `G.nodes`, and `G.edges` for a graph `G`.

### A.6.1 Graph attributes

Assign graph attributes when creating a new graph

```python
>>> G = nx.Graph(day="Friday")
>>> G.graph
{'day': 'Friday'}
```

Or you can modify attributes later

```python
>>> G.graph['day'] = 'Saturday'
>>> G.graph
{'day': 'Saturday'}
```
A.6.2 Node attributes

Add node attributes using add_node(), add_nodes_from(), or G.nodes

```python
>>> G.add_node(1, time='5pm')
>>> G.add_nodes_from([3], time='2pm')
>>> G.nodes[1]
{'time': '5pm'}
>>> G.nodes[1]['room'] = 714
>>> G.nodes.data()
NodeDataView({1: {'time': '5pm', 'room': 714}, 3: {'time': '2pm'}})
```

Note that adding a node to G.nodes does not add it to the graph, use G.add_node() to add new nodes. Similarly for edges.

A.6.3 Edge Attributes

Add/change edge attributes using add_edge(), add_edges_from(), or subscript notation.

```python
>>> G.add_edge(1, 2, weight=4.7)
>>> G.add_edges_from([(3, 4), (4, 5)], color='red')
>>> G.add_edges_from([(1, 2, {'color': 'blue'}), (2, 3, {'weight': 8})])
>>> G[1][2]['weight'] = 4.7
>>> G.edges[3, 4]['weight'] = 4.2
```

The special attribute weight should be numeric as it is used by algorithms requiring weighted edges.

A.7 Directed graphs

The DiGraph class provides additional properties specific to directed edges, e.g., DiGraph.out_edges(), DiGraph.in_degree(), DiGraph.predecessors(), DiGraph.successors() etc. To allow algorithms to work with both classes easily, the directed versions of neighbors() is equivalent to successors() while degree reports the sum of in_degree and out_degree even though that may feel inconsistent at times.

```python
>>> DG = nx.DiGraph()
>>> DG.add_weighted_edges_from([(1, 2, 0.5), (3, 1, 0.75)])
>>> DG.out_degree(1, weight='weight')
0.5
>>> DG.degree(1, weight='weight')
1.25
>>> list(DG.successors(1))
[2]
>>> list(DG.neighbors(1))
[2]
```

Some algorithms work only for directed graphs and others are not well defined for directed graphs. Indeed the tendency to lump directed and undirected graphs together is dangerous. If you want to treat a directed graph as undirected for some measurement you should probably convert it using Graph.to_undirected() or with

A.7. Directed graphs
A.8 Multigraphs

NetworkX provides classes for graphs which allow multiple edges between any pair of nodes. The `MultiGraph` and `MultiDiGraph` classes allow you to add the same edge twice, possibly with different edge data. This can be powerful for some applications, but many algorithms are not well defined on such graphs. Where results are well defined, e.g., `MultiGraph.degree()` we provide the function. Otherwise you should convert to a standard graph in a way that makes the measurement well defined.

```python
>>> MG = nx.MultiGraph()
>>> MG.add_weighted_edges_from([(1, 2, 0.5), (1, 2, 0.75), (2, 3, 0.5)])
>>> dict(MG.degree(weight='weight'))
{1: 1.25, 2: 1.75, 3: 0.5}
>>> GG = nx.Graph()
>>> for n, nbrs in MG.adjacency():
...     for nbr, edict in nbrs.items():
...         minvalue = min([d['weight'] for d in edict.values()])
...         GG.add_edge(n, nbr, weight = minvalue)
... >>> nx.shortest_path(GG, 1, 3)
[1, 2, 3]
```

A.9 Graph generators and graph operations

In addition to constructing graphs node-by-node or edge-by-edge, they can also be generated by

1. Applying classic graph operations, such as:

```
subgraph(G, nbunch) - induced subgraph view of G on nodes in nbunch
union(G1,G2) - graph union
disjoint_union(G1,G2) - graph union assuming all nodes are different
cartesian_product(G1,G2) - return Cartesian product graph
compose(G1,G2) - combine graphs identifying nodes common to both
complement(G) - graph complement
create_empty_copy(G) - return an empty copy of the same graph
to_undirected(G) - return an undirected representation of G
to_directed(G) - return a directed representation of G
```

2. Using a call to one of the classic small graphs, e.g.,

```python
>>> petersen = nx.petersen_graph()
>>> tutte = nx.tutte_graph()
>>> maze = nx.sedgewick_maze_graph()
>>> tet = nx.tetrahedral_graph()
```

3. Using a (constructive) generator for a classic graph, e.g.,

```python
>>> K_5 = nx.complete_graph(5)
>>> K_3_5 = nx.complete_bipartite_graph(3, 5)
>>> barbell = nx.barbell_graph(10, 10)
>>> lollipop = nx.lollipop_graph(10, 20)
```
4. Using a stochastic graph generator, e.g.,

```python
>>> er = nx.erdos_renyi_graph(100, 0.15)
>>> ws = nx.watts_strogatz_graph(30, 3, 0.1)
>>> ba = nx.barabasi_albert_graph(100, 5)
>>> red = nx.random_lobster(100, 0.9, 0.9)
```

5. Reading a graph stored in a file using common graph formats, such as edge lists, adjacency lists, GML, GraphML, pickle, LEDA and others.

```python
>>> nx.write_gml(red, "path.to.file")
>>> mygraph = nx.read_gml("path.to.file")
```

For details on graph formats see *Reading and writing graphs* and for graph generator functions see *Graph generators*.

### A.10 Analyzing graphs

The structure of \( G \) can be analyzed using various graph-theoretic functions such as:

```python
>>> G = nx.Graph()
>>> G.add_edges_from([(1, 2), (1, 3)])
>>> G.add_node("spam")  # adds node "spam"
>>> list(nx.connected_components(G))
[(1, 2, 3), {'spam'}]
>>> sorted(d for n, d in G.degree())
[0, 1, 1, 2]
>>> nx.clustering(G)
{1: 0, 2: 0, 3: 0, 'spam': 0}
```

Some functions with large output iterate over (node, value) 2-tuples. These are easily stored in a `dict` structure if you desire.

```python
>>> sp = dict(nx.all_pairs_shortest_path(G))
>>> sp[3]
{3: [3], 1: [3, 1], 2: [3, 1, 2]}
```

See *Algorithms* for details on graph algorithms supported.

### A.11 Drawing graphs

NetworkX is not primarily a graph drawing package but basic drawing with Matplotlib as well as an interface to use the open source GraphViz software package are included. These are part of the `networkx.drawing` module and will be imported if possible.

First import Matplotlib’s plot interface (pylab works too)

```python
>>> import matplotlib.pyplot as plt
```

You may find it useful to interactively test code using `ipython -pylab`, which combines the power of ipython and matplotlib and provides a convenient interactive mode.

To test if the import of `networkx.drawing` was successful draw \( G \) using one of

A.10. Analyzing graphs 737
>>> G = nx.petersen_graph()
>>> plt.subplot(121)
<matplotlib.axes._subplots.AxesSubplot object at ...>
>>> nx.draw(G, with_labels=True, font_weight='bold')
>>> plt.subplot(122)
<matplotlib.axes._subplots.AxesSubplot object at ...>
>>> nx.draw_shell(G, nlist=[range(5, 10), range(5)], with_labels=True, font_weight='bold')

when drawing to an interactive display. Note that you may need to issue a Matplotlib command if you are not using matplotlib in interactive mode (see Matplotlib FAQ).

>>> plt.show()

>>> options = {
...    'node_color': 'black',
...    'node_size': 100,
...    'width': 3,
...}
>>> plt.subplot(221)
<matplotlib.axes._subplots.AxesSubplot object at ...>
>>> nx.draw_random(G, **options)
>>> plt.subplot(222)
<matplotlib.axes._subplots.AxesSubplot object at ...>

(continues on next page)
You can find additional options via `draw_networkx()` and layouts via `layout`. You can use multiple shells with `draw_shell()`.

```python
>>> G = nx.dodecahedral_graph()
>>> shells = [[2, 3, 4, 5, 6], [8, 1, 0, 19, 18, 17, 16, 15, 14, 7], [9, 10, 11, 12, 13]]
>>> nx.draw_shell(G, nlist=shells, **options)
```
To save drawings to a file, use, for example

```python
>>> nx.draw(G)
>>> plt.savefig("path.png")
```

writes to the file path.png in the local directory. If Graphviz and PyGraphviz or pydot, are available on your system, you can also use `nx_aagraph.graphviz_layout(G)` or `nx_pydot.graphviz_layout(G)` to get the node positions, or write the graph in dot format for further processing.

```python
>>> from networkx.drawing.nx_pydot import write_dot
>>> pos = nx.nx_aagraph.graphviz_layout(G)
>>> nx.draw(G, pos=pos)
>>> write_dot(G, 'file.dot')
```

See *Drawing* for additional details.
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